Control of Harmonics in 6-Pulse Rectifiers

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CONTROL OF HARMONICS IN 6-PULSE RECTIFIERS

A Dissertation

Presented to

the Faculty of the Daniel Felix Ritchie School of Engineering and Computer Science

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Doctor of Philosophy

by

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Abstract

Harmonics are always present in electrical power systems. Harmonic distortion is harmless as long as its level is within the limit. However, with the recent rapid advancement of power electronics, i.e. non linear loads, the use of the variable speed drives are increasing day by day. Harmonics produced by non-linear loads are a potential risk if they are not evaluated, predicted, and controlled.

The power electronic switching devices like thyristor used in the rectifier circuits inject harmonic distortion to the utility grid in different applications. The harmonic distortion causes different problems in the power system. To minimize the unwanted effects of harmonic distortion, IEEE Std 519-1992 recommends the amount of harmonics that is acceptable in the power system. IEEE Std 519-1992 suggests that an individual harmonic distortion to be under 3% and the total harmonic distortion, THD, to be under 5% of the fundamental component.

Harmonic distortion can be mitigated using different methods. Based on the system configuration either active filters, passive filters, or phase shifting methods are used. In medium voltage high power applications, generally, phase shifting method is better suited.

In addition to harmonic distortion in AC side, AC-DC converter produce ripple in DC side. DC ripple can be mitigated by the use of filter circuits. However, when phase
shifting method is used in AC side for harmonic mitigation, a method called pulse multiplication can be used in DC side to mitigate DC ripple. Phase shifting and pulse multiplication methods are investigated in detail in this research.

A three-phase 6-pulse rectifier is modeled in Alternative Transients Program (ATP). Voltage and Current waveforms are obtained and the amount of harmonic distortion produced is calculated. It was found that the harmonic distortion produced by an ideal three-phase 6-pulse rectifier to be 31.1% and not under IEEE 519-1992 recommendation. Therefore a 12-pulse rectifier has been investigated. The analysis shows that a 12-pulse rectifier produces 15.3% THD and provides a window opportunity to be used in certain areas where the grid is comparatively stronger. For a rectifier to be able to be used without ac side filter and dc side filter, and to mitigate the THD under the IEEE std 519-1992 recommendation and the dc ripple under the specified value, a higher pulse rectifier will be needed. Further investigation is needed using 18-pulse and 24-pulse phase shifting rectifiers.
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Chapter One: Introduction

1.1 Background

The medium voltage high-power rectifiers find their use in various industrial plants. Application of 3-phase rectifiers are found for pipeline pumps in petrochemical industry, for steel rolling mills in metal industry, for pumps in water pumping stations, for fans in cement industry, for traction in locomotive industry, and in many other applications [1]-[8].

Fig 1.1 shows a general block diagram of a typical medium voltage high-power drive [9].

![Scope of Research](image)

**Figure 1.1 General block diagram of the MV drive**

The input is 3-phase utility voltage which is converted to dc voltage by the rectifier shown above. The dc voltage magnitude can be fixed or adjustable depending upon the power electronic switches that are used for switching. Multi-pulse silicon controlled rectifiers, SCR, multi-pulse diode rectifiers, or pulse-width-modulated (PWM) rectifiers are commonly used rectifier topologies.
For filtering the ripple from dc output, either a capacitor or an inductor can be used as dc filter. A capacitor can be used for providing a stiff voltage, while an inductor can be used for smoothing the current. Generally, the capacitors are used in voltage source drives while inductors are used in current source drives [9].

There are two types of inverters, voltage source inverter (VSI) and current source inverter (CSI). The voltage source inverters convert the rectified dc voltage to a three-phase ac voltage whereas the current source inverters convert the dc current to three-phase ac current. Generally, diode rectifiers are voltage sources; on the other hand, thyristor rectifiers are current sources. There are various inverter topologies present for the MV drive. Inverter topologies are not the scope of this research work.

1.1.1 State of the art

Multi-pulse converters for ac-dc conversion can mitigate the harmonic distortion on the ac side as well as reduce the ripple on the dc side. On ac side, these converters reduce harmonic current distortion and hence reduce the harmonic voltage distortion. These converters also reduce electro-magnetic interference, radio frequency interference, and reactive power burden at input ac mains. On dc side, these converters produce good quality dc with reduced dc ripple with unidirectional and bidirectional power flow for feeding loads from a few kilowatts to several hundred megawatts. These multi-pulse converters evolved in the last few decades with different configurations. Despite wide variety of configurations, all of these converts meet specific requirement of various applications. These converters maintain a high level of power quality both at input ac mains and at output dc loads.
For unidirectional power flow applications, multi-pulse converters are developed using diode rectifiers. These converters are built using two types of transformer topologies: isolated topology and non-isolated topology. Each of these topologies can have 12-pulse, 18-pulse, 24-pulse, 30-pulse or higher number of pulses. Higher the number of pulses, lower is the total harmonic distortion, THD, on input ac currents and lower is the ripple on output dc currents. Diode, unidirectional ac-dc converters can be classified into full-wave converters and bridge converters based on the number of pulses and isolated and non-isolated circuit between ac input and dc output. The concept of full-wave (mid-point) rectifiers with T-connection, zigzag, double star, and tapped winding in transformers are adopted to meet varying requirements of applications. Similarly, the concept of bridge rectifiers are used with compact autotransformers, pulse multiplication using inter-phase transformers, multiple secondary windings for phase shifting and pulse doubling, and additional devices are adopted to meet varying requirements of applications [1]-[8].

Some applications require bi-directional flow of power flow. These bi-directional converters are developed using thyristor rectifiers. These bi-directional rectifiers are used in few kilowatt dc motor drives to several megawatt high voltage dc, HVDC, transmission systems. To reduce total harmonic distortion, THD, in input ac mains, these bi-directional converters use multi-winding transformers and inter-phase transformers. Likewise, to reduce the ripple in output dc, these bi-directional converters use tapped reactors with the concept of pulse multiplication [1]-[8].

Phase-shifting process, which can be obtained through multiple transformer secondary windings, can be attributed to the major breakthrough in the technology of
multi-pulse converters. By the use of multi-secondary winding transformers, original three-phase ac supply can be converted to multiphase ac supply. This conversion results in a high number of steps in ac mains current and higher number of pulses in dc output. The higher number of steps in ac input currents makes the current close to sinusoidal with reduced and acceptable THD. Similarly, the higher number of pulses in dc output reduces the ripple in dc. Various topologies of transformer connections are in use for achieving the desired phase shift. The phase shift among the output of different secondary windings of a transformer is responsible to cancel, eliminate, and to reduce harmonics in input ac supply. Some of the topologies that are in use for phase shifting are polygon, T-connection, tapped winding, zigzag, plurality of windings of isolated multi-winding transformers and auto transformers. The weight, size, and cost of these transformers are cost prohibitive in many applications and attempts have been made to make these transformers cost effective and increase the acceptability of these converters in multi-pulse applications. The inductance of transformer primary winding works as a line inductor and reduces the total harmonic distortion, THD, in ac input side and thus improves the power quality. Multi-pulse technology eliminates some harmonics and mitigates other harmonics. This technology reduces electro-magnetic interference, radio frequency interference, and switching losses due to low-frequency switching by utilizing line commutation or natural commutation. The line/natural commutation results in high efficiency and low noise levels in the converter systems. Multi-pulse converters are more simple, reliable, and robust than those using pulse width modulations, PMW, technology. PMW technology uses high frequency forced commutation causing more switching losses and high level of noise in converter system [1]-[8].
Three-Phase Multi-pulse AC-DC Converters

Uncontrolled (Diode) Unidirectional

Controlled (Thyristor) Bidirectional

Isolated Full-Wave Bridge

Non-isolated Full-Wave Bridge

Isolated Full-Wave Bridge

Figure 1.2 Classification of three-phase multi-pulse ac-dc converter
Multi-pulse converters can be classified based on direction of power flow, number of switching pulses, and the topology, isolated and non-isolated. Fig. 1.2 shows a tree of such classification of three-phase multi-pulse ac-dc converters.

1.1.1.1 Unidirectional AC-DC Converters

As name suggests unidirectional ac-dc converters are those in which power flows in one direction i.e. from input ac to output dc. These converters are built using transformers, diodes, and other necessary components. Variable frequency ac drives for pumps, fans, compressors, electroplating, telecommunication, and power supply for water treatment plants etc are some of the applications of unidirectional converters. This type of converters can further be divided based on number of diodes used for switching, which also refers as number of pulses. Number of switches i.e. pulses are based on cost, reliability, and power quality requirements. Thus, unidirectional ac-dc converters can further be classified as follows [1]-[8]:

i. 12-Pulse AC-DC Converters

ii. 18-Pulse AC-DC Converters

iii. 24-Pulse AC-DC Converters

iv. Other high-Pulse AC-DC Converters

1.1.1.2 Bidirectional AC-DC Converters

In bidirectional converters power flow can flow from ac to dc or dc to ac. These converters are built using transformers, thyristors and other necessary components. Phase angle control technique is used in thyristor to control the wide varying dc output voltages. In these converters, reduction in THD is obtained by pulse multiplication using various types of transformer connections. Multiple secondary windings of transformer provide a
higher number of pulses in input ac side. Likewise tapped reactors and inter-phase reactors provide pulse multiplications in output dc side. The higher number of pulses reduces the THD in ac side and pulse multiplication reduces ripple in dc side. Some of the applications of bidirectional converters are dc motor drives, synchronous motor drive, and high voltage dc, HVDC, transmission system. Bidirectional converters can further be classified based on power quality, reliability, and overall cost of the system as follows [1]-[8]:

i. 12-Pulse AC-DC Converters

ii. 18-Pulse AC-DC Converters

iii. 24-Pulse AC-DC Converters

iv. 36-Pulse AC-DC Converters

v. 48-Pulse AC-DC Converters

### 1.2 Problem Statement

Solid state ac-dc converters, i.e., rectifiers have the problem of power quality in terms of harmonic distortion and poor power factor in ac side; and rippled output in dc side. Various methods are used to mitigate these problems in rectifiers. Normally filters are employed for controlling the problem in existing installations. However, in some installations, employment of filters becomes cost prohibitive. Therefore, in future installations, it is preferable to modify the converter topology in design stage to achieve required performance in terms of ac harmonics and dc ripple. IEEE standards 519-1992 [10] provides recommended practices and requirements for harmonic control in electrical power system and focuses on the point of common coupling (PCC) with the consumer-
utility interface. The recommendation attempts to reduce the harmonic effects by establishing limits on certain harmonic indices (currents and voltages). For an industrial plant, the point between the non-linear load and other loads is defined as PCC.

1.3 Objective

The objective of this research is to investigate harmonic distortion at ac input and dc ripple at dc output in a 3-phase, 6-pulse rectifier and a 3-phase 12-pulse rectifier and suggest a methodology for the improvement of harmonic distortion and dc ripple.

1.4 Scope of Research Work

A complete medium-voltage drive system is shown in Fig 1.1. This research is focused in AC to DC conversion stage as shown by dotted line. When AC is converted to DC, the converter produces harmonics in the ac system. In addition, the output current contains dc ripple component. This research investigates the available methods for mitigating harmonic distortion; a design procedure for limiting output dc ripple, and design of a negative feedback controller system for thyristor phase control in a 3-phase 6-pulse rectifier. Finally, a 12-pulse rectifier is modeled and the results are compared with those obtained from 6-pulse rectifier. It is recommended that by use of a 12-pulse rectifier in stead of a 6-pulse rectifier, we can improve the THD and dc ripple by a factor of 2; and thus the input ac side filter as well as output dc side filter can be eliminated unlike in a 6-pulse rectifier [11]-[15].

Existing techniques to mitigate the harmonic distortion are as follows:

1. Passive shunt filters [16-19]
2. Active harmonic injection [20-23]

3. Phase shifting [24-28]

Existing technique to mitigate the dc ripple are as follows:

1. Tapped reactor [29]
2. Inter-phase reactor [30]

1.5 Methodology

Computer model for various AC to DC converters are developed using a computer program called Alternative Transients Program, ATP [31]. ATP is used to obtain the voltage and current waveforms. First, the analyses are carried out for single-phase converters. Then, the knowledge of single-phase converters has been applied to the three-phase converters.

1.6 Chapter Organization

Chapter 2 investigates the theoretical detail on various aspects of harmonics and harmonic distortion. Chapter 3 presents the detailed description of Alternative Transients Program, ATP; the program that is used to create computer model of various rectifier. Chapter 4 presents the design work for thyristor phase control principle, DC output ripple filter, 3-phase 6-pulse rectifier, 3-phase 12-pulse rectifier, and a negative feedback integral controller for controlling the firing angle of thyristors. Chapter 5 presents results, conclusion, and future work. References are given at the end. Appendix A includes published papers and Appendix B includes ATP file.
Chapter Two: Harmonic Distortion

2.1 Introduction

Harmonic distortion is the distortion in the power system due to harmonics. Harmonics of a signal are the signals that are an integer multiple of the fundamental frequency. If the fundamental frequency is 60 Hz, the harmonics will have frequency of 2x60, 3x60, and 4x60 and so on. Harmonics in the electrical power system are unwanted effect due to various reasons. They produce undesirable effects in the power system. If the level of harmonics is above the limit, they must be mitigated to operate the power system satisfactorily.

Medium-voltage, high-power industrial drives consists of power electronic components like diodes, IGBTs, power thyristors etc. The switching frequency of thyristor in phase control mode is not very high but the switching frequency of IGBT in pulse width control mode can be as high as 10 kHz. Thus, these devices when switched at certain frequency produce harmonics. These devices are non-linear loads to the utility [9].

Harmonic distortion in the power system can give rise to a variety of problems including reduced power factor, deteriorating performance of electrical equipment, equipment overheating, the incorrect operation of protective relays, interference with communication devices etc. In some cases, circuit resonance causes dielectric failure of electric equipment and other type of severe damage [1]-[8].
The amount of total harmonic distortion produced by these power electronic drives should be under recommended value per IEEE standard 519-1992. Criteria specify that any individual harmonics should be equal or less than 3% and total harmonic distortion (THD) should be equal or less than 5% [10].

Majority of high-power drive manufacturers around the world increasingly use multi-pulse rectifier in their drive at the front end converter in order to meet the harmonic requirements set by IEEE519-1992 [10]. These converters can be manufactured as 12-pulse, 18-pulse, or 24-pulse rectifiers. When the number of pulses is higher than 6, as in 12-, 18-, or 24-, these converters are powered by a transformer with multiple secondary windings. The number of secondary windings is equal to the number of 6-pulse converters used and each 6-pulse converters is fed by a separate secondary winding. To achieve 12-pulse converter, two secondary windings of the transformers are used with a voltage separation of $30^\circ$. Likewise, for 18-pulse converter, three secondary windings with a voltage separation of $20^\circ$, and for 24-pulse converter, four secondary windings of transformer with a voltage separation of $15^\circ$ are used [9].

The main feature of multi-pulse rectifier’s is its ability by which it reduces the line harmonic distortion. The lower order harmonic currents generated by the multi-pulse rectifiers are cancelled through the phase-shifting transformer. In general, the line current distortion decreases with increased number of rectifier pulses, i.e. higher the number of rectifier pulses, the lower the line current distortion is [9].

Multi pulse rectifier provides multifold advantages over filters as former eliminates the need of using any active or passive filter and also eliminates possible LC
resonance in case of filters. The common-mode voltages generated by the rectifier get blocked by the phase shifting transformer. If the common-mode voltage is not blocked, it could lead to premature failure of winding insulation of electrical equipment.

The multi-pulse rectifier can be manufactured using diodes or thyristors. In case of diodes, dc output voltage cannot be controlled. On the other hand, thyristor provides the ability to control the dc output voltage. Thus the rectifier with diodes as switching device are termed as voltage source while the rectifier with thyristor as switching device are termed as current source.

2.2 Production of Harmonics

If a load draws current that is proportional to the applied voltage and the shape of current waveform is identical to the shape of the voltage waveform, the load is called to be a linear load. Resistance heater, incandescent lamps, etc are the examples of linear loads. But, if a load draws current that is not proportional to the applied voltage and the shape of current waveform is not identical to the shape of applied voltage, the load is referred to as a non-linear load. The examples of non-linear loads are computers, discharge lighting, variable speed drives etc. The majority of non-linear loads utilize semiconductors for power conversion e.g. rectifier.

In non-linear loads, the current waveform that are not sinusoidal contains harmonic current in additional to the sinusoidal fundamental current. The presence of harmonic component is responsible for the distortion of the sinusoidal shape of the current. Harmonic components are integer multiple of the fundamental component. In a
60 Hz system, 3\textsuperscript{rd} harmonics will have a frequency of 180 Hz; 5\textsuperscript{th} harmonics will have a frequency of 300 Hz; the 7\textsuperscript{th} harmonics will have a frequency of 420 Hz and so on.

A symmetrical waveform, in which the positive portion of the non sinusoidal waveform is identical to the negative portion, contains only odd harmonics, like 3\textsuperscript{rd}, 5\textsuperscript{th}, 7\textsuperscript{th} etc. The waveform produced by a full-wave rectifier is an example of symmetrical waveform. On the other hand, an asymmetrical waveform, in which the positive portion of the non sinusoidal waveform is not identical to the negative portion, contains both even and odd harmonics like 2\textsuperscript{nd}, 3\textsuperscript{rd}, 4\textsuperscript{th}, 5\textsuperscript{th} etc. The waveform produced by a half-wave rectifier is an example of an asymmetrical waveform.

