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Zachary Gann
Metropolitan State University of Denver

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Mr. Zachary Gann,
Economics Major Student, College of Business, Metropolitan State University of Denver, P. O. Box 173362, Denver, CO 80217-3362.

Abstract

This paper intends to apply the Hubbert curve to the production of rare earth elements by the United States, China, and total global production. The goal of this research is to see if the production of rare earth elements follows the predicted production forecasted by the Hubbert curve and to observe if the curve can create usable predictions of future production. Global demand for rare earth elements has drastically increased in the modern era due to their unique properties. Global production has increased as well to meet this increased demand.

Rare earth elements are a collection of seventeen chemical elements that are used in the production of advanced technologies. The demand for rare earth elements has increased in the modern era with new applications for them being discovered and the increasing demand for green energy which requires rare earth elements in its production.

The United States was chosen to be examined due to its long history of producing rare earth elements. The United States was also the largest supplier of rare earth elements before China overtook them in rare earth element production. Ever since China became the top producer of rare earth elements, the United States' production of rare earth has declined. Production reached zero in 2016 due to the lone company that mines rare earth elements in the country filing for bankruptcy. This caused their only mine to be put on care and maintenance. This meant that the United States had to import all of the rare earth metals it requires until the mine reopens or until new mines are created.

China was chosen as the other country to analyze because it has the largest known reserves of rare earth metals and is the largest supplier of rare earth elements in the world market today. China's supply of rare earth metals for the market is also affected by its own increasing demand for rare earth due to its rising industrial sector and their government trying to preserve their reserves of rare earth metals.

It was concluded that observed REE production does follow the trend predicted by the Hubbert curve, but the Hubbert curve does not create accurate predictions of future REE productions due to its simplicity.

The first section of this paper is a literature review that scrutinizes previous research done about rare earth elements and the Hubbert curve. The reasoning behind this analysis is to get a better understanding of the state of the rare earth elements market and to create a basis for the research of this paper to be conducted on. Correspondingly in this section, the equation of the Hubbert curve and the theoretical implications of its results will also be discussed. The data and regressions will be described that look at the application of the Hubbert curve to the United States' rare earth element production, China's rare earth element production and global rare earth production in the next section. The results of this research will be thoroughly described in the conclusion alongside what implications these results have as well. A bibliography citing all material used within this project will be the last part of this paper.

The Hubbert Curve and Rare Earth Elements Production

Section 1:

Rare earth elements are integral to the future advancement and production of technology. This paper applies the Hubbert curve which was originally used to analyze oil production to the production of rare earth elements by the United States, China, and the global production as a whole. This is done in order to see if the past production of rare earth elements follows the Hubbert curve as derived by Hubbert (1956) and to see if the Hubbert curve can be used to create accurate predictions of future production. Seventeen chemical elements compose the group known as rare earth elements. They have unique properties that make them desirable in the production of advanced technology. The demand for rare earth elements has drastically increased in the modern era due to new applications for them being discovered and because of the increasing demand for green energy which requires rare earth elements in its production.

The United States will be examined due to its long history of producing rare earth elements and due to them being the largest supplier of rare earth elements in the past. This changed in the last few decades when China overtook the United States in rare earth element production. In that time period the United States' production of rare earth has declined with it reaching zero in 2016 due to the only company that produces rare earth elements in the country filing for bankruptcy and causing their only mine to be put on care and maintenance. This meant that the United States had to import all of the rare earth metals it required.

China was chosen because she has the largest reserves of rare earth metals and is the largest supplier of rare earth elements in the world market in the modern times. China's supply of rare earth metals for the market is also affected by its own increasing demand for rare earth due to its rising industrial sector and their government trying to preserve their reserves of rare earth metals.

The first section of this paper is a literature review that examines previous research done surrounding rare earth elements and the Hubbert curve. This is done to get a better understand of the rare earth elements market and to create a foundation for the research of this paper to be conducted on. This section also describes the equation of the Hubbert curve and the theoretical implications of its results. The next section is the research section. This is where the data and regressions will be described that look at the application of the Hubbert curve to the United States' rare earth element production, China's rare earth element production and global rare earth production. The results of this research is thoroughly described in the conclusion with what repercussions these results have. The final piece of this paper is a bibliography citing all references used within this project.