A complex harmonic can be formed by adding all harmonics to the fundamental signal. Fig 2.1 shows an example of complex waveform consisting of the fundamental, 1\textsuperscript{st} harmonic, and 3\textsuperscript{rd} harmonic.
AC to DC conversion using full-wave rectifiers generates harmonic currents. The idealized characteristic harmonic currents can be given by the formula:

\[ h = np \pm 1 \] (2.1)

where: 
- \( h \) = order of harmonics
- \( n \) = an integer 1, 2, 3…
- \( p \) = number of current pulses per cycle

The magnitude of the harmonic current in an idealized harmonic is given as the reciprocal of the harmonic number, i.e.:

\[ I = \frac{1}{h} \] (2.2)

Thus,

- 2\(^{nd}\) harmonic current should represent 50.00% of the fundamental current
- 3\(^{rd}\) harmonic current should represent 33.33% of the fundamental current
- 4\(^{th}\) harmonic current should represent 25.00% of the fundamental current
- 5\(^{th}\) harmonic current should represent 20.00% of the fundamental current
- 6\(^{th}\) harmonic current should represent 16.67% of the fundamental current
- 7\(^{th}\) harmonic current should represent 14.29% of the fundamental current
- 8\(^{th}\) harmonic current should represent 12.50% of the fundamental current
- 9\(^{th}\) harmonic current should represent 11.11% of the fundamental current
- 10\(^{th}\) harmonic current should represent 10.00% of the fundamental current
- 11\(^{th}\) harmonic current should represent 9.09% of the fundamental current
- 12\(^{th}\) harmonic current should represent 8.33% of the fundamental current
- 13\(^{th}\) harmonic current should represent 7.69% of the fundamental current
14\textsuperscript{th} harmonic current should represent 7.14\% of the fundamental current
15\textsuperscript{th} harmonic current should represent 6.67\% of the fundamental current
16\textsuperscript{th} harmonic current should represent 6.25\% of the fundamental current
17\textsuperscript{th} harmonic current should represent 5.88\% of the fundamental current
18\textsuperscript{th} harmonic current should represent 5.56\% of the fundamental current
19\textsuperscript{th} harmonic current should represent 5.26\% of the fundamental current

Thus, the harmonic components that will be present in a single-phase half-wave rectifier, which has the number of pulses per cycle equal to one, are 0, 1\textsuperscript{st}, 2\textsuperscript{nd}, 3\textsuperscript{rd}, 4\textsuperscript{th}, 5\textsuperscript{th} and so on. In a single-phase full-wave rectifier, which has the number of pulses per cycle is equal to 2, are 1\textsuperscript{st}, 3\textsuperscript{rd}, 5\textsuperscript{th}, 7\textsuperscript{th}, 9\textsuperscript{th} and so on. In a three-phase 6-pulse rectifier, the harmonic components that will be present are 5\textsuperscript{th}, 7\textsuperscript{th}, 11\textsuperscript{th}, 13\textsuperscript{th}, 17\textsuperscript{th}, 19\textsuperscript{th}, and so on. Theoretically, the total harmonic distortion, THD, of a 3-phase 6-pulse rectifier can be given as

\[
\%\text{THD} = \sqrt{(I_5^2 + I_7^2 + I_{11}^2 + I_{13}^2 + I_{17}^2 + I_{19}^2 + \ldots)}
\]

\(\%\text{THD} = \sqrt{(20^2 + 14.29^2 + 9.09^2 + 7.69^2 + 5.88^2 + 5.26^2 + \ldots)}\)

\(\%\text{THD} = \sqrt{(400 + 204.2 + 82.63 + 59.14 + 34.57 + 27.67 + \ldots)}\)

\(\%\text{THD} = \sqrt{(808.21\ldots)}\)

\(\%\text{THD} = 28.43\ldots\%\)
2.3 Line Notching

Line notching is a phenomenon associated with the thyristor based phase-controlled rectifiers. Commutation notches can be present in diode rectifiers too but to a lesser extent than the notches associated with thyristor rectifiers. The line notching can impact the supply system and other equipment seriously. A 3-phase full-wave thyristor rectifier network supplying a DC load is shown in Fig 2.2.

![Figure 2.2 Three Phase 6-Pulse Rectifier Bridge](image)

Fig 2.3 illustrates an example of line notching at the terminals of the thyristor rectifier. As the notching is at the input terminals, the circuit presents a minimal amount of inductance in the circuit without any source impedance. The voltage notches can be seen at the moments when the continuous line current commutates from one phase to another. During the commutation period though for a very short duration, the two phases are short-circuited through the rectifier bridge and the AC source impedance.
The result is that the voltage, as illustrated, reduces to almost zero as the current increases, limited only by the source impedances.

![Figure 2.3 Example of Line Notching](image)

The number of notches is equal to the number of pulses. In this case, 6 notches can be seen as Fig 2.3 represents line notching for a 6-pulse rectifiers. The location of notches represents the firing instants of thyristors, as the phase angle of thyristor varies depending upon the needed output voltage. The disturbances associated with line notching reduce as we head towards a stiff source. An impedance of relatively low value with relatively high source circuit capacity is called a stiff source.


2.4 Effects of Harmonics

Harmonics produce many unwanted effects in the system. The most prominent effect is the increased equipment heating caused by increase in iron losses and copper losses. Single-phase nonlinear loads present increased the peak-to-peak voltage magnitudes, increasing the stress on the rectifiers.

Many electronic controls are based on zero crossing principle. Where line notching occurs such as in thyristor phase controlled load, additional zero crossing of the input signal may appear. The control loops can become unstable due to the combined effect of harmonic distortion and line notching. When the drives are at low speed and heavy load, the effect of line notching is more pronounced.

Isolation transformers and/or commutating reactors are often installed between the line and the rectifier to attenuate the voltage notches. The isolation transformer or commutating reactors are used for rectifier above 10 HP. This also helps reduce the effect of line notching and harmonics impressed on the drive. However, if the line notching or harmonics level are significant, thyristor misfiring can result. The result of thyristor misfiring is blowing of fuse or the circuit breaker tripping.

Increasing the number of pulse multiplication i.e. 12-pulses, 18-pulses, or 24-pulses, reduces the effects of harmonics as well as line notching at the drive terminals [9].

2.5 Source of Harmonics:

A common type of 3-phase non-linear load is 3-phase 6-pulse rectifier bridge. The nonlinear loads produce a significant amount of harmonics in the system. Not only the
nonlinear loads but also linear equipment such as rotating machines and transformers produce harmonics. However, the magnitude of harmonics produced by linear loads is relatively small compared to those produced by nonlinear loads. Major sources that produce harmonics are given in more detail below.

2.5.1 Thyristor drives:

The harmonics produced by a thyristor drive are a function of the pulse number, i.e. pulse number \( \pm 1 \), i.e. for 6-pulses, 5\(^{th}\), 7\(^{th}\), 11\(^{th}\), 13\(^{th}\) etc. The magnitude of harmonic current is found to be at a maximum value at full load. The effect of loading on harmonic distortion is described in more detail in section 2.6. A typical current waveform of a 3-phase 6-pulse rectifier is shown in Fig 2.4 and a typical harmonic spectrum is shown in Fig 2.5.

Figure 2.4 Typical 3-phase 6-Pulse Thyristor Rectifier Current Waveform
2.5.2 Rotating Machines

Linear loads like generators and motors are also source of harmonics. These sources produce relatively small amount of harmonics in comparison with electronic nonlinear loads.

![Harmonic Current Spectrum of Typical 3-phase 6-Pulse Thyristor Rectifier](chart.png)

Figure 2.5 Harmonic Current Spectrum of Typical 3-phase 6-Pulse Thyristor Rectifier

The magneto-motive force (MMF) in rotating machines is not evenly distributed as windings are embedded into the slots. Because of this, distortion will occur which produces harmonics. In larger machines, coil spanning is used to attenuate the 5\textsuperscript{th} and 7\textsuperscript{th} harmonics. However, rotating machine harmonics are relatively small in magnitude and rarely troublesome compared to those in nonlinear loads.

2.5.3 Transformers

As a result of the nonlinear relationship between currents and voltage and magnetic materials used, transformers also produce harmonics. Transformer magnetizing
current, due to the nonlinear relationship between current and voltage and magnetic material used, are not sinusoidal and contain harmonics, especially 3rd and other triplens. If the input voltage is perfectly sinusoidal, the current will be nonlinear and would contain harmonics. On the other hand, if the magnetizing current is sinusoidal, the output voltage will be nonlinear. However, it should be noted that the harmonics produced in transformers are rarely problematic and are almost negligible, similar to rotating machines, compared to harmonics from electronic nonlinear sources.

2.6 The Effect of Loading on Harmonic Distortion

The magnitude of the characteristic harmonic currents is proportional to the load current in non-linear load. The harmonic current distortion (I_{thd}) is at highest at rated load, except under resonant conditions. At resonant condition, specific harmonic currents can have significantly higher value.

The total voltage distortion (V_{thd}), which is dependent on the magnitude of each harmonic current at its specific harmonic frequency, tends to decrease the farther it is measured from the harmonic-producing load except in resonance condition. It is usually the highest nearer to the harmonic load and progressively reduces due to the voltage drop in cables, transformers, and other impedances. Higher the magnitude of harmonic current, higher is the voltage distortion for given impedance.
2.7 Harmonics and System Power Factor

2.7.1 Power Factor in Systems with only Linear Loads

In power systems containing only linear loads, the vector relationship between voltage and current, the power factor, \( \cos \phi \), can be illustrated by the figure 2.6

\[
\cos \phi = \frac{P}{S} = \frac{kW}{kVA}
\]

(2.4)

The apparent power, \( S (kVA) = \sqrt{P^2 + Q^2} = \sqrt{(kW^2 + kVAR^2)} \)

(2.5)

where, 
- \( P \) = Active Power in kW
- \( S \) = Apparent Power in kVA
- \( Q \) = Reactive Power in kVAR

![Figure 2.6 Power Factor Components in System with Linear Load](image)

2.7.2 Power Factor in Power System with Nonlinear Loads

In power systems consisting of also nonlinear loads, there are two power factors; the true power factor, which is a measure of the power factor of both the fundamental and...
harmonic component in power system, and the displacement power factor, i.e. the power factor of the fundamental component. These two power factors are illustrated in Fig. 2.7

![Figure 2.7 Power Factor Components in System with Harmonics](image)

With reference to the Fig. 2.6 the power factor \( \cos \phi = \frac{P}{S} \neq \frac{kW}{kVA} \)

In Fig. 2.7 the apparent power, \( S \) (kVA) = \( \sqrt{(P^2 + Q^2 + D^2)} \)

\[ = \sqrt{(kW^2 + kVAR^2 + kVAR_H^2)} \]  \hspace{1cm} \text{(2.6)}

The active power, \( P \), can be calculated as:

\[ P = \sum_{h=0}^{\infty} V_h I_h \cos \phi_h = P_0 + P_1 + \sum_{h=2}^{\infty} V_h I_h \cos \phi_h \]  \hspace{1cm} \text{(2.7)}

where \( \phi_h \) is the phase of the \( n \)-th harmonic

Similarly, the reactive power, \( Q \), can be given as

\[ Q = \sum_{h=1}^{\infty} V_{H_{rms}} I_{H_{rms}} \sin \phi_h = Q_1 + \sum_{h=2}^{\infty} Q_h \]  \hspace{1cm} \text{(2.8)}

where \( P_0 = \) DC component of active power

\( P_1/Q_1 = \) (active power/reactive power) of fundamental component

\( P_h/Q_h = \) (active power/reactive power) of individual harmonics

The apparent power, \( S \), can also be given as follows:

\[ S = V_{rms} I_{rms} \]  \hspace{1cm} \text{(2.9)}

23
\[
S=\sqrt{\left(\sum_{r=1}^{\infty} V_{\text{rms}}^2 I_{\text{rms}}^2\right)} \quad \text{---------------------------------------- (2.10)}
\]
\[
S=V_{\text{rms}} I_{\text{rms}} \sqrt{(1+\text{THD}_v^2 \sqrt{(1+\text{THD}_i^2})} \quad \text{---------------------------------------- (2.11)}
\]
\[
S=S_1 \sqrt{(1+\text{THD}_v^2 \sqrt{(1+\text{THD}_i^2})} \quad \text{---------------------------------------- (2.12)}
\]

where \(S_1\) is the apparent power of the fundamental component.

Now, since active power, \(P\), reactive power, \(Q\), and apparent power, \(S\), are defined, the distortion power, \(D\), can be given as
\[
D^2=S^2 - (P^2+Q^2) \quad \text{---------------------------------------- (2.13)}
\]

Thus the power factor for a system containing nonlinear loads can be defined as

Power factor: \(\cos \varphi = P/S = P/\{S_1 \sqrt{(1+\text{THD}_v^2) \sqrt{(1+\text{THD}_i^2})}\}
\]
\[
= \cos \varphi_{\text{disp}} \cdot \cos \varphi_{\text{dist}} \quad \text{---------------------------------------- (2.14)}
\]

Displacement power factor: \(\cos \varphi_{\text{disp}} = P/S_1 \quad \text{---------------------------------------- (2.15)}
\]

Distortion power factor: \(\cos \varphi_{\text{dist}} = 1/\{\sqrt{(1+\text{THD}_v^2) \sqrt{(1+\text{THD}_i^2})}\}
\]
\[
= V_{\text{rms}} I_{\text{rms}} / V_{\text{rms}} I_{\text{rms}} \quad \text{---------------------------------------- (2.16)}
\]

Many drive manufacturers sell their drives stating a high power factor at all loads as an advantage but this high power factor is only displacement power factor and not the true power factor.

**2.7.3 Improvement in power factor by mitigating harmonics**

It can be seen from Fig 2.7 that the true power factor improves if the distortion power factor is reduced. Any type of harmonic mitigation, active, passive, or phase-shifting, will increase the true power factor.
On a 3-phase, 6-pulse rectifier, the true power factor will vary, depending upon also the conduction angle of the rectifier, due to changing speed. On this type of drive, the use of passive (L-C) filters tuned to a specific harmonic frequency like 5th, 7th, 11th, 13th etc can be used to mitigate the harmonics. These filters not only mitigate harmonics but also improve power factor via filter capacitors. However, we must be careful at light load to prevent a leading power factor.

Power factor corrections can also be achieved by active filters. The active filters provide harmonic cancellation current into the power system. Thus, the power system source then supplies only fundamental component to the nonlinear loads. In additional to the harmonics cancellation, active filters also provide reactive current for correcting leading or lagging power factor at any load.

2.8 Influence of Source Impedance and kVA on Harmonics

Based on impedance there are 2 types of sources; stiff source and soft source. Both sources have a significant effect on the nonlinear currents drawn by the load and the resultant voltage distortion. Stiff sources have lower impedance but higher short circuit current and are often associated with transformers where the source impedance is in the order of 5% to 10%. On the other hands, soft source have comparatively higher impedance and lower short circuit current and are often associated with the generators. In generators the sub-transient reactance, \(X_d\)” is in the range of 0.1 per unit to 0.2 per unit.

In case of stiff sources, nonlinear loads draw a higher value of harmonic current as the current is not limited by the leakage reactance of the source but the voltage
distortions are lower. On the other hand, in case of soft sources, loads draw a lower value of harmonic current but significantly higher levels of voltage distortion is present.

To reduce the harmonic voltage distortion from a soft source, generators would need a low value of sub-transient reactance, Xd”, which can be achieved by over-sizing the kVA rating. By over-sizing the generator, it supplies increased fault current and turning the generator into a relatively stiffer source.

For a given load, the kVA rating also has an impact on the magnitude of harmonic currents and associated voltage distortion. The short circuit capability and associated I_l/Isc (load current to short circuit current ratio) varies with a variation in kVA. The harmonic current distortion I_{thd} are higher for higher value of kVA or MVA but the resultant harmonic voltage distortion, V_{thd} will be lower, provided constant source impedance.

2.9 Effect of Unbalance and Background Voltage Distortion

The voltage unbalance introduces uncharacteristic harmonics and possibly DC component in the power system. In addition, the unbalance currents are responsible for increased thermal stress on the drive. Further, an increase in the production of uncharacteristic harmonics and the total harmonic current distortion are possible due to misfiring of thyristor. The uncharacteristic harmonics are rarely considered in the design of equipment as these harmonics are difficult to predict. It is easier to address the cause of the uncharacteristic harmonics than designing a system to mitigate these uncharacteristic harmonics.
2.9.1 Voltage Unbalance and 3-Phase, 6-Pulse Rectifier

The performance of phase-shift in harmonic mitigation system gets poorer by voltage unbalance and voltage distortion. A 30° phase shift cancels the 5th and 7th harmonics. However, due to voltage unbalance, there will be residual 5th and 7th harmonic current present even after mitigation.

2.9.2 Background Voltage Distortion and 3-Phase, 6-Pulse Rectifier

Background voltage distortion produces undesirable performance in multi-pulse drive system. They can cause damage to the front end capacitors used for power factor correction and harmonic filter. The background voltage distortion also degrades the performance of multi-pulse drive system that depends on phase shift for harmonic mitigation.

2.10 Effect of Resonance

A resonance in the power system results in high voltage or high current depending upon series or parallel resonance respectively. The high voltage or high current can damage the electrical equipment. The reactive impedances are frequency dependent. The capacitive reactance is inversely proportional to the frequency, whereas the inductive reactance is directly proportional to the frequency. When the inductive reactance is equal to capacitive reactance, the resonance is said to be occurred.

In series resonance, the total impedance reduces to the resistance component only. Where resistive impedance is low, a high value of current at resonance frequency can flow through the circuit and damage the equipment.
In parallel resonance, the total impedance increases to the resistance component only. At resonance frequency a high voltage can be present across the equipment and damage the equipment.

Series resonance is less common than parallel resonance because the majority of equipment is connected in parallel. Major problems due to resonance are overhearing of equipment, equipment damage, nuisance relay tripping, and capacitor fuse failure.

2.11 Mitigation of Harmonics

In order to limit the harmonic currents and associated voltage distortion within the limits, the majority of nonlinear loads associated with bulk power often needs addition of mitigation equipment. Depending upon the solution desired, the mitigation may be an integral unit with nonlinear load or a discrete unit installed in the switchboard. On a system with multiple nonlinear loads, some harmonic cancellation occurs due to phase angle diversification between the multiple harmonic sources.