Section 2:

There has been research done on the Hubbert curve. In 1956, M. King Hubbert presented his paper to the American Petroleum Institute. In it he concluded that the rate of oil production of an oil reserve follows a bell shaped curve and that United States oil production would peak between 1965 and 1970. In their 2016 paper, Istemi Berk and Volkan n Ş. Ediger applied the Hubbert curve to the coal production of Turkey's lignite fields to forecast the future coal production. They determined that the largest lignite fields would start to decline in production in the near future and that most of the coal reserves would remain unused if the trend continued into the future. In his 2011 paper, Brian Gallagher created an idealized Hubbert curve for global oil production. He predicted peak oil production would occur in 2009 with 83.2 million barrels a day and deduced that the idealized Hubbert curve could be a useful tool for prediction. Cavallo examined if the Hubbert curve should be used for determining peak oil given that it has it has failed in the past. He concluded in his 2004 paper that the Hubbert curve can't be used to predict ultimate oil reserves, but it has importance as an econometric model. In their 2015 paper, Chavez-Rodriguez, Szklo, and Frossard Pereria de Lucena applied the Hubbert curve to oil production in Peru. They found that a multi-Hubbert approach had a better fit to Peru's oil production and that Peru could reach a second peak oil after the first peak in 1982, but this would depend on oil production in Amazonia.

There also has been numerous research done on rare earth metals and predictions for their future production. In 2015, Schlinkert and van den Boogaart constructed an economic model in their paper to depict what the past and future of the rare earth element market looks like. They found that the mining and separation of rare earth elements could be focused in China and the market itself could change into an oligopoly due in part to increasing demand for rare earth elements. Paulick and Machacek analyzed the rare earth element exploration boom that lasted from 2010 to 2014 and analyzed the mineral resource definitions that occurred. In their 2017 paper, they found that rare earth element mineral resources outside of China reached 98 million metric tons by 2015 and that a large percentage of those resources have a higher heavy rare earth element to light rare earth element ratio than other rare earth element minerals. Klossek, Kullik, and van den Boogaart (2016) examined the problems with the rare earth market that caused China to dominate the rare earth market. They identified four systematic problems which were competing political economic models, resource nationalism, lack of market transparency, and absence of trust. Rui Wan and Jean-François, in their 2017 paper, created a three country trade model of a rare earth elements market to examine how the three different polices of a downstream subsidy, an upstream export tariff, and an upstream pollution tax would affect welfare and the market. They determined that equilibrium policies and their outcomes depend on the coefficient of the local environmental cost function, the coefficient of the global environmental benefit function, and the amount of competitors in the market downstream.

Now it is time to explain the Hubbert curve. M. King Hubbert theorized that the cumulative production of a nonrenewable resource follows this logistic curve over time (Cavallo, 2004):

$$Q(t) = \frac{Q_{max}}{(1 + ae^{bt})}$$

In this equation $Q(t)$ is the cumulative production at time t, Q_{max} is the total amount of the resource available and a and b are constants. According to this model, production of the resource begins slowly, then production grows exponentially till it reaches the maximum yearly production were it declines afterwards. This equation also makes the curve symmetric about the point of maximum yearly production. Two further equations can be used to determine the maximum yearly production and the year that it occurs. Maximum annual production (P_{max}) can be calculated by the equation (Cavallo, 2004):

$$P_{max} = \frac{Q_{max}|b|}{4}$$

The year of maximum annual production (t_{max}) is calculated by the equation (Cavallo, 2004):

$$t_{max} = \left(\frac{1}{b}\right) \ln\left(\frac{1}{a}\right)$$

Section 3:

To test how well the Hubbert curve fits the data, the logistic equation will be plotted against data of cumulative rare earth element production and time (years) for the United States, China and Global production respectively. The data were gathered from the United States Geological Survey and put into an Excel spreadsheet. All rare earth element amounts throughout are in metric tons (t). These data, however, only have annual production data, not cumulative data. To correct this, the earliest point of data was called year 1 and cumulative production was calculated by adding each annual production together up to the desired year. These data were then put into Stata and a nonlinear regression was run to determine the values of Q_{max}, a and b. The resulting values were then put into the logistic equation and ran through Excel to determine the theoretical cumulative production of earth elements. Theoretical yearly production for a given year was calculated by subtracting the theoretical production for the previous year from the theoretical production of the desired year.