There are three major methods for mitigating the harmonics. The method utilized in any particular application depends upon the nature and magnitude of the mitigation needed and power system configuration.

2.11.1 Active Filters

Active filters are mainly used for mitigating triplen harmonics in four-wire systems. The active filters monitor the load currents in 3-phases and neutral wire. The notch filter is used to remove the fundamental component. The remainder is the harmonic distortion current. This remainder distortion is phase reversed (180° out of phase) and
injected into the load as harmonic cancellation current. Thus, the harmonic component is supplied to the load by active filter and the source supplies only fundament component.

2.11.2 Passive Filters

Passive filters are used to mitigate harmonic distortion for multiple applications. A passive filter consists of a capacitor, inductor and occasionally resistors. Their operation depends on resonance phenomenon. At series resonance, inductive reactance and capacitive reactance cancel each other leaving only resistive impedance in the circuit. The series filters are usually connected in parallel with the nonlinear load. These filter offer very low impedance to the harmonic frequencies that need to be mitigated. For example, for mitigating, 5\textsuperscript{th} and 7\textsuperscript{th} harmonic distortion, one set of filters would be needed to mitigate 5\textsuperscript{th} harmonics, and another set of filters would be needed to mitigate 7\textsuperscript{th} harmonics. Change in sources and load impedances have impact in the performance of passive filters. Since passive filters attract harmonics from other sources, the phenomenon must be taken into account in design.

2.11.3 Phase Shifting

For high power operation, generally 400 HP motors or larger, phase-shifting technique is used to mitigate the input harmonic current. The phase-shifting is achieved by using multiple converters. The way multiple converters are connected together facilitate in canceling certain harmonics produced by one converter to those produced by other converter. Certain harmonics, determined by number of converter bridges used, are eliminated at the input i.e. primary side of the phase shift transformer.
The phase shifting technique is also called multi-pulsing. So the converter that utilizes phase shifting transformer for harmonics mitigation is termed as multi pulse drive, i.e. 6-pulse drive, 12-pulse drive, 18-pulse drive, 24-pulse drive etc. A single unit 3-phase rectifier is called 6-pulse rectifier. Thus, a 12-pulse rectifier will have 2x6-pulse rectifier, 18-pulse will have 3x6-pulse and so on. Phase-shifting not only reduce the harmonic input current but also reduces the ripple on the DC output of the rectifier.

The phase shift angle is a function of the number of the rectifiers that are used [9]:

\[
\text{Phase shift in degrees} = \frac{60}{\text{Number of rectifier bridges}}
\]

For example, a 12-pulse rectifier will need a phase shift of \(\frac{60^0}{2}=30^0\). Similarly an 18-pulse rectifier will need a phase shift of \(20^0\). Due to stringent harmonic requirement of IEEE 519-1992, 6-pulse rectifiers are finding decreased use as the harmonics produced by a 6-pulse rectifier is above the IEEE 519-1992 recommendation. However, the use of 18-pulse rectifier is increasing in the USA.

2.12 Harmonic Limit Recommendation

Total harmonic distortion is calculated in a range of 50\(^{th}\) harmonics. In this range, the total harmonic voltage distortion should be equal or less than 5\% as measured at any point of common coupling (PCC) with any individual harmonic voltage distortion not exceeding 3\% of the fundamental [10].

On a dedicated system, higher level of harmonic distortion may be permissible as long as the equipment can operate safely at the higher limits. This method is generally not practical in a system where industrial loads are supplied form a common power point.
In a power system all operating modes should be considered for harmonic limits. If certain operating modes last for only a small fraction of time but produce higher harmonic distortion, it may not be necessary to provide additional harmonic mitigation technique for that short duration.
Chapter Three: Alternative Transients Program, ATP

3.1 Introduction

Alternative Transients Program, ATP is a universal program for digital simulation of transient phenomenon of power electronics, control systems, and power systems. ATP can simulate complex networks and control systems of arbitrary structure. It is one of the most powerful programs available for digital simulations [31].

3.2 Operating Principles

ATP uses trapezoidal rule of integration to solve the differential equations of system component in time domain. Therefore output waveform can be plotted for a desired length of time. Non-zero initial conditions can be determined either automatically by a steady-state phasor solution or the initial conditions can be entered by the user for simpler components. Interfacing capability to the program modules like TACS (Transient Analysis of Control Systems) and MODELS (a simulation language) enables modeling of control system and components with nonlinear characteristics such as power electronics, arcs, and corona. Symmetric or un-symmetric disturbances are allowed such as faults, lightning surges, and all kind of switching operations including power electronics switching [31].
3.3 Program Capabilities

ATP can simulate the system up to 6,000 buses; 10,000 branches; 1,200 switches; 900 sources; 2,250 nonlinear elements, and 90 synchronous machines. ATP is capable of providing solutions of a system that is a combination of electrical and mechanical systems [31].
Chapter Four: Modeling, Simulation, Results and Analysis

4.1 Thyristor Phase Control

Rectifiers with diode switches are not controllable. In many applications, it is necessary for the dc voltage to be controllable. For controllability, diodes are replaced by thyristors. Unlike in diodes, in thyristors, conduction does not happen only after exceeding the threshold voltage when forward biased but also thyristors need a triggering signal at the gate. Anode, cathode, and gate of a thyristor are shown in Fig. 4.1.

By the use of phase control, average values of load voltage can be controlled and varied [11]. The application of a triggering pulse at the thyristor gate at any desired instant during the period when the thyristor is forward biased to control the magnitude of the dc output voltage is called phase control [11].

The system being investigated in this section is single-phase full-wave controlled bridge rectifier. Circuit diagram, mathematical expressions, and voltage and current waveforms are presented for each rectifier when feeding the following loads:

1. Resistive load, R
2. Resistive, R, and inductive, L, load
3. Resistive, R, Inductive, L, and EMF
The load electro-magnetic force, EMF, may be either a battery bank or back emf of a dc motor.

### 4.1.1 Rectifier with R Load

Fig. 4.1 shows a single-phase bridge rectifier with R load. Anode, cathode, and gate are marked for thyristor T1. Similar is true for thyristors T2, T3, and T4. Once thyristor is fired at gate, while it is forward biased, thyristor starts conducting. Thyristor turns off when current being conducted reaches zero value.

![](image)

**Figure 4.1** Single-phase full-wave bridge rectifier with R load

Vs is root mean square, RMS, value of source voltage and Vo is average dc output voltage. Vs is given by \( V_m \sin \omega t \) where \( V_m \) is peak value of Vs.

In the circuit shown in Fig. 4.1, T1 and T2 are fired simultaneously at firing angle, \( \alpha \), in positive half cycle of the source. Similarly, T3 and T4 are fired simultaneously at firing angle, \( \pi + \alpha \), in negative half cycle. Since the load is purely resistive, the voltage and current both go to zero at \( \pi, 2\pi \), and so on.
The source voltage and firing signals are shown in Fig 4.2. It can be seen that one cycle of the supply is 16.667 ms i.e. the supply frequency is 60 Hz. Positive cycle is fired at 2.0833 ms i.e. at $45^0$ and, $2\pi+45^0$, $4\pi+45^0$, and so on. Similarly negative half cycle is fired at 10.4167 ms i.e. at $180^0+45^0$ and, $2\pi+180^0+45^0$, $4\pi+180^0+45^0$, and so on.

![Figure 4.2 Source voltage and firing pulses](image)

The output voltage (plotted as $v$) and output current (plotted as $c$) are shown in Figure 4.3. The thyristor that is turned on at $\alpha$ is turned off at $\pi$ and the thyristor that is turned on at $\pi+\alpha$ is turned off at $2\pi$. During positive half cycle of the source voltage the thyristor pair $T_1$ and $T_2$ are turned on after the instant of firing pulse and are turned off when the current becomes zero. Thyristor turn off process is known as commutation. Similar is true for the thyristor pair $T_3$ and $T_4$ during negative half cycle.

The average output voltage $V_0$ and the average load current $I_0$ can be given as follows [11]:

![Average output voltage and current](image)
Figure 4.3 Load voltage and current for purely resistive load

\[
V_o = \frac{1}{2\pi} \left[ \int_{\alpha}^{\pi} V_m \sin \omega t \, d(\omega t) + \int_{\pi+\alpha}^{2\pi} V_m \sin \omega t \, d(\omega t) \right] \tag{4.1}
\]

After solving we get:

\[
V_o = \frac{V_m}{\pi} (1+\cos \alpha) \tag{4.2}
\]

and the average load current can be given as:

\[
I_o = \frac{V_o}{R} \tag{4.3}
\]

From equations (4.2) and (4.3), it can be seen that the maximum value of Vo and hence of Io occurs when \(\cos \alpha = 1\) i.e. \(\alpha = 0^0\).

The firing angle may thus be defined as the angle corresponding to the duration between the instant of triggering that gives largest average output voltage to the instant that gives any desired average output voltage.
4.1.2 Rectifier with R and L Load

A single phase controlled full-wave bridge rectifier feeding a RL load is shown in Fig 4.4. The output waveform for this circuit depends on the value of the inductance, L, and the firing angle, $\alpha$.

![Figure 4.4 Single-phase full-wave bridge rectifier with R and L load](image)

Since, the current cannot change instantaneously in an inductor; thyristor T1 and T2 will continue to conduct even after the voltage zero until the current becomes zero. If the value of inductance is large, T1 and T2 may not turn off at all until thyristors T3 and T4 is fired. Firing of T3 and T4 reverse biases and turns T1 and T2 off. The load current will shift from pair T1 and T2 to pair T3 and T4.

The voltage and current waveforms for an arbitrary RL load are shown in Figure 4.5. The load voltage $V_o$ becomes equal to the source voltage $V_s$ immediately after thyristors are fired. However, because of the load inductance, the current starts rising gradually from its zero value and attains maximum value after sometime depending upon the value of inductor. It can be seen that at 8.33 ms i.e. $\pi$ radian ($180^0$), $I_o$ is not zero.
even though Vo is zero. At some angle β (9.3 ms), Io reduces to zero and thyristors T1 and T2 turned off. During the period between $\omega t = \pi$ and $\omega t = \beta$, i.e. when the voltage is negative and the current is positive, the magnetic energy stored in the inductor is delivered back to the supply. After $\omega t = \beta$, Vo=0 and Io=0. Angle β is called extinction angle. Thus, the conduction angle y can be given as [11]:

$$y = (\beta - \alpha) \quad \text{--------------------------------------------}$$

(4.4)

Figure 4.5 Load voltage and current for resistive and inductive load

It can be seen from Fig 4.5 that the output current is zero when $\omega t = \alpha$ and $\omega t = \beta$.

If the inductance value is large, output current becomes continuous and comparatively ripple-free.

The average output voltage Vo can be given as [11]:

$$Vo = \frac{2Vm}{\pi} \cos \alpha \quad \text{--------------------------------------------}$$

(4.5)
4.1.3 Rectifier with R, L, and E Load

A single phase controlled full-wave bridge rectifier feeding a RLE load is shown in Fig 4.6. Voltage E corresponds to a battery emf or a back emf of a dc motor. Thyristor pair T1 and T2 is fired simultaneously at $\alpha$ while the pair T3 and T4 are fired after $\pi$ radians in each cycle. When pair T1 and T2 is ON, the output voltage is same as the supply voltage. When the pair T1 and T2 is OFF and pair T3 and T4 are not turned ON yet, the output voltage is equal to emf E. Fig 4.7 shows the output voltage and current waveforms. The presence of a voltage source in the load tends to reverse bias the thyristor during the period when $\omega t = 0$ and $\omega t = \alpha$. The thyristor will not turn on for a firing angle smaller than a certain value called critical angle, $\alpha_c$, can be given as [11]:

$$\alpha_c = \sin^{-1}\left(\frac{E}{V_m}\right) \quad \text{------------------------------------------} \quad (4.6)$$

In this model, for $E=50V$ and $V_m=\sqrt{2} \times 120 \text{ V}=169.71 \text{ V}$, $\alpha_c = 17.14^0$. Since a firing angle of $45^0$ is used, there is no problem for thyristor to turn on.

Figure 4.6 Single-phase full-wave bridge rectifier with R, L, and E load
The average value of the output voltage is given below [14]:

\[ V_o = \frac{2V_m}{\pi} \cos \alpha \]  
(4.7)

### 4.1.4 Summary

The variation of output voltage, \( V_o \), with respect to firing angle \( \alpha \) is plotted in Fig. 4.8. It can be noted that the output voltage is positive until the firing angle is less than \( \pi/2 \) (90\(^\circ\)) and the output voltage is negative for values of \( \alpha \) greater than \( \pi/2 \). The operation of the converter takes two different forms. For values \( \alpha \) smaller than \( \pi/2 \), the converter operates as a rectifier. During this operation power flows from the supply to the load. But, if \( \alpha \) is greater than \( \pi/2 \) and the load circuit emf \( E \) is reversed, it is possible to transfer the dc power from battery or a running dc motor to the ac supply as ac power.

This is equivalent to inversion. Thus, the same converter can operate as a rectifier as well as an inverter depending upon the value of \( \alpha \) and the polarity of the dc source. The
converter operating as an inverter makes use of the line voltage for commutation. Therefore, it is called line-commutated inverter.

![Variation of Vo with \( \alpha \)](image)

Figure 4.8 Variation of output voltage with firing angle

From Fig. 4.8 a table as shown below can be created. TABLE 4.1 below shows an interesting relationship between firing angle, \( \alpha \), and output voltage, \( V_o \).

**Table 4.1 Firing angle vs. Output Voltage**

<table>
<thead>
<tr>
<th>Firing Angle</th>
<th>Output Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>2 ( V_m /\pi )</td>
</tr>
<tr>
<td>60°</td>
<td>( V_m /\pi )</td>
</tr>
<tr>
<td>90°</td>
<td>0</td>
</tr>
<tr>
<td>120°</td>
<td>- ( V_m /\pi )</td>
</tr>
<tr>
<td>180°</td>
<td>- 2 ( V_m /\pi )</td>
</tr>
</tbody>
</table>
In rectifier operation, the average value of output voltage $V_o$ must be greater than the load circuit emf $E$, while during inverter operation, the load circuit emf must be greater than the average value of the output voltage.

### 4.2 Filter Design

When ac power is converted to dc power using rectifiers, dc output contains undesirable ac components called ripple. Many rectifier applications require that the ripple do not exceed a specified value. If the ripple exceeds the specified value, different unwanted effects appear in the system. Some of the unwanted effects are stray heating, audible noise etc. Ripple can be mitigated using an output filter.

The ripple factor is a ratio of the rms value of the ripple voltage $V_{rms}$ to the average value $V_0$ at the output of a rectifier filter [14]. Fig 4.9 indicates the parameters needed to determine the ripple factor graphically. It is assumed that the ripple voltage has sinusoidal waveform. The formula for determining the percentage of ripple is

\[
\text{Percentage of ripple} = \left( \frac{\text{RMS value of ripple}}{\text{Average DC output}} \right) \times 100
\]

\[
= \left( \frac{V_{rms}}{V_0} \right) \times 100
\]

where; $V_{rms} = 0.707 \times V_p$

$V_p = \text{peak value of ripple voltage}$
A circuit that minimizes or eliminates the ripple component from the rectified output is called a filter. Filter systems in general are composed of a capacitor or an inductor. Capacitor filters are used for lower-power applications. On the other hand, inductor filters are used in high-power applications [14]. Depending upon the passive element used, the filters can be classified as

1. Capacitor filter
2. Inductor filter

In this section, each of above filters is modeled for full-wave bridge rectifier.
4.2.1 Capacitor Filter

A rectifier circuit without a filter produces pulses at the output. The fluctuations can be reduced if some of the energy can be stored in a capacitor while the rectifier is producing pulses and is allowed to discharge from the capacitor between pulses.

![Diagram of Full-wave bridge rectifier with a capacitor filter](image)

Figure 4.10 Full-wave bridge rectifier with a capacitor filter

Fig 4.10 shows a full-wave bridge rectifier with a capacitor filter. Vs is rms value of source voltage and is equal to 120 V. Frequency of the source is 60 Hz. Time varying source current is represented by $i_s$ and load current by $i_o$. Diodes are represented as D1, D2, D3, and D4. During positive half cycle of the source, D1 and D2 conduct. During negative half cycle, D3 and D4 conduct. Filter capacitor is represented by C and the load is represented by R. The value of load is 10 Ohm. Value of C is calculated in this section for different values of ripple. The output waveforms are plotted in ATP for 3% ripple and the ATP model is verified with the formula for calculating the ripple.
Fig. 4.11 Full-wave output without a filter

Fig. 4.11 shows the voltage output of the full-wave bridge rectifier across point POS and NEG of Fig. 4.10 without the filter capacitor. This pulsating voltage is applied to the filter capacitor represented by C in Fig. 4.10. The capacitor will react to any change in circuit voltage. Because the rate of capacitor charging is limited only by the impedance in the source side which is pretty low, the voltage across the capacitor can rise nearly as fast as the half sine wave voltage from the rectifier. In other words, the RC charge time is relatively short. The charge on the capacitor represents storage of energy. When the rectifier output drops to zero, the voltage across the capacitor does not fall immediately. Instead, the energy stored in the capacitor is discharged through the load during the time that the rectifier is not supplying energy.

The voltage across the capacitor and the load falls off very slowly if it is assumed that a large capacitor and relatively large value of load resistance are used. In other words, the RC discharge time is relatively long. Therefore, the amplitude of the ripple is greatly decreased as shown in Fig. 4.12. Waveform v:POS–NEG represents the output without a filter and waveform v:LOAD–NEG represents the output with the filter.
After the capacitor has been discharged, the rectifier does not begin to pass current until the output voltage of the rectifier exceeds the voltage across the capacitor. This occurs at 10 ms in Fig 4.12. Current flows in the rectifier until slightly after the peak of the half sine wave at 13 ms. At this time the sine wave is falling faster than the capacitor can discharge. A short pulse of current beginning at 10 ms and ending at 13 ms is therefore supplied to the capacitor by power source.

![Figure 4.12 Output for full-wave bridge rectifier](image)

The average voltage, $V_o$, of the full-wave bridge rectifier output is $0.636xV_m$ [10]. $V_m$ is peak value of $V_s$ and is equal to $\sqrt{2} \times V_s$. From Fig 4.11 it can be seen that the average value is $0.636x\sqrt{2}\times120$ V = 108 V. Because the capacitor absorbs energy during the pulse and delivers this energy to the load between pulses, the output voltage can never fall to zero. Hence the average voltage of the filter output as shown in Fig 4.12 is greater than that of the unfiltered output shown in Fig 4.11. However, if the resistance of the load
is small, a heavy current will be drawn by the load and the average output voltage will fall. Also, the filter capacitor acts like a short circuit across the rectifier while the capacitor is being charged. Due to these reasons, a simple capacitor filter is not suitable for rectifiers in higher power applications.

In practice, the ripple factor can be found from

$$RF = \frac{1}{\sqrt{2}} \left(\frac{2f_r RC-1}{} \right)$$

(4.8)

where $f_r$ is the output ripple frequency.

From Fig 4.11, it can be seen that there are 2 output pulses for each cycle of supply voltage. Therefore the frequency of output ripple, $f_r$, is 120 Hz. Using equation (4.8) the value of capacitor is calculated for different values of ripple and the results are tabulated in Table 4.2.

<table>
<thead>
<tr>
<th>RF %</th>
<th>R Ohms</th>
<th>fr Hz</th>
<th>C mF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>120</td>
<td>29.880</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>120</td>
<td>15.148</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>120</td>
<td>10.238</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>120</td>
<td>7.782</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>120</td>
<td>6.309</td>
</tr>
</tbody>
</table>

Using the capacitor value of 10.238 mF for 3% ripple, the voltage waveforms are plotted in Fig 4.13.
From Fig 4.13, $V_{\text{max}} = 170$ Volts, $V_{\text{min}} = 156$ Volts, $V_o = 163$ Volts, and, $V_p = 7$ Volts

$V_{\text{rms}}$ of ripple = $0.707 \times 7$ Volts$= 4.95$ Volts.

Ripple percentage = \( \frac{V_{\text{rms}}}{V_o} \times 100 \)

\[ = 3\% \]

The same value of percentage ripple is obtained from simulation as well as from the equation (4.8).

### 4.2.2 Inductor Filter

From above we know that a capacitor is a device that reacts to variation in voltage and are connected across the load. The inductor is a device that reacts to changes in current. The inductor causes delay in current. Since the current is same in all parts of the series circuit, an inductor $L$ is connected in series with the load as shown in Fig 4.14.

Circuit nomenclature and circuit parameters are the same as those in capacitor filter. Output of the rectifier without the inductor filter will be the same as shown in Fig 4.11.
The sequence of operation of diodes is also the same as those in capacitor filter. In this section the value of inductor required for different value of ripple are calculated and results are tabulated in Table 4.3. The output waveforms are plotted using ATP for 3% ripple.

The use of an inductor prevents the current from building up or dying down too quickly. If the inductor is made large enough, the current becomes continuous and nearly constant. The inductor prevents the current from ever reaching the peak value which would otherwise be reached without a filter inductor. Consequently, the output voltage never reaches the peak value of the applied sine wave. Thus, a rectifier whose output is filtered by an inductor cannot produce as high a voltage as that could be produced by a rectifier filtered by a capacitor. However, this disadvantage is partly compensated because the inductor filter permits a larger current without a serious change in output voltage. This is the reason that an inductor filter is suitable for high power applications.

Figure 4.14 Full-wave bridge rectifier with an inductor filter
Typical output waveforms are shown in Fig 4.15. Waveform that has a peak value of 17 A is the current without the inductor filter. The waveform that has a peak value of 12.6 A is the current with the inductor filter.

In practice, the ripple factor can be found from

$$RF = \frac{0.4714}{\sqrt{1 + (4\pi f_i L/R)^2}}$$

(4.9)

where $f_i$ is the input frequency

It should be noted that in equation (4.9) for inductor filter, input line frequency is used for calculation unlike the case of capacitor filter where output ripple frequency was used in equation (4.8). Using formula shown above, the value of inductor is calculated for different values of ripple and the results are tabulated in Table 4.3.
Table 4.3 Percentage Ripple vs. Inductance

<table>
<thead>
<tr>
<th>RF %</th>
<th>R Ohms</th>
<th>( f_i )</th>
<th>L mH</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>60</td>
<td>624.821</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>60</td>
<td>312.200</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>60</td>
<td>207.898</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>60</td>
<td>155.677</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>60</td>
<td>124.287</td>
</tr>
</tbody>
</table>

Using the inductor value of 207.898 for 3\% ripple, the current waveform is plotted as shown in Fig 4.16. From Fig 4.16, \( I_{\text{max}} = 11.25 \text{ A} \), \( I_{\text{min}} = 10.34 \text{ A} \), \( I_o = 10.79 \) A, \( I_p = 0.46 \)

\[
\text{Irms of ripple} = 0.707 \times 0.46 \text{ A} = 0.325 \text{ A}
\]

\[
\text{Ripple percentage} = \left(\frac{\text{Irms}}{I_o}\right) \times 100
\]

\[
= 3 \%
\]

The same value of percentage ripple is obtained from computer simulation as that was calculated using equation (4.9).

Figure 4.16 Output current waveform for 3\% ripple with 207.898 mH inductor
4.2.3 Summary

The values of capacitor or inductor needed for various type of filter to limit the dc ripple to 3% is shown in Table 4.4.

<table>
<thead>
<tr>
<th>Filter Type</th>
<th>Capacitor in mF</th>
<th>Inductor in mH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacitor Filter</td>
<td>10.238</td>
<td>N/A</td>
</tr>
<tr>
<td>Inductor Filter</td>
<td>N/A</td>
<td>207.898</td>
</tr>
</tbody>
</table>

4.3 Multi-pulse Rectifier

In order to meet the harmonic requirement set by IEEE standard 519-1992, major high-power drive manufactures around the world are increasingly using multi-pulse rectifier in their drive at front end converter. These rectifiers can be configured as 12-, 18-, and 24-pulse rectifiers and powered by a transformer with a number of secondary windings. Each 6-pulse rectifier is fed by a secondary winding. To achieve 12-pulse rectifier two 6-pulse rectifiers are used. Likewise, for 18-pulse, three 6-pulse rectifiers, and for 24-pulse, four 6-pulse rectifiers are used.

The multi-pulse rectifier’s main feature is its ability to reduce the line harmonic distortion. Some of the lower order harmonic currents generated by the six-pulse rectifiers are cancelled through the phase-shifting transformer. In general, as the number of rectifier pulses increases, the line current distortion decreases i.e. higher the number of rectifier pulses, the lower the line current distortion is.
4.3.1 Three-phase 6-pulse Rectifier

Fig 4.17 shows a simplified circuit diagram of a three-phase six-pulse thyristor rectifier. The inductance $L_s$ is the total inductance including the line inductance, transformer reactance, and line reactor between the utility and the rectifier. For the ideal rectifier $L_s$ is assumed to be zero. On the dc side a choke $L_d$ is used to make the dc current ripple free [12]. The RC snubber circuits [33] for thyristor are not shown in Fig 4.17 but are considered in the computer model created in alternative transients program (ATP) [31].

![Simplified circuit diagram of a six-pulse thyristor rectifier](image)

Figure 4.17 Simplified circuit diagram of a six-pulse thyristor rectifier

Fig 4.18 shows typical waveforms of the rectifier, where $v_a$, $v_b$, and $v_c$ are the phase voltages of the utility supply, P1 thru P6 are the gate firing pulses for thyristors T1 thru T6 respectively and $\alpha$ is the firing angle of the thyristors.
Figure 4.18 Waveform of an ideal six-pulse thyristor rectifier operating at $\alpha=30^0$
During interval I, thyristors T1 and T6 are conduct assuming T6 was conducting prior to turn on of T1. The positive dc voltage is $v_p$ with respect to ground is $v_a$ and the negative bus voltage $v_n$ i.e. equal to $v_b$. The dc output voltage can be found from $v_d = v_p - v_n = v_{ab}$. The line currents can be given as $i_a = I_d$, $i_b = -I_d$, and $i_c = 0$.

During interval II, thyristor T6 is turned off after T2 turns on and the dc current $I_d$ is commuted from T6 to T2. Thus T1 and T2 are conducting. The positive dc voltage $v_p$ is still the same i.e. $v_p = v_a$ but the negative bus voltage $v_n$ is equal to $v_c$. The dc output voltage can be found from $v_d = v_p - v_n = v_{ac}$. The line currents can be given as, $i_a = I_d$, $i_b = 0$, and $i_c = -I_d$.

Following the same procedure all the voltage and current waveforms in other interval can be obtained. It should be noted that the number of thyristor and gate is also the sequence of their firing.

The average dc output voltage can be given as [13]

$$V_d = 1.35 V_{LL} \cos \alpha \quad \text{------------------------------------------} \quad (4.10)$$

The equation (4.10) depicts that the rectifier dc output voltage $V_d$ is positive when the $\alpha$ is less than $\pi/2$ and becomes negative for an $\alpha$ greater than $\pi/2$. But irrespective of the polarity of the dc output voltage, the dc current $I_d$ is always positive. The technique to control the dc output voltage by firing angle, $\alpha$, is called phase-control technique [14].

The power is delivered from utility to the load when the rectifier produces positive dc voltage. With a negative dc voltage, the rectifier operates as an inverter and the power is fed from the load back to utility. This often takes place during rapid deceleration when the kinetic energy of the rotor and its mechanical load is converted to
the electric energy by the converter working in inverter mode and are used for dynamic braking.

The line current $i_a$ in Fig 4.18 can be expressed in a Fourier series as [13]

$$i_a = \frac{2\sqrt{2}}{\pi} I_d \{ \sin(\omega t - \phi) - \frac{1}{5} \sin(5\omega t - \phi) - \frac{1}{7} \sin(7\omega t - \phi) + \frac{1}{11} \sin(11\omega t - \phi) + \frac{1}{13} \sin(13\omega t - \phi) - \frac{1}{17} \sin(17\omega t - \phi) - \frac{1}{19} \sin(19\omega t - \phi) + \ldots \}$$

where, $\phi$ is the phase angle between the supply voltage $v_a$ and the fundamental frequency line current $i_{a1}$.

The rms value of $i_a$ can be given as [13]

$$I_a = \sqrt{\frac{2}{3}} I_d = 0.816 I_d$$

From which the total harmonic distortion, THD, for the line current $i_a$ is [13]

$$\text{THD} = \frac{\sqrt{I_{a1}^2 - I_{a1}}}{I_{a1}} = \frac{\sqrt{(0.816I_d)^2 - (0.78I_d)^2}}{0.78I_d} = 0.311$$

where $I_{a1}$ is the rms value of $i_{a1}$.

i.e. THD = 31.1%

4.3.1.1 Summary

It can be seen that the theoretical calculation of % THD from equation (2.3) is approximately 28.43% but the total % THD from equation (4.13) is 31.1%. The error in %THD is 8.59%. The total harmonic distortion produced by a three-phase ideal 6-pulse rectifier, which is 31.1%, is above the IEEE 519-1992 guideline [10].
### 4.3.2 Three-Phase 12-Pulse Rectifier

A 12-pulse rectifier is modeled using two 6-pulse rectifier. Fig 4.19 shows ac input voltage, input currents, thyristor firing pulses, and dc output voltage for 12-pulses and 6-pulses. It can be seen that the input currents are much more close to sinusoidal wave than those in 6-pulse rectifier shown in Fig. 4.18. Similarly, the output voltage for 12-pulse rectifier has double the number of pulses than those in 6-pulse rectifier, resulting in 50% less ripple.

![Figure 4.19 Waveforms for a 12-pulse rectifier](f ile 12pulse.pl4; x-var t)

Vin AC  
Vout DC  
12-Pulse  
6-Pulse  
Va  
Vb  
Vc  
i_a  
i_b  
i_c  
P1-P6  
P7-P12  

Figure 4.19 Waveforms for a 12-pulse rectifier
Using equations 2.1 and 2.2, the total harmonic distortion can be calculated as

\[ \% \text{THD} = \sqrt{I_{11}^2 + I_{13}^2 + I_{23}^2 + I_{25}^2 + I_{35}^2 + I_{37}^2 + \ldots} \]

\[ \% \text{THD} = \sqrt{(9.09^2 + 7.69^2 + 4.35^2 + 4.00^2 + 2.86^2 + 2.70^2 + \ldots)} \]

\[ \% \text{THD} = \sqrt{(82.64 + 59.14 + 18.92 + 16.00 + 8.18 + 7.29 + \ldots)} \]

\[ \% \text{THD} = 15.3\% \]  

4.3.2.1 Summary

It can be seen from equation (4.14) that the % THD is 15.3%. The total harmonic distortion produced by a three-phase 6-pulse rectifier is 31.1%. Thus by using a 12-pulse rectifier instead of a 6-pulse rectifier both the % THD and dc ripple can be lowered by 50%. If the grid is relatively stronger, a 12-pulse rectifier provides a window opportunity per IEEE 519-1992 guideline. Thus, by use of a 12-pulse rectifier, the ac side filter as well as the dc side filter can be removed from the drive.

4.4 Controller Design

In section 4.3.1, the thyristor firing pulses were generated from the independently running pulse generators. For an automatic firing pulses generation in a phase-controlled rectifier, it is necessary to control the phase of the thyristor firing pulses. To control the phase of the firing pulses, a negative feedback closed-loop control is presented in this section.

In a negative feedback system, the function of the firing pulse generator is to deliver correctly timed, properly shaped, firing pulses to the gates of the thyristors. The phase of the firing pulses, relative to the rectifier input voltage is controlled, almost
invariably, in accordance with an analog reference signal by one of the following methods [34]:

i. Phase-locked Oscillator:

ii. Integral Control:

Phase-locked oscillator technique for timing the firing pulses is based upon the fact that under steady state conditions (with a steady d-c output), the pulses for successive thyristors are produced at evenly spaced intervals of time. Thus, the time period between any two consecutive commutations is equal to the period of the input voltage waveform, divided by the pulse number of the converter. In practice, it is not possible to achieve a free-running firing circuit that precisely maintains the desired phase control of firing pulses. But, with additional hardware and increased firing circuit complexity, it may be possible to use phase-locked oscillator technique. This technique is not considered in detail.

Another method, which is integral control, is presented here. The integral control technique suggests a simple firing pulse timing principle that generates a firing pulse each time the integral of the output ripple voltage waveform becomes instantaneously equal to zero. This technique is presented in detail below.

4.4.1 Integral Control

Rectifier output voltage waveform and integrated output voltage waveform of a 6-pulse rectifier with a negative feedback integral control is shown in Fig 4.19. The basic principle of the integral control method can be explained by the Fig 4.19.
Examining the rectified output voltage waveform, it is seen that during the interval between any two successive firing points, the net voltage-time integral of this wave is zero. In other words, area A, above the Vref, and area B, below Vref are equal. If rectified output voltage is applied to the input of an integrator circuit, integrated output voltage waveform as shown in Fig 4.19 will be obtained at the output. The integrated output voltage waveform shows that output of the integrator circuit is zero at each firing point.

Since the mean output voltage is equal to the reference voltage, the rectified output voltage waveform can be obtained by subtracting the reference from the actual output waveform. This scheme ensures that each and every segment of the output ripple voltage has zero mean value, and therefore between every two firing points the mean value of the output voltage is equal to reference voltage. Thus a very tight pulse by pulse control is obtained over the rectified output voltage waveform and in fact this principle automatically provides a closely regulated closed-loop control of the output voltage.
The integral control principle offers two important advantages. First, it is insensitive to changes in the supply frequency because the firing pulses are generated at the zero values of the integral of the rectified output voltage. This means that, although the amplitude of the waveform of the integral of the output voltage changes with changing supply frequency, its zero values always correspond with the desired firing instants. Second, any spikes which appear on the output voltage waveform of the rectifier do not have any noticeable effect upon the timing of the firing pulses, since the integral value of the output ripple voltage is hardly influenced by these spikes. This is not the case with other types of pulse timing control, in which the timing of the firing pulses is determined from the instantaneous intersection of a timing waveform with the reference voltage.

A functional diagram of a 6-pulse firing pulse generator using the integral control principle is shown in Fig. 4.20. The difference between the reference voltage and the rectified output voltage wave of the rectifier is fed as an input to the integrator. The output of the integrator is a replica of the output ripple voltage and therefore the output voltage of the integrator is the integral of the rectifier ripple voltage which has a zero-value at each firing point. This zero-value, by the action of a comparator, is translated into a timing clock pulse and the clock pulse generator. The clock pulses are fed as the trigger input to a 6-stage firing pulse generator circuit. The function of the firing circuit is both to shape and distribute the output firing pulses. Each successive clock pulse produces a firing pulse in regular sequence, one after another [15].
Figure 4.20 Integral Control Firing Pulse Generator
Chapter Five: Conclusion and Future Work

5.1 Conclusion

Total harmonic distortion, THD, produced by an ideal three-phase 6-pulse rectifier is 31.1% and that produced by an ideal three-phase 12-pulse rectifier is 15.3%. Table 5.1 below shows the current distortion limit that a consumer can inject to the grid.