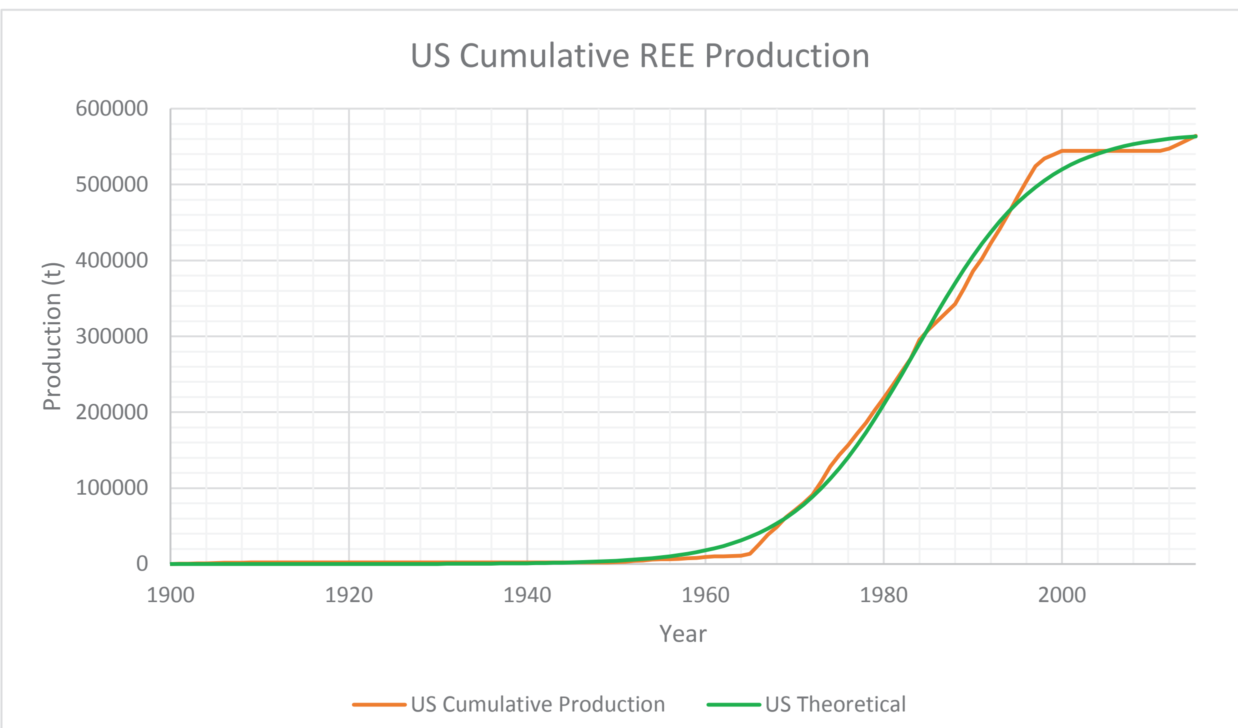
Table 1

Source	**SS**	**df**	**MS**	**Number of obs**	116
Model	8.110e+12	3	2.7034e+12	**R-squared**	0.9986
Residual	1.141e+10	113	101002701	**Adj R-squared**	0.9986
Total	8.122e+12	116	7.0015e+10	**Root MSE**	10050.01
				Res. Dev.	2464.111

Constant	**Coef.**	**Std. Err.**	**t**	**P>t**	**[95% Conf.**	**Interval]**
Q_{max}	569720.2	3429.438	166.13	0.000	562925.8	576514.5
a	203251.5	45435.1	4.47	0.000	113236.3	293266.7
b	-.144263	.0027636	-52.20	0.000	-.1497382	-.1387878

The data for the United States can be found in the appendix. The data were of annual rare earth production from 1900 to 2015. There were some missing production data for a few years with most occurring between 1911 and 1949. The results of the regression for the United States can be found in the Table 1 above. The determined value for Q_{max} was 569,720.2 metric tons and the determined values for constants a and b being 203,251.5 and -.144263 respectively. The R-squared and adjusted R-squared were 0.9986 and 0.9986 meaning that a lot of the variation in U.S. cumulative REE production is explained by Hubbert's model. This is clearly visible in Figure 1 where U.S. cumulative production is graphed alongside U.S. theoretical production across time. The theoretical cumulative production closely follows the actual cumulative production. The high t-statistics for Q_{max}, a and b illustrates that these values are significant at the 95% level. This means that there is some type of relationship occurring here.