Table 5.1 IEEE Std 519-1992 Maximum odd harmonic current limits

<table>
<thead>
<tr>
<th>Isc/IL</th>
<th>n&lt;11</th>
<th>11≤n&lt;17</th>
<th>17≤n&lt;23</th>
<th>23≤n&lt;35</th>
<th>35≤n</th>
<th>THD</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 20</td>
<td>4.0</td>
<td>2.0</td>
<td>1.5</td>
<td>0.6</td>
<td>0.3</td>
<td>5.0</td>
</tr>
<tr>
<td>20-50</td>
<td>7.0</td>
<td>3.5</td>
<td>2.5</td>
<td>1.0</td>
<td>0.5</td>
<td>8.0</td>
</tr>
<tr>
<td>50-100</td>
<td>10.0</td>
<td>4.5</td>
<td>4.0</td>
<td>1.5</td>
<td>0.7</td>
<td>12.0</td>
</tr>
<tr>
<td>100-1000</td>
<td>12.0</td>
<td>5.5</td>
<td>5.0</td>
<td>2.0</td>
<td>1.0</td>
<td>15.0</td>
</tr>
<tr>
<td>&gt;1000</td>
<td>15.0</td>
<td>7.0</td>
<td>6.0</td>
<td>2.5</td>
<td>1.4</td>
<td>20.0</td>
</tr>
</tbody>
</table>

Above limits are in % for general distribution system, 120 V to 69 kV

The limits for even harmonics are 25% of the odd harmonics

The limits for general distribution system at 69.001 kV to 161 kV are 50% of the values listed above

The first column in Table 5.1 shows the ratio of short circuit current to the load current. The first row represents the weakest grid, gradually turning to stronger as we
move down the table and the last row represents the strongest grid. The last column represents the % THD. A 6-pulse rectifier, which produces 31.1% total current distortion, does not meet the % THD criteria in any of the rows. The highest value of distortion that can be injected where the grid is strongest is 20%. However, a 12-pulse rectifier which produces 15.3% meets the criteria for the bottom two rows when the value of $I_{sc}/I_L$ is between 100-1000 or above 1000. Thus, a 12-pulse rectifier provides a window opportunity.

Similarly, Table 5.2 shows the voltage distortion limit that the utility company must maintain to operate.

**Table 5.2 IEEE Std 519-1992 Voltage distortion limits**

<table>
<thead>
<tr>
<th>Bus Voltage at PCC</th>
<th>Individual Harmonics</th>
<th>THD</th>
</tr>
</thead>
<tbody>
<tr>
<td>69 kV and lower</td>
<td>3.00%</td>
<td>5.00%</td>
</tr>
<tr>
<td>69.001 kV to 161 kV</td>
<td>1.50%</td>
<td>2.50%</td>
</tr>
<tr>
<td>Above 161 kV</td>
<td>1.00%</td>
<td>1.50%</td>
</tr>
</tbody>
</table>

Voltage distortion, $V_h = (I_h/I_{base})x(h)(Z_{sys})x100\%$

Total Harmonic Distortion, THD = $\sqrt{\sum V_h^2}$

**5.2 Future Work**

A practical circuit with cable inductance and transformer inductance should be considered for an 18-pulse rectifier. To achieve an 18-pulse rectifier, three 6-pulse rectifiers can be used. Resultant %THD should be calculated and waveforms for line currents should be obtained. If 18-pulse rectifier does not produce current distortion less than 5%, a 24-pulse rectifier should be investigated.
References


Appendix A: List of Published Papers


Implementation of AC to DC converter Using Thyristor in ATP

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Abstract: Silicon diodes are widely used for converting ac power into dc power. Diodes start conducting when they are forward biased and start producing dc voltage at the output but the output voltage is uncontrolled. With the use of a thyristor, instead of a diode, the output voltage can be controlled to a desired level. A thyristor needs a triggering pulse at the gate, when forward biased, to conduct. By controlling the triggering (firing) angle, the output dc voltage can be controlled effectively. A single phase full-wave controlled bridge ac to dc converter, rectifier, using thyristors is presented in this paper. Behavior of rectifier feeding different kinds of loads is investigated. To obtain the voltage and current waveforms, a program called ATP. Alternative Transients Program, has been utilized. ATP is world’s most widely used electro-magnetic transients program and is available for use to the licensed users free of charge. In this paper, for a phase-controlled thyristor based rectifier, it has been shown that the average value of dc output voltage is controllable and is a function of triggering angle.

Keywords: AC to DC converter, ATP, Phase control, Rectifier, Thyristor

I. Introduction

Present day power electronic semiconductor switches with increased power capability, reduced cost, and increased controllability have made power converters cost effective solution for a number of power conversion applications. To understand the applicability of these power converters, it is necessary to understand the voltage and current capability of currently available power electronic switches. Power electronic semiconductor switches can be classified into following two groups according to their degree of controllability:

1. Uncontrollable - Diodes
2. Controllable - Thyristors

The controllable switches category includes several device types including bipolar junction transistors (BJTs), metal-oxide-semiconductor field effect transistors (MOSFETs), and insulated gate bipolar transistors (IGBTs). Some of these are suitable for low power and low voltage application, while some of these are suitable for high power and high voltage application.

Power electronics and semiconductor power switches in fact can be defined as a branch of electrical engineering that deals with conversion and control of electric power. It is estimated that at least half of the electric power generated in the USA flows through power electronic converters and an increase in this share to almost 100% is expected in the following few decades [1].

Block diagram of electric power conversion is shown in Fig. 1 [1]. It can be seen from Fig. 1 that a power converter converts constant magnitude and frequency input to a variable magnitude and/or frequency output.

As shown in Fig. 1, ac-to-de conversion is accomplished using rectifiers. Rectifiers find wide-range of application in industries, transportation, power transmission, and so on.

![Figure 1. Electric power conversion](image-url)
Implementation of AC to DC converter Using Thyristor in ATP

The silicon diodes are widely used for rectifiers. It is well known fact that a diode starts conduction as soon as its anode to cathode voltage exceeds the threshold voltage when the diode is forward biased. But rectifiers with diodes are not controllable. In some application such as battery charger, it is necessary for the dc voltage to be controllable. When the diodes are replaced by thyristors, conduction does not happen merely after exceeding the threshold voltage when forward biased but also they need a triggering signal at the gate. Anode, cathode, gate of a thyristor are shown in Fig. 2.

A thyristor controlled rectifier works as an uncontrolled diode rectifier when the firing angle, α, of thyristor is zero. Therefore, in this paper, diode converters are not presented. Uncontrollable diode rectifiers are a subset of the controlled rectifiers. By the use phase control, average values of load voltage can be controlled and varied [2]. The application of triggering pulse at the thyristor gate at any desired instant during the period when the thyristor is forward biased to control the magnitude of the dc output voltage is called phase control [3].

Unless otherwise specified, a firing angle, α, of 45° is used in this paper.

The system being investigated in this paper is single-phase full-wave controlled bridge rectifier. Circuit diagram, mathematical expressions, and voltage and current waveforms are presented for each rectifier when feeding the following loads:
1. Resistive load, R
2. Resistive, R, and inductive, L, load
3. Resistive, R, Inductive, L, and EMF

The load electro-magnetic force, EMF, may be either a battery or back emf of a dc motor. Alternative Transients Program, ATP, has been used to model the system and obtain the waveforms [4].

II. Rectifier With R Load

Fig. 2 shows a single-phase bridge rectifier with R load. Anode, cathode, and gate are marked for thyristor T1. Similar is true for thyristors T2, T3, and T4. Once thyristor is fired at gate, while it is forward biased, thyristor starts conducting. Thyristor turns off when current being conducted reaches zero value. V is root mean square, RMS, value of source voltage and Vo is average dc output voltage. Vs is given by £m sin α where £m is peak value of Vs.

In the circuit shown in Fig. 2, T1 and T2 are fired simultaneously at firing angle, α, in positive half cycle of the source. Similarly, T3 and T4 are fired simultaneously at firing angle, π−α, in negative half cycle. Since the load is purely resistive, the voltage and current both go to zero at π, 2π, and so on.

![Figure 2: Single-phase full-wave bridge rectifier with R load.](image-url)

The source voltage and triggering signals are shown in Figure 3a. It can be seen that one cycle of the supply is 16.667 ms i.e. the supply frequency is 60 Hz. Positive cycle is fired at 2.0833 ms i.e. at 45°, 2π+45°, 4π+45°, and so on. Similarly negative half cycle is fired at 10.4167 ms i.e. at 180°+45°, 2π+180°+45°, 4π+180°+45°, and so on.
The output voltage (plotted as $v$) and output current (plotted as $i$) are shown in Figure 3b. The thyristor that is turned on at $\alpha$ is turned off at $\pi$ and the thyristor that is turned on at $2\pi-\alpha$ is turned off at $2\pi$. During positive half cycle of the source voltage thyristor pair T1 and T2 are turned on after the instant of firing pulse and are turned off when the current becomes zero. Thyristor turn off process is known as commutation. Similar is true for thyristor pair T3 and T4 during negative half cycle.

The average output voltage $V_o$ and the average load current $I_o$ can be given as follows [5]-[7]:

$$V_o = \frac{1}{2\pi} \int_\alpha^{2\pi-\alpha} \frac{V_m}{2} \sin \omega t \, d(\omega t) + \int_\alpha^{2\pi-\alpha} V_m \sin \omega t \, d(\omega t)$$

(1)

After solving we get:

$$V_o = \frac{2V_m}{\pi} (1+\cos \alpha)$$

(2)

and the average load current can be given as:

$$I_o = \frac{V_o}{2}$$

(3)

From equations (2) and (3), it can be seen that the maximum value of $V_o$ and hence of $I_o$ occurs when $\cos \alpha = 1$ i.e. $\alpha = 0^\circ$.

The firing angle may thus be defined as the angle corresponding to the duration between the instant of triggering that gives largest average output voltage to the instant that gives any desired average output voltage.

### III. Rectifier With R and L Load

A single phase controlled full-wave bridge rectifier feeding a RL load is shown in Figure 4. The output waveform for this circuit depends on the value of the inductance, $L$, and the firing angle, $\alpha$. 
Since the current cannot change instantaneously in an inductor, thyristor $T_1$ and $T_2$ will continue to conduct even after the voltage zero until the current becomes zero. If the value of inductance is large, $T_1$ and $T_2$ may not turn off at all until thyristors $T_3$ and $T_4$ are fired. Firing of $T_3$ and $T_4$ reverse biases and turns $T_1$ and $T_2$ off. The load current will shift from pair $T_1$ and $T_2$ to pair $T_3$ and $T_4$.

The voltage and current waveforms for an arbitrary $RL$ load are shown in Figure 5. The load voltage $V_o$ becomes equal to the source voltage $Vs$ immediately after thyristors are fired. However, because of the load inductance, the current starts rising gradually from its zero value and attains maximum value after sometime depending upon the value of inductor. It can be seen that at $3.33 \text{ ms}$ i.e. $\pi$ radians $(180^\circ)$, $I_o$ is not zero even though $V_o$ is zero. At some angle $\beta$ $(9.3 \text{ ms})$, $I_o$ reduces to zero and thyristors $T_1$ and $T_2$ turned off. During the period between $\alpha = \pi$ and $\alpha = \beta$, i.e. when the voltage is negative and the current is positive, the magnetic energy stored in the inductor is delivered back to the supply. After $\alpha = \beta$, $V_o=0$ and $I_o=0$. Angle $\beta$ is called extinction angle. Thus, the conduction angle $\gamma$ can be given as $(5)-(7)$:

$$\gamma = (\beta - \alpha)$$

It can be seen from Fig 5 that the output current is zero when $\alpha = \pi$ and $\alpha = \beta$. If the inductance value is large, output current becomes continuous and comparatively ripple free.

The average output voltage $V_o$ can be given as below $(5)-(7)$:

$$V_o = \frac{220}{\pi} \cos \alpha$$

**IV. Rectifier With R, L, And E Load**

A single phase controlled full-wave bridge rectifier feeding a $RLE$ load is shown in Figure 6. Voltage $E$ corresponds to a battery emf or a back emf of a dc motor. Thyristor pair $T_1$ and $T_2$ are fired simultaneously at a while the pair $T_3$ and $T_4$ are fired after $\pi$ radians in each cycle. When pair $T_1$ and $T_2$ is ON, the output voltage is same as the supply voltage. When the pair $T_1$ and $T_2$ is OFF and pair $T_3$ and $T_4$ are not turned ON yet, the output voltage is equal to emf $E$. Figure 7 shows the output voltage and current waveforms. The presence of a
voltage source in the load tends to reverse bias the thyristor during the period when \( \alpha_t = 0 \) and \( \alpha_e = \alpha \). The thyristor will not turn on for a firing angle smaller than a certain value called critical angle, \( \alpha_c \), can be given as

\[
\alpha_c = \sin^{-1} \left( \frac{E}{V_m} \right)
\]

(6)

In this paper, for \( E=50\text{V} \) and \( V_m = V_2 = 120 \text{V} = 169.71 \text{V} \), \( \alpha_c = 17.14^\circ \). Since a firing angle of 45° is used, there is no problem for thyristor to turn on.

![Figure 6. Single-phase full-wave bridge rectifier with R, L, and E load](image)

The average value of the output voltage is given below [5]-[7]:

\[
V_o = \frac{2V_m}{\pi} \cos \alpha
\]

(7)

V. Results

The variation of output voltage, \( V_o \), with respect to firing angle \( \alpha \) is plotted in Fig. 8. It can be noted that the output voltage is positive until the firing angle is less than \( \pi/2 \) (90°) and the output voltage is negative for values of \( \alpha \) greater than \( \pi/2 \). The operation of the converter takes two different forms. For values \( \alpha \) smaller than \( \pi/2 \), the converter operates as a rectifier. During this operation power flows from the supply to the load. But, if \( \alpha \) is greater than \( \pi/2 \) and the load circuit emf \( E \) is reversed, it is possible to transfer the dc power from battery or a running dc motor to the ac supply as ac power. This is equivalent to inversion. Thus, the same converter can operate as a rectifier as well as an inverter depending upon the value of \( \alpha \) and the polarity of the dc source. The converter operating as an inverter makes use of the line voltage for commutation. Therefore, it is called line-commutated inverter.

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![Graph showing Variation of Vo with α](image)

Figure 8. Variation of output voltage with firing angle.

From Fig. 8 a table as shown below can be created. The TABLE 1 below shows an interesting relationship between firing angle, α, and output voltage, Vo.

<table>
<thead>
<tr>
<th>Firing Angle</th>
<th>Output Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>2 Vm/α</td>
</tr>
<tr>
<td>60°</td>
<td>Vm/α</td>
</tr>
<tr>
<td>90°</td>
<td>0</td>
</tr>
<tr>
<td>120°</td>
<td>-Vm/α</td>
</tr>
<tr>
<td>180°</td>
<td>-2 Vm/α</td>
</tr>
</tbody>
</table>

Table 1: Results

In rectifier operation, the average value of output voltage Vo must be greater than the load circuit emf E, while during inverter operation, the load circuit emf must be greater than the average value of the output voltage. Implementation of the inverter circuit is not presented in this paper and is left as a future work.

VI. Conclusion

Single-phase full-wave controlled bridge rectifier with thyristor was implemented successfully in ATP. It was noted that the average value of output voltage is a function of firing angle. Thus, by controlling the firing angle, the output voltage can be controlled effectively. This research will help seniors, graduate students, and design engineers to understand the modeling and working principle of ac to dc converters i.e. rectifiers.

References

[4] [pdf](https://www.jstor.org) accessed September 2012
Ripple Control in AC to DC Converter
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Abstract: When a sinusoidal voltage is converted into dc, the output voltage waveform inherently contains ripples. Ripple is an unwanted ac component in dc output. Smaller value of ripple factor is desirable. Ideal value of ripple factor is zero. Zero ripple factor means a perfectly dc quantity. Undesirable effects of the ripple include equipment heating, increased losses, and reduced equipment life among others. Ripple factor of a single-phase half-wave uncontrolled rectifier is 1.21. This value is unacceptably high. To overcome the problem, a single-phase full-wave uncontrolled rectifier is proposed with a ripple factor of 0.48.

In this paper, the ripple factor for above mentioned rectifiers with a resistive load is presented mathematically and pictorially. The rectifiers are modeled and waveforms are obtained using a computer program called Alternative Transients program, ATP. At the end, results are presented and compared. Ripple factor was improved by a factor of 40% by using a 2-pulse rectifier instead of a 1-pulse rectifier. Theoretically, if the number of pulses is increased to infinity, the ripple factor will reduce to zero giving a perfect dc output. Subsequently, 6-pulse, 12-pulse, and 18-pulse rectifiers will be modeled and advanced studies will be carried out. This paper will help understand the one of the power quality components called ripple in dc output. The paper will be useful for college seniors, graduated students, and power electronics design engineers.

Keywords: AC to DC converter. Rectifier. Diode. Ripple Factor. ATP

I. INTRODUCTION

Power quality components can easily be found for a sinusoidal voltage and current of same frequency. However, in power electronic, the switching devices like diodes are not on for entire cycle. The switches are on for some portion of the cycle and off for other portion. Therefore, the output waveforms from power electronic devices like rectifiers are periodic, but not sinusoidal. Therefore the equations available for pure sinusoidal waveforms cannot be applied in power electronics.

To compare the performance of different types of rectifiers of the same class, an index called ripple factor is investigated in this paper in detail. It is desirable for a rectifier that the voltage and current ripple in the output waveform be as low as possible to maintain a good quality of the output. The ripple factor is ratio of root mean square (rms) value of ac component to average value of dc output [1]. The ac component of the output waveform is obtained by subtracting dc component from the output waveform [2]. Thus, mathematically, ripple factor is given by

\[
R = \frac{\sqrt{V^2 + I^2 \cos^2 \phi}}{2I_a} = \frac{1}{\sqrt{1 + (\frac{I_R}{I_a})^2}}
\]

where, \(I_a\): RMS value of output current
\(I_R\): Average value of output current

II. RIPPLE EFFECTS

Ripple quantity results in many unwanted effects in a dc system. Some of the known effects are explained below [3]

A. IR Loss: For a perfectly dc current, the current will be distributed uniformly across the cross-section of the conductor. However, when the current is alternating or has ac component, the current tends to concentrate closer to the conductor surface. The effect is called skin effect. Skin effect offers higher resistance to the ripple current resulting in higher surface temperature and conductor losses.