Figure 1



This is further supported by the p-value of 0.000 for all of the tested variables which goes further to say that each variable is significant at the 99.99% level.

Figure 2

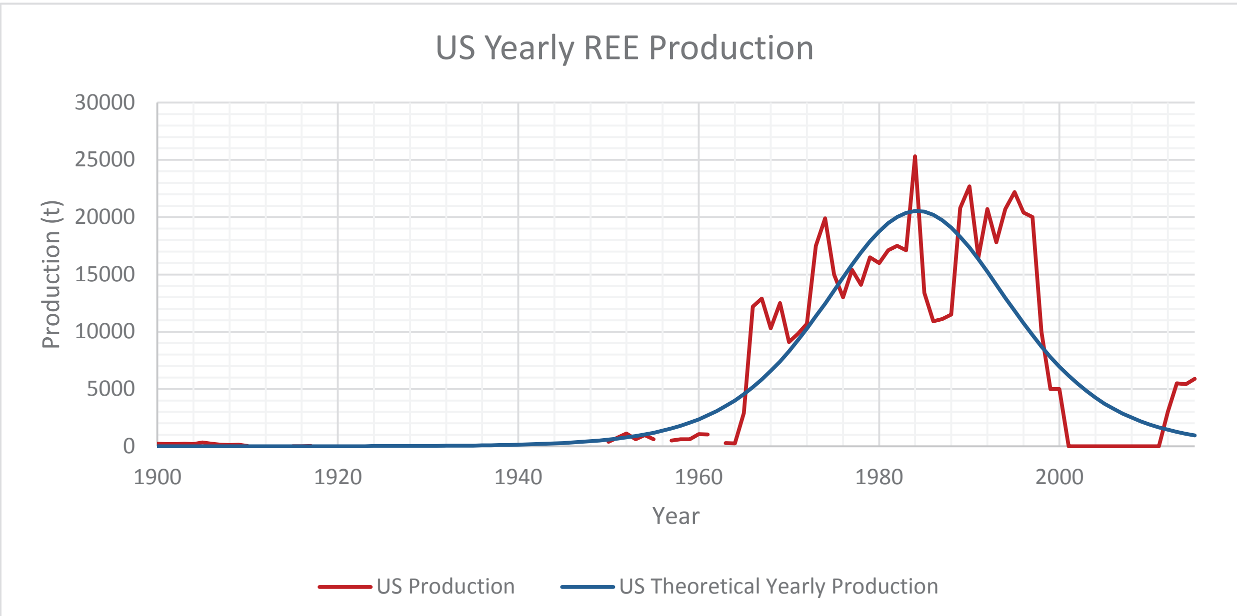


Figure 2 shows U.S. yearly production and U.S. theoretical yearly production over time. This graph illustrates that while theoretical cumulative production follows actual cumulative production, actual yearly production is much more volatile than the predicted yearly production. According to Hubbert's equations, maximum annual production should have been roughly 20,547.39 metric tons of REE and it should have occurred between 1983 and 1984. Actual maximum annual production was 25,300 metric tons and occurred in 1984. While the time period is accurate, the estimated value for maximum production is off by almost 20%. This discrepancy is clear with other data points in Figure 2. The figure also shows that yearly production started to significantly increase in 1965 and decreased to 0 metric tons in 2001. This corresponds to the production of the Mountain Pass Mine. This mine was the sole producer of rare earth elements in the United States (Castor, 2008). The mine was forced to shut down in