B. Stray Heating: Ripple current induces current in the neighboring metal structure or piping per Faraday's law of electromagnetic induction and causes induction heating. Avoiding the induction coupling by increasing the clearance may not always be possible. However, it can be reduced by reducing ripple current itself or by placing a low-impedance shield such that induced current does not produce a overheating in the shield conductor or as a result of lower resistance.

C. Instrumentation and Communication: Induced current in instrumentation cable causes noise. The ratio of signal to noise should be within acceptable level. The induction current can be reduced by using shielded cable.
D. Audible Noise: Changing ripple current causes the audible noise in metal laminations. Sometimes, the ripple frequency matches the natural frequency of the structure and magnifies the mechanical vibration noise. Mechanical noise can be reduced by damping or by reducing ripple current itself.

III. RIPPLE CURRENT MITIGATION

In most applications the effect of ripple current as mentioned in section II should be below a specified value. If the ripple current is above the specified value, it should be mitigated. Following are some of the ways to mitigate the ripple effect from dc output.

A. Increasing the pulse number of rectifier: Higher the number of pulses, lower is the ripple magnitude [4]-[5].

B. Using an Output Filter: If a capacitor is used across the load and an inductor is used in series with the load, the load current will be smoother and ripple effect will be lowered [6].

In this paper, ripple magnitude has controlled by the number of pulses. Higher the number of pulses, lower is the ripple magnitude. In a single-phase half-wave uncontrolled rectifier, the number of pulses is one. Likewise, in a single-phase full-wave uncontrolled rectifier, the number of pulses is two. It is shown in this paper that the ripple factor is 1.21 for a 1-pulse rectifier and 0.48 for a 2-pulse rectifier. To model the rectifier and to obtain voltage and current waveforms, a computer program called Alternative Transients Program (ATP) has been used [7]. Subsequently, 6-pulse, 12-pulse, and 18-pulse rectifiers will be modeled and advanced studies will be carried out. Use of output filter for ripple mitigation is left as a future work and is not presented in this paper.

IV. SINGLE-PHASE HALF-WAVE UNCONTROLLED RECTIFIER

A single-phase half-wave uncontrolled rectifier feeding a resistive load is shown in Figure 1. The diode D conducts during the positive half cycle of supply voltage. Vs and stops conducting during negative half-cycle of the supply voltage forming a 1-pulse rectifier. Source current and load current are represented by i_s and i_o respectively. V_o average output voltage.

![Figure 1. Single-phase half-wave uncontrolled rectifier with R load]

The instantaneous value of Vs is given by

\[ V_s = V_m \sin \omega t \]

where \( V_m \) is the peak value of Vs

Similarly, the instantaneous value of V_o is given by

\[ V_o = V_m \sin \omega t \left\{ \text{0 to } \pi, \text{2} \pi \text{ to } 3\pi, \text{etc.} \right\} \]

And, the instantaneous value of voltage across diode \( V_D \) is given by

\[ V_D = V_s - \left( V_m \sin \omega t \right) \left\{ \text{0 to } \pi, \text{2} \pi \text{ to } 3\pi, \text{etc.} \right\} \]

Voltage input waveform is shown in Figure 2. Output voltage and output currents are shown in Figure 3.
Ripple Control in AC to DC Converter

Figure 2. Supply voltage waveform

Figure 3. Half-wave Rectifier Output Voltage and Current

The average value, \( I_a \), of the load current \( i(t) \) which is periodic in \( 2\pi \) is given by [8]-[10]

\[
I_a = \frac{1}{2\pi} \int_{0}^{2\pi} i(t) \, dt
\]

(7)

After solving above equation we get,

\[
I_a = \frac{V_a}{X} \left( \frac{1}{2R} \right)
\]

(8)

where \( I_a \) is peak value of \( i(t) \).

For the load current \( i(t) \) periodic in \( 2\pi \), the rms current can be given as [7]

\[
I = \frac{1}{2\pi} \int_{0}^{2\pi} i(t)^2 \, dt
\]

(9)

\[
I = \frac{V_a}{2R} = \frac{I_a}{\sqrt{2}}
\]

(10)

It should be noted that the corresponding rms value of the load current for sinusoidal operation is \( I_a/\sqrt{2} \). Thus the degree of distortion, Ripple Factor, in a single-phase half-wave rectified current waveform can be calculated using equation (2).

\[
R = \sqrt{\left( \frac{I_a/2}{I_a/\sqrt{2}} \right)^2 - 1}
\]

(11)

\[
R = 1.21
\]

(12)

If the output wave is perfectly dc, the ripple factor will be zero. Ripple factor of 1.21 is unacceptably high for many industrial applications.

V. SINGLE-PHASE FULL-WAVE UNCONTROLLED RECTIFIER
A single-phase full-wave uncontrolled rectifier feeding a resistive load is shown in Figure 4. Diodes D1 and D2 conduct during the positive half cycle of supply voltage Vs and diodes D3 and D4 conduct during negative half cycle of the supply voltage forming a 2-pulse rectifier. Source current and load current are represented by is and iL respectively. V0 average output voltage. Output voltage and output currents are shown in Figure 5.

![Diagram of single-phase full-wave uncontrolled rectifier with R load](image)

**Figure 4. Single-phase full-wave uncontrolled rectifier with R load**

The instantaneous value of load current can be given as,

\[ i_L(t) = \frac{V_s(t)}{R} \quad (0 \leq t \leq 2\pi) \]

The average value of load current which is periodic in 2π is given by [7]

\[ I_L = \frac{1}{2\pi} \int_0^{2\pi} i_L(t) \, dt \]

After solving above equation we get, \[ I_L = \frac{2V_s}{\pi R} \]

It can be seen that the average value of output current in case of full-wave rectifier is twice the value of in case of half-wave rectifier.

The rms value of the load current can be given as [7]

\[ I = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} i_L^2(t) \, dt} \]

\[ = \frac{V_s}{\sqrt{2R}} \]

\[ = I_m/\sqrt{2} \]

It should be noted that the corresponding rms value of the load current for half-wave rectifier is \( I_m/\sqrt{2} \).

Thus the degree of distortion, Ripple Factor, in a single-phase full-wave rectified current waveform can be calculated using equation (2).
Ripple Control in AC to DC Converter

\[
= \sqrt{\left(\frac{(L_0/2)}{(2L_\infty/\alpha)}\right)^2 - 1}
\]

= 0.48

(19)

Ripple factor of 0.48 is significantly better than that in half-wave rectifier with a ripple factor of 1.21.

VI. RESULTS

The variation of average load current, rms load current, and ripple factor of load current are presented below in a tabular form.

<table>
<thead>
<tr>
<th>Property</th>
<th>Half-wave rectifier</th>
<th>Full-wave rectifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average load current</td>
<td>(I_{0}=27) A</td>
<td>(2I_{0}=54) A</td>
</tr>
<tr>
<td>RMS load current</td>
<td>(I_{rms}=42.4) A</td>
<td>(I_{rms}=60) A</td>
</tr>
<tr>
<td>Ripple factor of load current</td>
<td>(\sqrt{\left(\frac{1}{L_0}\right)^2-1}) = 1.21</td>
<td>(\sqrt{\left(\frac{1}{L_\infty}\right)^2-1}) = 0.48</td>
</tr>
</tbody>
</table>

VII. CONCLUSION

Single-phase 2-pulse rectifier offered better ripple factor than a single-phase 1-pulse rectifier. The ripple factor was improved by 0.48/1.21 = 40%. Theoretically, if the number of pulses is increased to infinity, the ripple factor will reduce to zero giving a perfect dc output. Subsequently, 6-pulse, 12-pulse, and 18-pulse rectifiers will be modeled and advanced studies will be carried out. This research will help seniors, graduate students, and design engineers to understand the modeling and working principle of ac to dc converters i.e. rectifiers.

REFERENCES

Harmonic Analysis for a 6-pulse Rectifier

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Abstract: When sinusoidal voltage is converted into dc voltage in medium-voltage high-power applications, the switching devices used in the rectifier circuit inject harmonic component to the utility grid. The harmonic component causes various problems in the power system. IEEE std 519-1992 limits the amount of harmonics that is acceptable in the power system so that the undesirable effects of harmonic distortion are minimized.

Among various harmonic distortion mitigation techniques, the pulse multiplication technique is investigated in this paper. A 6-pulse rectifier is modeled in Alternative Transients Program (ATP). Voltage and Current waveforms are obtained and the amount of harmonic distortion is calculated. 6-pulse rectifier is a building block for higher order rectifier like 12-pulse, 18-pulse, 24-pulse rectifiers. Results suggest that the total harmonic distortion (THD) produced by an ideal 6-pulse rectifier is higher than the IEEE limit and therefore higher order pulse rectifier must be used to control the THD.

Keywords: AC to DC converter, Rectifier, THD, Thyristor, ATP

I. INTRODUCTION

Medium-voltage high-power drives are widely used in the industry. They are used in the petrochemical industry for pipeline pumps, in the cement industry for fans, in water pumping stations for pumps, in transportation industry for traction applications, mills in the metal industry for steel rolling and in many other industries for various applications [1-8].

These drives are made up of power electronic components like power diodes, power IGBT, power thyristor etc. When these devices are turned on and off at certain frequency, these devices produce non-sinusoidal components called harmonics. These devices act as non-linear load to the utility [9].

IEEE standard 519-1992 requires that the amount of harmonic distortion produced by these drives should be under specified value. Criteria dictate that any individual harmonics should not be more than 3% and total harmonic distortion (THD) should not be more than 5% [10].

Presence of harmonics in power system can give rise to a variety of problems including equipment overheating, reduced power factor, deteriorating performance of electrical equipment, the incorrect operation of protective relays, interference with communication devices, and in some cases, circuit resonance to cause electric apparatus dielectric failure and other type of severe damage [11].

In order to meet the harmonic requirement set by IEEE standard 519-1992, major high-power drive manufacturers around the world are increasingly using multi-pulse rectifier in their drive at front end converter. The rectifiers can be configured as 12-, 18-, and 24-pulse rectifiers powered by a transformer with a number of secondary windings. Each secondary winding feeds a 6-pulse rectifier. To achieve 12-pulse rectifier two 6-pulse rectifiers are used in parallel. Likewise, for 18-pulse, three 6-pulse rectifiers, and for 24-pulse, four 6-pulse rectifiers are connected in parallel [12][15].

The main feature of the multi-pulse rectifier is its ability to reduce the line harmonic distortion. This is achieved by the phase-shifting transformer through which some of the lower order harmonic currents generated by the six-pulse rectifiers are cancelled. In general, higher the number of rectifier pulses, the lower the line current distortion [12][15].

Multi pulse rectifier eliminates the need of using any LC filter or power factor compensators which leads to the elimination of possible LC resonance. The use of phase shifting transformer provides an effective means to block common-mode voltages generated by the rectifier which would otherwise appear on electric motor terminals leading to premature failure of winding insulation [16].

The multi-pulse rectifier can be built using diodes or thyristors as switching device. In case of diodes, dc output voltage cannot be controlled. On the other hand, thyristor provides the ability to control the dc output voltage. Fig 1 shows a general block diagram of medium-voltage drive [17].
II. IDEAL SIX-PULSE RECTIFIER

Fig 2 shows a simplified circuit diagram of a six-pulse thyristor rectifier. The inductance \( L_a \) is the total inductance including the line inductance, transformer reactance, and line reactor between the utility and the rectifier. For the ideal rectifier \( L_a \) is assumed to be zero. On the dc side a choke \( L_d \) is used to make the dc current ripple free [19]. The RC snubber circuits for thyristors are not shown in Fig 2 but are considered in the computer model created in alternative transients program (ATP) [19].

![Simplified circuit diagram of a six-pulse thyristor rectifier](image)

Fig 3 shows typical waveforms of the rectifier, where \( v_a \), \( v_b \), and \( v_c \) are the phase voltages of the utility supply, \( P1 \) thru \( P6 \) are the gate firing pulses for thyristors \( T1 \) thru \( T6 \) respectively and \( \alpha \) is the firing angle of the thyristors.

During interval I, thyristors \( T1 \) and \( T6 \) are conducting assuming \( T6 \) was conducting prior to turn on of \( T1 \). The positive dc voltage is \( v_a \) with respect to ground is \( v_a \) and the negative bus voltage \( v_b \) is equal to \( v_a \). The dc output voltage can be found from \( v_a = v_a - v_b = 0 \). The line currents can be given as \( i_a = i_b = 0 \) and \( i_c = 0 \).

During interval II, thyristor \( T6 \) is turned off after \( T2 \) turns on and the dc current \( i_a \) is commutated from \( T6 \) to \( T2 \). Thus \( T1 \) and \( T2 \) are conducting. The positive dc voltage \( v_a \) is still the same i.e., \( v_a = v_b \), but the negative bus voltage \( v_b \) is equal to \( v_a \). The dc output voltage can be found from \( v_a = v_a - v_b = 0 \). The line currents can be given as \( i_a = i_b = 0 \) and \( i_c = 0 \).

Following the same procedure all the voltage and current waveforms in other interval can be obtained. It should be noted that the number of thyristor and gate is also the sequence of their firing.

The average dc output voltage can be given as [17]

\[ V_d = 1.35 V_{LL} \cos \alpha \]

The equation (1) depicts that the rectifier dc output voltage \( V_d \) is positive when the \( \alpha \) is less than \( \pi/2 \) and becomes negative for an \( \alpha \) greater than \( \pi/2 \). However, the dc current \( I_d \) is always positive irrespective of the polarity of the dc output voltage. The technique to control the dc output voltage by firing angle, \( \alpha \), is called phase-control technique [20].

When the rectifier produces positive dc voltage, the power is delivered from utility to the load. With a negative dc voltage, the rectifier operates as an inverter and the power is fed from the load back to utility. This often takes place during rapid deceleration when the kinetic energy of the rotor and its mechanical load is converted to the electric energy by the converter working in inverter mode and are used for dynamic braking.

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The line current $i_l$ in Fig 3 can be expressed in a Fourier series as [17]

$$i_l = \frac{2L}{\pi} I_d \left( \sin(\alpha - \varphi) - \frac{1}{3} \sin(3(\alpha - \varphi)) - \frac{1}{5} \sin(5(\alpha - \varphi)) - \frac{1}{7} \sin(7(\alpha - \varphi)) - \frac{1}{11} \sin(11(\alpha - \varphi)) - \frac{1}{13} \sin(13(\alpha - \varphi)) - \frac{1}{17} \sin(17(\alpha - \varphi)) - \frac{1}{19} \sin(19(\alpha - \varphi)) + \cdots \right)$$

where $\varphi$ is the phase angle between the supply voltage $V_s$ and the fundamental frequency line current $I_d$.

The rms value of $i_l$ can be given as [17]

$$I_l = \frac{2}{\sqrt{2}} I_d = 0.816 I_d$$

From which the total harmonic distortion, THD, for the line current $i_l$ is [17]

$$\text{THD} = \frac{\sqrt{\sum_{n=2}^{\infty} I_n^2}}{I_1} = \frac{0.816 I_d}{0.707 I_d} = 0.311$$

where $I_n$ is the rms value of $i_n$.

i.e. THD = 31.1%

III. CONCLUSION AND FUTURE WORK

The waveforms of line currents $i_1$, $i_2$, and $i_3$ are not sinusoidal as shown in Fig 3. If these waveforms were perfectly sinusoidal, the total harmonic distortion, THD, would be zero. From equation (4) it can be seen that the THD of an ideal six-pulse thyristor rectifier is 31.1%. This value is significantly higher than the IEEE limit of 5%.

In low-voltage applications, THD can also be mitigated using shunt reactor but in a medium-voltage application, using shunt reactor to control the THD is not cost effective, therefore pulse multiplication method is widely used. However, reactance due to cable inductance and transformer inductance will always be there in medium-voltage application and the THD will be mitigated to some extent. But to achieve the THD value under IEEE limit, pulse multiplication must be used.

In future, a practical circuit with cable inductance and transformer inductance will be considered for a 12-pulse and 18-pulse rectifier. To achieve a 12-pulse rectifier, two 6-pulse rectifiers will be connected in...
parallel and to achieve an 18-pulse rectifier, three 6-pulse rectifiers will be connected in parallel. Resultant THD for both the cases will be calculated and waveforms for line currents will be obtained. If neither of these rectifiers produces less than 5% THD, a 24-pulse rectifier should be investigated.

REFERENCE


Filter Design for AC to DC Converter

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University of Denver

ABSTRACT: AC power is available commercially at low cost. DC power is relatively expensive to produce. Therefore a method for changing ac to dc is needed in relatively less expensive way as a source of dc power. The method presented in this paper to convert ac power to dc power is a full-wave bridge rectifier. But the output from the rectifier contains undesirable ripple. In many applications, the ripple has to be under specified limit. To control the ripple, the method suggested in this paper is utilizing a filter circuit. Filter circuits use either capacitor or inductor or both to limit the ripple. Mathematical formula to calculate the value of filter capacitor or inductor is available when they are used alone but when both capacitor and inductor are used together, there is no mathematical formula available to calculate their values for controlling the amount of ripple. In this paper, the ATP model for the case when a filter consists of either only capacitor or only inductor is verified with mathematical formulas. After verification of the computer model with mathematical formula, a methodology is presented to design a combined capacitor-inductor filter to control the ripple at specified level.

Keywords: AC to DC converter, ATP, Diode, Filter, Rectifier, Ripple Factor

I. INTRODUCTION

AC power is available commercially at low cost. DC power is more expensive to produce; therefore a method of changing ac to dc is needed as an inexpensive dc source. AC power can be converted to DC power using rectifiers [1]. When ac power is converted to dc power using rectifiers, dc output contains undesirable ac components called ripple. Many rectifier applications require that the ripple do not exceed a specified value. If the ripple exceeds the specified value, different unwanted effects appear in the system. Some of the unwanted effects are stray heating, audible noise etc [2]. Ripple can be mitigated using an output filter [3]. When a capacitor is used alone or an inductor is used alone as a filter, there is mathematical expression available for calculating the values of capacitor or inductor for controlling the ripple under the specified value, but when both capacitor and inductor are used together, there is no mathematical formula available to calculate their values [3]. A computer model is developed and verified in this paper to find out the values of capacitor and inductor when used together for controlling the ripple under specified value.

The ripple factor is a ratio of the rms value of the ripple voltage Vrms to the average value Va at the output of a rectifier filter [4]. Fig 1 indicates the parameters used for determining the ripple factor graphically. It is assumed that the ripple voltage has sinuosoidal waveform. The formula for determining the percentage of ripple is:

\[ \text{Percentage of ripple} = \left( \frac{\text{RMS value of ripple}}{\text{Average DC output}} \right) \times 100 \]

where, \( V_{\text{rms}} = 0.707 \times V_p \)
\[ V_p = \text{peak value of ripple voltage} \]

A circuit that minimizes or eliminates the ripple component from the rectified output is called a filter. Filter systems in general are composed of a capacitor, an inductor, or both. Capacitor filters are used for low-power applications. On the other hand, inductor filters are used in high-power applications [5]. Depending upon the passive element used, the filters can be classified as
1. Capacitor filter
2. Inductor filter
3. Capacitor-Inductor filter

In this paper, each of above filters is modeled for full-wave bridge rectifier and waveforms are obtained in Alternative Transients Program (ATP) [6].

II. CAPACITOR FILTER

A rectifier circuit without a filter produces pulses at the output. The fluctuations can be reduced if some of the energy can be stored in a capacitor while the rectifier is producing pulses and is allowed to discharge from the capacitor between pulses [4].

![Figure 2a. Full-wave bridge rectifier with a capacitor filter](image)

Fig 2a shows a full-wave bridge rectifier with a capacitor filter. \( V_s \) is rms value of source voltage and is equal to 120 V. Frequency of the source is 60 Hz. Time varying source current is represented by \( i_s \) and load current by \( i \). Diodes are represented as D1, D2, D3, and D4. During positive half cycle of the source, D1 and D2 conduct. During negative half cycle, D3 and D4 conduct. Filter capacitor is represented by \( C \) and the load is represented by \( R \). The value of load is 10 Ohm. Value of \( C \) is calculated in this section for different values of ripple. The output waveforms are plotted in ATP for 3% ripple and the ATP model is verified with the formula for calculating the ripple.

![Figure 2b. Full-wave output without a filter](image)

Fig 2b shows the voltage output of the full-wave bridge rectifier across point POS and NEG of Fig 2a without the filter capacitor. This pulsating voltage is applied to the filter capacitor represented by \( C \) in Fig 2a. The capacitor will react to any change in circuit voltage. Because the rate of capacitor charging is limited only
by the impedance in the source side which is pretty low, the voltage across the capacitor can rise nearly as fast as the half sine wave voltage from the rectifier. In other words, the RC charge time is relatively short. The charge on the capacitor represents storage of energy. When the rectifier output drops to zero, the voltage across the capacitor does not fall immediately. Instead, the energy stored in the capacitor is discharged through the load during the time that the rectifier is not supplying energy.

The voltage across the capacitor and the load falls off very slowly if it is assumed that a large capacitor and relatively large value of load resistance are used. In other words, the RC discharge time is relatively long. Therefore, the amplitude of the ripple is greatly decreased as shown in Fig 2c. Waveform v-POS -NEG represents the output without a filter and waveform v-LOAD -NEG represents the output with a filter.

After the capacitor has been discharged, the rectifier does not begin to pass current until the output voltage of the rectifier exceeds the voltage across the capacitor. This occurs at 10 ms in Fig 2c. Current flows in the rectifier until slightly after the peak of the half sine wave at 13 ms. At this time the sine wave is falling faster than the capacitor can discharge. A short pulse of current beginning at 10 ms and ending at 13 ms is therefore supplied to the capacitor by power source.

The average voltage, $V_0$, of the full-wave bridge rectifier output is $0.636V_m$ [5]. $V_m$ is peak value of $V_a$ and is equal to $\sqrt{2}V_a$. From Fig 2b it can be seen that the average value is $0.636V_m=(120 V)= 108 V$. Because the capacitor absorbs energy during the pulse and delivers this energy to the load between pulses, the output voltage can never fall to zero. Hence the average voltage of the filter output as shown in Fig 2c is greater than that of the unfiltered output shown in Fig 2b. However, if the resistance of the load is small, a heavy current will be drawn by the load and the average output voltage will fall. Also, the filter capacitor acts like a short circuit across the rectifier while the capacitor is being charged. Due to these reasons, a simple capacitor filter is not suitable for rectifiers in higher power applications.

In practice, the ripple factor can be found from [5]

$$RF = \frac{1}{\sqrt{2}} \left(\frac{2}{RC-1}\right)$$

(1)

where $f$ is the output ripple frequency.

From Fig 2b, it can be seen that there are 2 output pulses for each cycle of supply voltage. Therefore the frequency of output ripple, $f$, is 120 Hz. Using equation (1) the value of capacitor is calculated for different values of ripple and the results are tabulated in Table 1.

<table>
<thead>
<tr>
<th>RF %</th>
<th>$R$ Ohms</th>
<th>$f$ Hz</th>
<th>C mF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>120</td>
<td>29.88</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>120</td>
<td>15.148</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>120</td>
<td>10.238</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>120</td>
<td>7.782</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>120</td>
<td>6.309</td>
</tr>
</tbody>
</table>

Using the capacitor value of 10.238 mF for 3% ripple, the voltage waveforms are plotted in Fig 2d.
Filter Design for AC to DC Converter

![Diagram of a circuit with a capacitor and inductor]

Figure 2d. Output voltage waveform for 3% ripple with 10.238 mF capacitor.

From Fig 2d, $V_{max} = 170$ Volts, $V_{min} = 156$ Volts, $V_o = 163$ Volts, $V_p = 7$ Volts.
Rms of ripple = 0.707 x 7 Volts = 4.95 Volts.
Ripple percentage = ($V_{rms}/V_o$) x 100

Since the same value of percentage ripple is obtained from simulation as well as from the equation (1), the ATP model has been verified with formula given in equation (1).

III. INDUCTOR FILTER

From above we know that a capacitor is a device that reacts to variation in voltage and are connected across the load. The inductor is a device that reacts to changes in current. The inductor causes delay in current since the current is same in all parts of the series circuit, an inductor $L$ is connected in series with the load as shown in Fig 3a. Circuit nomenclature and circuit parameters are the same as those in capacitor filter. Output of the rectifier without the inductor filter will be the same as shown in Fig 2b. The sequence of operation of diodes is also the same as those in capacitor filter. In this section the value of inductor required for different value of ripple are calculated and results are tabulated in Table 2. The output waveforms are plotted using ATP for 3% ripple and the ATP model has been verified with the formula given in equation (2).

![Diagram of a full-wave bridge rectifier with an inductor filter]

Figure 3a. Full-wave bridge rectifier with an inductor filter

The use of an inductor prevents the current from building up or dying down too quickly. If the inductor is made large enough, the current becomes continuous and nearly constant. The inductor prevents the current from ever reaching the peak value which would otherwise be reached without a filter inductor. Consequently, the output voltage never reaches the peak value of the applied sine wave. Thus, a rectifier whose output is filtered by an inductor cannot produce as high a voltage as that could be produced by a rectifier filtered by a capacitor. However, this disadvantage is partly compensated because the inductor filter permits a larger current without a serious change in output voltage. This is the reason that an inductor filter is suitable for high power applications.

A typical output waveforms are shown in Fig 3b. Waveform that has a peak value of 17 A is the current without the inductor filter. The waveform that has a peak value of 12.6 A is the current with the inductor filter.

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In practice, the ripple factor can be found from [5]

$$RF = \frac{0.4714}{\sqrt{1 + 4(nf/LR)^2}}$$

where $f$ is the input frequency.

It should be noted that in equation (2) for inductor filter, input line frequency is used for calculation unlike the case of capacitor filter where output ripple frequency was used in equation (1). Using formula shown above, the value of inductor is calculated for different values of ripple and the results are tabulated in Table 2.

<table>
<thead>
<tr>
<th>RF %</th>
<th>R Ohms</th>
<th>f</th>
<th>L mH</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>60</td>
<td>624.821</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>60</td>
<td>312.200</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>60</td>
<td>207.898</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>60</td>
<td>155.677</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>60</td>
<td>124.287</td>
</tr>
</tbody>
</table>

Using the inductor value of 207.898 for 3% ripple, the current waveform is plotted as shown in Fig 3c. From Fig 3c, $I_{max} = 11.25$ A, $I_{min} = 10.34$ A, $I_{p} = 10.79$ A, $I_p = 0.46$

I rms of ripple $= 0.707 \times 0.46$ A $= 0.325$ A

Ripple percentage $= (I_{rms}/I_{p}) \times 100$

$= 3\%$

Since the same value of percentage ripple is obtained from computer simulation as that was calculated using equation (2), the ATP model has been verified with the formula.
IV.  CAPACITOR-INDUCTOR FILTER

A capacitor-inductor filter is used to improve the filtering action of rectified voltage and current. We saw in above sections that the capacitor alone or the inductor alone cannot perform the filter action satisfactorily as former is suitable for low-power applications and the latter is suitable for high-power applications. However, if both the capacitor and inductor are combined, they produce high quality dc voltage and current. The function of the capacitor is to smooth out the variations in voltage while the inductor is used to smooth out the variations in current. Because of the uniform flow of current, the capacitor-inductor filter is used widely in high-power applications. Fig 4a shows a full-wave bridge rectifier with a capacitor-inductor filter.

![Figure 4a. Full-wave bridge rectifier with a capacitor-inductor filter](image)

From previous sections we know that, if used alone, a 10.238 mF capacitor is needed to reduce the ripple to 3% in the circuit under consideration. Similarly, if used alone, a 207.898 mH inductor is needed to reduce the ripple to 3%. The aim of this section is to find out the value of a capacitor and an inductor used together in the way shown in Fig 4a to reduce the ripple to 3%.

The approach taken to design the filter is to select an arbitrary value of capacitor, about 25% of the value that was needed to reduce the ripple to 3% when used alone and then vary the value of inductor until a 3% ripple is obtained. The values are obtained by trial and error using the verified ATP model. The results from the simulations are tabulated in Table 3. Terminologies used in Table 3 are same as those shown in Fig 1. Terminologies in Fig 1 are shown for voltage ripple but they are also applicable for current ripple.

<table>
<thead>
<tr>
<th>C (mF)</th>
<th>L (mH)</th>
<th>I_max Amp</th>
<th>I_min Amp</th>
<th>I_avg Amp</th>
<th>Peak ripple</th>
<th>I rms Amp</th>
<th>I ripple %</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>5</td>
<td>13.20</td>
<td>11.30</td>
<td>12.35</td>
<td>0.85</td>
<td>0.60</td>
<td>4.87</td>
</tr>
<tr>
<td>2.5</td>
<td>10</td>
<td>11.38</td>
<td>10.28</td>
<td>10.83</td>
<td>0.55</td>
<td>0.39</td>
<td>2.59</td>
</tr>
<tr>
<td>2.5</td>
<td>15</td>
<td>11.18</td>
<td>10.46</td>
<td>10.82</td>
<td>0.36</td>
<td>0.25</td>
<td>2.35</td>
</tr>
<tr>
<td>2.5</td>
<td>20</td>
<td>11.08</td>
<td>10.55</td>
<td>10.82</td>
<td>0.26</td>
<td>0.19</td>
<td>1.73</td>
</tr>
<tr>
<td>2.5</td>
<td>25</td>
<td>11.02</td>
<td>10.60</td>
<td>10.81</td>
<td>0.21</td>
<td>0.15</td>
<td>1.37</td>
</tr>
</tbody>
</table>

Current ripple in percentage as a function of inductor in mH is plotted in Fig 4b. From Fig 4b it is clear that 3% current ripple is obtained when a capacitor of 2.5 mF and an inductor of 12.5 mH are used. Using this value the voltage and current waveforms are obtained from ATP as shown in Fig 4c.

![Figure 4b. Percentage Current ripple vs Inductance](image)
In Fig. 4c there are three different waveforms plotted. There are two voltage waveforms and one current waveform. Voltage waveforms are plotted on left y-axis and the current waveform is plotted on right y-axis. For clarity, v-axis is shown only between 0.05 sec to 0.10 sec. The waveform v: LOAD → NEG is the output voltage waveform from the rectifier before the filter. The waveform v: LOAD → NEG is the load voltage waveform after the filter. The waveform v: LOAD → NEG is the current waveform through the load. It should be noted that, since the load is purely resistive, the load current waveform follows the load voltage waveform. For percentage ripple calculation, we can use either load voltage waveform or load current waveform; both will end up with the same result.

Using voltage waveform from figure 4c, we have:
- \( V_{\text{max}} = 112.57 \text{ Volts} \), \( V_{\text{min}} = 103.90 \text{ Volts} \), \( V_o = 108.24 \text{ Volts} \), \( V_p = 4.34 \text{ Volts} \).
- \( \text{Rms of ripple} = 0.707 \times 4.34 \text{ Volts} = 3.07 \text{ Volts} \).
- \( \text{Ripple percentage} = \left( \frac{\text{Rms of ripple}}{V_o} \right) \times 100 = 3\% \)

Using current waveform from Fig. 3c, we have:
- \( I_{\text{max}} = 11.26 \text{ A} \), \( I_{\text{min}} = 10.39 \text{ A} \), \( I_o = 10.83 \text{ A} \), \( I_p = 0.44 \).
- \( \text{Rms of ripple} = 0.707 \times 0.44 \text{ A} = 0.311 \text{ A} \).
- \( \text{Ripple percentage} = \left( \frac{\text{Rms of ripple}}{I_o} \right) \times 100 = 3\% \)

Percentage ripple obtained from both the voltage waveform as well as the current waveforms are same. Therefore, it can be noted that the Fig. 1 is applicable for both the voltage ripple as well as the current ripple.

V. RESULTS

The values of capacitor and/or inductor needed for various type of filter to limit the dc ripple to 3% is shown in Table 4. It is clear from Table 4 that significantly larger value of capacitor or inductor is needed when they are used alone to achieve the same result than their values when they are used together. Capacitor needed when used alone is 10.238 mF and Inductor needed when used alone is 207.898 mH for 3% ripple. On the other hand, when they are used together, the value of capacitor needed is 2.5 mH (compared to 10.238 mF) and the value of inductor needed is 12.5 mH (compared to 207.898 mH) for the same percentage of ripple.

<table>
<thead>
<tr>
<th>Filter Type</th>
<th>Capacitor in mF</th>
<th>Inductor in mH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacitor Filter</td>
<td>10.238</td>
<td>N/A</td>
</tr>
<tr>
<td>Inductor Filter</td>
<td>N/A</td>
<td>207.898</td>
</tr>
<tr>
<td>Capacitor-Inductor Filter</td>
<td>2.500</td>
<td>12.500</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

ATP model and the mathematical equations for percentage ripple for capacitor filter or inductor filter were verified. Based on verified ATP model, a capacitor-inductor filter was designed to achieve the specified ripple percentage. It has been shown that the effective control of ripple can be achieved by choosing proper values of capacitor and inductor for a filter in AC to DC rectifier.
Since there is no mathematical formula available for designing a combined capacitor-inductor filter, the verified ATP model will help seniors, graduate students, and design engineers to design a filter to limit the ripple to a specified value in an AC to DC converter.

REFERENCES
Feedback Controller for a 3-Phase 6-Pulse Rectifier

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ABSTRACT

In all phase controlled thyristor bridges, the dc output voltage is a function of supply line to line voltage and the phase angle of gate firing signal. In on ac to dc converter, it is desired to obtain a constant dc output voltage, in spite of disturbances. Some of the disturbances are due to change in supply voltage, supply frequency, or load current, and due to harmonics produced by converter itself. The values of circuit parameters are also within a certain tolerance. So in high-volume production of rectifiers, the rectifier output voltage lies in some distribution. It is desired to have the output voltage within a range. However, this is not practical to achieve the dc output voltage within the range without the use of a negative feedback controller. Thus, we cannot just set a fixed firing angle for thyristors and obtain a desired dc output voltage under all conditions. The idea behind the use of negative feedback controller is to design a circuit that automatically adjusts the firing angle to obtain desired dc output voltage regardless of the disturbances. In this paper a 3-phase 6-pulse rectifier is designed first with the independently running pulse generators and then a negative feedback integral control is applied to the rectifier. Alternative Transients Program ATP has been used to model the rectifier and negative feedback system, and to obtain thyristor firing signals, input voltage and current waveforms, and output voltage waveform.

KEYWORDS: Integral Control, Negative Feedback, Phase Control, Rectifier, Thyristor.

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I. INTRODUCTION

The medium-voltage high-power rectifiers find their use in various industrial plants. Their application are found for pipeline pumps in petrochemical industry, for steel rolling mills in metal industry, for pumps in water pumping stations, for fans in cement industry, for traction in locomotive industry, and in many other applications [1-8].

Fig 1.1 shows a general block diagram of a typical medium-voltage high-power drive [9].

![General block diagram of the MV drive](image)

Figure 1. General block diagram of the MV drive

The input is 3-phase utility supply which is converted to dc voltage by the rectifier shown above. The dc voltage magnitude can be fixed or adjustable depending upon the power electronic switches that are used for switching. Multipulse silicon controlled rectifiers, SCR, multi-pulse diode rectifiers, or pulse-width-modulated (PWM) rectifiers are commonly used rectifier topologies.

In multipulse silicon controlled rectifier, SCR, or thyristor, the output dc voltage, \( V_d \), is a function of input line to line voltage, \( V_{lin} \), and the firing angle, \( \alpha \), of thyristor as given below [10]

\[
V_d = 1.35 \frac{V_{lin}}{2} \cos \alpha
\]

Thyristor firing pulses can be generated from the independently running pulse generators. In practice, it is not possible to achieve an independently running firing circuit that precisely maintains the desired phase control of...
firing pulses over the time. As time passes, the control angle either advances or retards gradually by increasing or decreasing the dc output voltage. In order to overcome this problem, a negative feedback integral control system can be used so that a closely regulated dc output voltage can be achieved and the output will not change gradually over the time. In negative feedback integral control system, the dc output voltage is compared with the desired voltage i.e. reference voltage and the error signal is processed to get firing angles such that a desired dc output voltage is obtained. In a negative feedback integral control system, the function of the firing pulse generator is to deliver correctly timed, properly shaped, firing pulses to the gates of the thyristors.

II. 3-PHASE 6-PULSE RECTIFIER

Fig 2 shows a simplified circuit diagram of a three-phase six-pulse thyristor rectifier. The inductance $L_a$ is the total inductance including the line inductance, transformer reactance, and line reactor between the utility and the rectifier. For the ideal rectifier $L_a$ is assumed to be zero. On the dc side a choke $L_d$ is used to make the dc current ripple free [10-13]. The RC snubber circuits [14] for thyristor are not shown in Fig 2 but are considered in the computer model created in alternative transients program (ATP) [15].

![Figure 2. Simplified circuit diagram of a 3-phase 6-pulse thyristor rectifier](image)

Fig 3 shows a typical dc output voltage of a 3-phase 6-pulse rectifier shown in Fig 2. The equation (1) in section 1 depicts that the rectifier dc output voltage $V_d$ is positive when $\alpha$ is less than $\pi/2$ and becomes negative for a greater than $\pi/2$. The technique to control the dc output voltage by controlling the phase of firing pulse, $\alpha$, is called phase-control technique [11].

![Figure 3. Typical dc output voltage with ac ripple](image)

The power is delivered from utility to the load when the rectifier produces positive dc voltage. With a negative dc voltage, the rectifier operates as an inverter and the power is fed from the load back to utility. This often takes place during rapid deceleration when the kinetic energy of the rotor and its mechanical load is
converted to the electric energy by the converter working in inverter mode and are used for dynamic braking. The irrespective of the polarity of the dc output voltage, the dc current $I_d$ is always positive.

### III. NEGATIVE FEEDBACK CONTROLLER

There are two main types of feedback control system, a positive feedback system and a negative feedback system. In positive feedback system, the output variable and the control set point are added. In negative feedback system, the output variable is subtracted from the set point. The negative feedback system is more stable than positive feedback system [16]. In a negative feedback system, a Proportional Integral Derivative, PID, control is the most widely used control system in industry. In proportional branch, the error signal, the difference between the reference and the output, is multiplied by a constant. The proportional control is responsible for controlling the peak overshoot in the system. In integral branch, the error signal is integrated. The integral control is responsible for controlling steady-state error. In derivative branch, the error signal is differentiated. The derivative control is responsible for response time or settling time of the system. A negative feedback with integral control is presented in this paper to control the steady-state error in dc output voltage of a 3-phase 6-pulse rectifier by controlling the thyristor firing angle. Rectifier output voltage waveform and the waveform of integrator output is shown in Fig 4. The basic principle of the integral control method can be explained by the Fig 4.

![Figure 4](image)

**Figure 4. Basic Principle of the Integral Control**

Examining the ac ripple of rectified output voltage, it is seen that during the interval between any two successive firing points, the net voltage-time integral of this wave is zero. In other words, area A, above the $V_{ref}$, and area B, below $V_{ref}$ are equal. If the ac ripple is applied to the input of an integrator circuit, the output waveform of the integrator will be same as the one given in Fig 4. The output waveform from integrator shows that its value is zero at each firing point. Since the mean output voltage is equal to the reference voltage, the ac ripple waveform can be obtained by subtracting the reference voltage from the actual rectified output waveform. This scheme ensures that each and every segment of the output ripple voltage has zero mean value, and therefore between every two firing points the mean value of the output voltage is equal to reference voltage. Thus a very tight pulse by pulse control is obtained over the rectified output voltage waveform and in fact this principle automatically provides a closely regulated closed-loop control of the output voltage [17].

The integral control principle offers two important advantages. First, it is insensitive to changes in the supply frequency because the firing pulses are generated at the zero values of the integral of the ac ripple voltage. This means that, although the amplitude of the waveform of the integral of the error voltage changes with changing supply frequency, its zero values always correspond with the desired firing instants. Second, any spikes which appear on the output voltage waveform of the rectifier do not have any noticeable effect upon the timing of the firing pulses, since the integral value of the output ripple voltage is hardly influenced by these spikes. This is not the case with other types of pulse timing control, in which the timing of the firing pulses is determined from the instantaneous intersection of a timing waveform with the reference voltage [17]. A functional diagram of a 3-phase 6-pulse firing pulse generator using the integral control principle is shown in Fig. 5. The difference between the reference voltage and the rectified output voltage wave of the rectifier is fed as an input to the integrator. The output of the integrator is a replica of the output ripple voltage and therefore the output voltage of the integrator is the integral of the rectifier ripple voltage which has a zero-value at each firing point. This zero-value, by the action of a comparator, is translated into a timing clock pulse and the clock
Feedback Controller for a 3-Phase 6-Pulse Rectifier

The clock pulses are fed as the trigger input to a 6-stage firing pulse generator circuit. The function of the firing circuit is to shape and distribute the output firing pulses. Each successive clock pulse produces a firing pulse in regular sequence, one after another [17].

IV. RESULTS

Fig. 6 shows waveforms of the rectifier shown in Fig. 2 with a negative feedback integral control, where \( v_u, v_d, \) and \( v_c \) are the input phase voltages of the utility supply, \( P1 \) thru \( P6 \) are the gate firing pulses for thyristors \( T1 \) thru \( T6 \) respectively and \( \alpha \) is the firing angle of the thyristors. It should be noted that the number of thyristors and gate is also the sequence of their firing. The value of line to line voltage, \( V_{UL} \), is 460 \( V_{ac} \), firing angle of 30°, and from equation (1) the reference voltage is set at 538 \( V_{ac} \).

![Figure 5. Integral Control Firing Pulse Generator](image)

During interval I, thyristors \( T1 \) and \( T6 \) are conducting assuming \( T6 \) was conducting prior to turn on of \( T1 \). The positive dc voltage \( v_u \) with respect to ground is \( V_u \) and the negative bus voltage \( v_d \) i.e. equal to \( V_d \). The dc output voltage can be found from \( v_u = v_d - v_{nab} \). The line currents can be given as \( i_1 = i_L, i_2 = i_L \), and \( i = 0 \).

During interval II, thyristor \( T6 \) is turned off after \( T2 \) turns on and the dc current \( i_3 \) is commutated from \( T6 \) to \( T2 \). Thus \( T1 \) and \( T2 \) are conducting. The positive dc voltage \( v_u \) is still the same i.e. \( v_u = v_d \), but the negative bus voltage \( v_d \) is equal to \( v_d \). The dc output voltage can be found from \( v_{nab} = v_u - v_{nab} \). The line currents can be given as \( i_1 = i_L, i_2 = i_L, \) and \( i_3 = i_L \).

Following the same procedure all the voltage and current waveforms in other interval can be obtained.

V. CONCLUSION

A negative feedback control system with integral control principle is explained and applied to a 3-phase 6-pulse rectifier. The control system as well as the rectifier is modeled in ATP. The controller is designed to control the dc output voltage by adjusting the phase angle of thyristor firing signals. The generated firing pulses, the dc output voltage, as well as input current and voltage waveforms obtained from ATP are in agreement with the theory presented. Thus a successful implementation of negative feedback control system is achieved to control the steady-state error of a 3-phase 6-pulse rectifier by integral control method.

![Graphs showing Input Voltage, Integrator output, Gate Signal P1-P6](image)
Feedback Controller for a 3-Phase 6-Pulse Rectifier

![Diagram of a 3-Phase 6-Pulse Rectifier](image)

Figure 6. Waveform of an ideal six-pulse thyristor rectifier operating at α=30°

REFERENCES

Appendix B: ATP File
--- 75 cards of disk file read into card cache cells. 1 onward.

Consult the 860-page ATP Rule Book of the Can/Am EMTP User Group in Portland, Oregon, USA. Source code date is 1091331 INTEGER words. VARDIM List Sizes follow: 6002
10K 10K 900 400K 1200 12K
220K 2000 720 2K 7200 510 90K 800 90 254 120K 100K 3K 10K 12K 12K
45K 260K 600 210K 410 1900

Descriptive interpretation of input data cards. Input data card images are shown below, all 80 columns, character by character:

01234567890123456789012345678901234567890123456789012345678901234567890

Comment card. KOMPAR > 0.

C

data: C:\\USER\SUSE\DES\A\PERSONAL\FORTH\PAPERS\5P\SCHEMATIC\ATP
Marker card preceding new EMTP data case. BEGIN NEW DATA CASE

Comment card. KOMPAR > 0.

C Generated by ATPDRAW July, Sunday
21, 2013
Comment card. KOMPAR > 0. C A Bonneville Power Administration program
Comment card. KOMPAR > 0. C by H. K. Hendalen at SFS/HYDRO -
NORWAY 1999-2009
Comment card. KOMPAR > 0.

C

Comment card. KOMPAR > 0.

C dT = Tnat = Xopt = < Opt >

Misc. data. 5.000E-06 1.000E-01 0.000E+00 5.00E-06
Misc. data. 500 1 1 1 1 1 0 1 0 1 0 0 500 1 1 1 1
Printout : 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Electric network, too. But TACS data first...

TACS source. 1.000E-00 5.000E-04 1.667E-02 123P1
.TACS source. 1.000E-00 5.000E-04 1.667E-02 123P1
.TACS source. 1.000E-00 5.000E-04 1.667E-02 123P1
.TACS source. 1.000E-00 5.000E-04 1.667E-02 123P1
.TACS source. 1.000E-00 5.000E-04 1.667E-02 123P1
.TACS source. 1.000E-00 5.000E-04 1.667E-02 123P1
.TACS source. 1.000E-00 5.000E-04 1.667E-02 123P1
.TACS source. 1.000E-00 5.000E-04 1.667E-02 123P1

Names of TACS variables for output vector.

Names of TACS variables for output vector.

Names of TACS variables for output vector.

Names of TACS variables for output vector.

Comment card. KOMPAR > 0. C

C 1 2 3

6 7 8

Comment card. KOMPAR > 0.

C 345768901234567890123456789012345678901234567890123456789012345678901234567890
Blank card terminating all TACS data cards.

Comment card. KOMPAR > 0.

C < n1 <= n2 > ref1 <= ref2 <= R > = L <=
C >

Comment card. KOMPAR > 0.

C < n1 <= n2 > ref1 <= ref2 <= R > = A <=

Series R-L-C. 2.500E+03 0.000E+00 1.000E-08 | POS VIHA 2500.
.
1
Series R-L-C. 2.500E+03 0.000E+00 1.000E-08 | POS VIHA 2500.
.
1
Series R-L-C. 2.500E+03 0.000E+00 1.000E-08 | POS VIHA 2500.
.
1
Series R-L-C. 2.500E+03 0.000E+00 1.000E-08 | VIHA NEG 2500.
.
1
Series R-L-C. 2.500E+03 0.000E+00 1.000E-08 | VIHA NEG 2500.
| SERIES R-L-C | 5.000E+00 | 0.000E+00 | 0.000E+00 | LOAD | NEG | 5. |
| SERIES R-L-C | 0.000E+00 | 5.000E+03 | 0.000E+00 | POS | LOAD | 5. |
| SERIES R-L-C | 1.000E+09 | 0.000E+00 | 0.000E+00 | POS | NEG | 1.E+9 |
| SERIES R-L-C | 2.500E+03 | 0.000E+00 | 1.000E-08 | POS | 2500. |
| SERIES R-L-C | 2.500E+03 | 0.000E+00 | 1.000E-08 | POS | VINC | 2500. |
| SERIES R-L-C | 2.500E+03 | 0.000E+00 | 1.000E-08 | POS | 2500. |
| SERIES R-L-C | 2.500E+03 | 0.000E+00 | 1.000E-08 | NEG | 2500. |
| SERIES R-L-C | 2.500E+03 | 0.000E+00 | 1.000E-08 | VINC | NEG | 2500. |
| SERIES R-L-C | 2.500E+03 | 0.000E+00 | 1.000E-08 | NEG | 2500. |
| SERIES R-L-C | 1.000E-06 | 1.000E-04 | 0.000E+00 | X0001AVINA | 1.E-6 | .1 |
| SERIES R-L-C | 1.000E-06 | 1.000E-04 | 0.000E+00 | X0001BVINB | 1.E-6 | .1 |
| SERIES R-L-C | 1.000E-06 | 1.000E-04 | 0.000E+00 | X0001CVIN | 1.E-6 | .1 |

**Blank card ending branches.**  
| IRF, NOUT = 18 10 | BLANK BRANCH |

**Comment card.**  
| KOPPAR = 0. |

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<th>Te</th>
<th>VRF/CLIP</th>
<th>type</th>
</tr>
</thead>
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<td>Valve</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
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<td>Valve</td>
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<td>0.000E+00</td>
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<tr>
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**Blank card ending switches.**  
| KSWITCH = 0. |

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<th>&lt;=Phase/70&lt;</th>
<th>A1</th>
<th>&lt;= T1</th>
<th>&lt;= TSTART</th>
<th>&lt;= TSTOP</th>
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<td>3.76E+02</td>
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<td></td>
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</table>

**Blank card ends electric sources.**  
| KCONST = 3 |

**Blank SOURCE.**  

**List of input elements that are connected to each node.**  
Only the physical connections of multi-phase lines are shown (capacitive and inductive coupling are ignored). Repeated entries indicate parallel connections. Sources are included, although sources (including rotating machinery) are omitted -- except that U.M. usage produces extra, internally-defined nodes "UXXX".
Sinusoidal steady-state phasor solution, branch by branch. All flows are away from a bus. The real part, magnitude, or "P" is printed above the imaginary part, the angle, or "Q". The first solution frequency = 5.00000000E+01 hertz.

Bus K

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<tr>
<th>Power Flow</th>
<th>Phasor node Voltage</th>
<th>Phasor branch current</th>
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<td>Polar Rectangular</td>
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<td>-413687126-17</td>
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102
Output for steady-state phasor switch currents.

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<th>Node-M</th>
<th>Degree</th>
<th>VINA</th>
<th>Power</th>
<th>Reactive</th>
<th>I-real</th>
<th>I-imag</th>
<th>I-magn</th>
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</tr>
</tbody>
</table>

Solution at nodes with known voltage. Nodes that are shorted together by switches are shown as a group of names, with the printed result applying to the composite group. The entry "MV" is sqrt(P^2 + Q^2) in units of power, while "P.F." is the associated power factor.
105
-11.065
0.0 0.0 0.0 0.0 0.0
0.0
16000 .08 528.74926 438.889671 -359.00325 279.116712 79.8864193 79.8864193
-359.00325 510.749583 -125.74337 105.739629 0.0 .002871835
0.0
5000 .0823 550.086783 486.007631 -117.34484 368.662791 -251.31779 368.662791
-117.34484 110.017357 -110.017357 -.003838323
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0
17500 .0856 564.251621 527.756674 270.765721 153.495681 -374.26099 153.495681
-374.26099 112.850324 -.003459921 -112.84876
221.841341
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0
18000 .0871 570.32207 563.642759 375.717251 -187.9792 -187.92501 375.717251
-187.92501 114.024414 114.024414 -114.01607
-.00396988
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0
18500 .09 567.92744 593.277322 220.246206 -373.03113 152.73929 220.246206
-373.03113 113.53849 113.53849 -.003883164
-113.53849
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0
19000 .0925 555.464669 616.273956 -315.79999 -250.00896 366.264992 366.264992
-250.00896 113.128934 -.00391812 111.130304
-113.128934
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0
19500 .095 534.08269 632.300267 -354.37037 80.4615747 278.020254 278.020254
-354.37037 106.816538 -.106.816538 106.816538
-.00537046
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0
20000 .0975 500.54097 641.349026 -309.50496 331.844601 -.63.287095 331.844601
-309.50496 100.108196 100.10724 100.104702
0.0 0.0 0.0 0.0 1.0 1.0 1.0 1.0
0.0
**% % % % Final time step, PLTFL dumps plot data to "PLT" disk file. Gone dumping plot points to C-like disk file.**
0.0
--- End of plot points to C-like disk file. ---
0.0
--- Times of maxima ---
0.0
10 0.02633 0.02633 0.02633 0.02633 0.02633 0.02633 0.02633 0.02633 0.02633 0.02633
0.0
10 0.06522 0.008335 0.011115 0.00278 0.0056 0.01389
0.0
--- Times of minima ---
0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0
--- Blank card terminating all plot cards. ---
--- Blank plot ---
--- Actual list sizes for the preceding solution follow. ---
--- Nov-18 10.00 11.0000
--- Size 1-10: 10 18 18 5 10 12 50 0 0 0 0 0 0 10 0 0 0
--- Size 11-20: 0 0 0 0 0 0 0 0 0 0 10 0 0 0 0 0
--- Size 21-30: 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
--- Seconds for overlays 1-5: 0.000 0.000 0.000 0.000 0.000 (CP: wait; Real)
--- Seconds for overlays 6-11: 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
--- Seconds after DELTA-loop:
--- Totals: 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
---