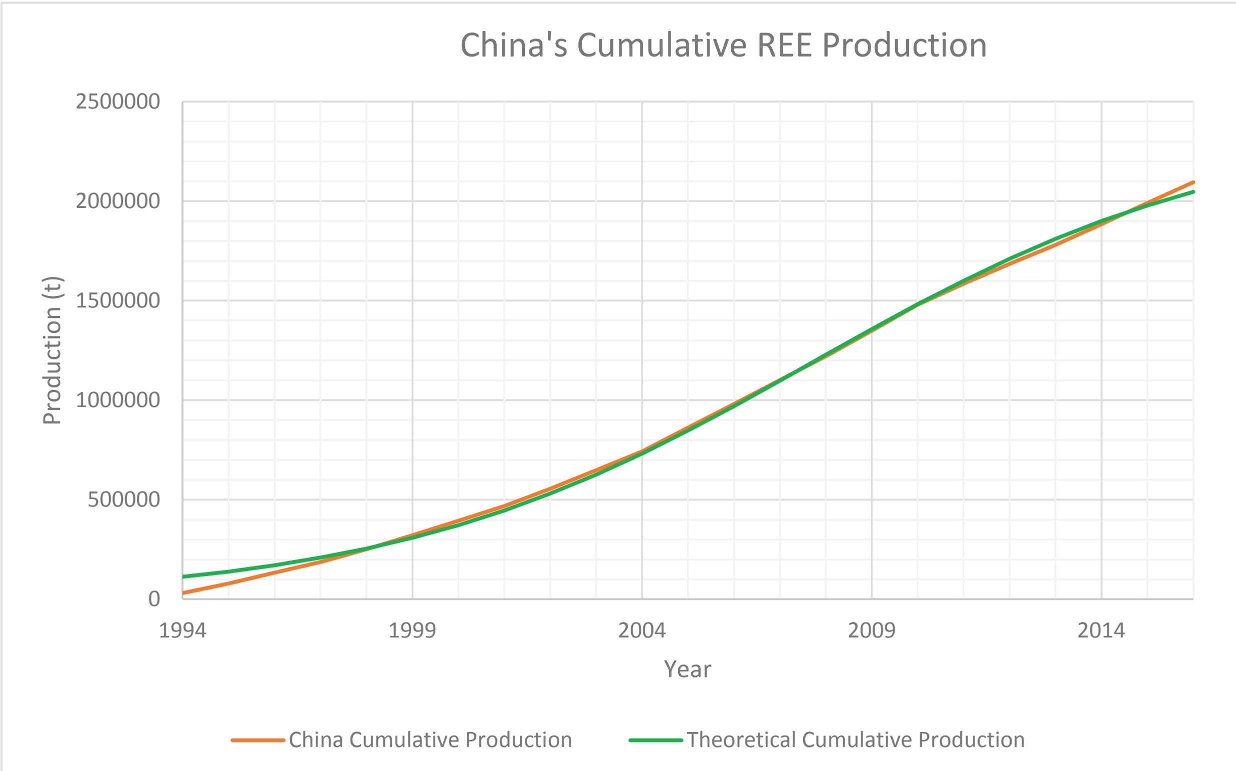
2002 due to competition from China and environmental constraints. Production resumed in 2012 but stopped again in 2016. Now the mine is under new ownership that plans to reopen the mine. This illustrates a problem with the Hubbert curve, its simplicity. The model has no variables to adjust for political and technological factors. There is no way the model could have predicted the Mountain Pass mine would be forced to close down with its limited parameters. This result suggests that the annual production of REE calculated by the Hubbert curve is not suitable to accurately predict the actual annual REE production.

Table 2

Source	**SS**	**df**	**MS**		**Number of obs**	23
Model	3.070e+13	3	1.0234e+13		**R-squared**	0.9994
Residual	1.976e+10	20	987998558		**Adj R-squared**	0.9993
Total	3.072e+13	23	1.3358e+12		**Root MSE**	31432.44
					Res. Dev.	538.4141
Constant	**Coef.**	**Std. Err.**	**t**	**P>t**	**[95% Conf.**	**Interval]**
Q_{max}	2380213	56607.42	42.05	0.000	2262132	2498294
a	24.89001	1.47325	16.89	0.000	21.81686	27.96316
b	-.2185454	.0069499	-31.45	0.000	-.2330426	-.2040481

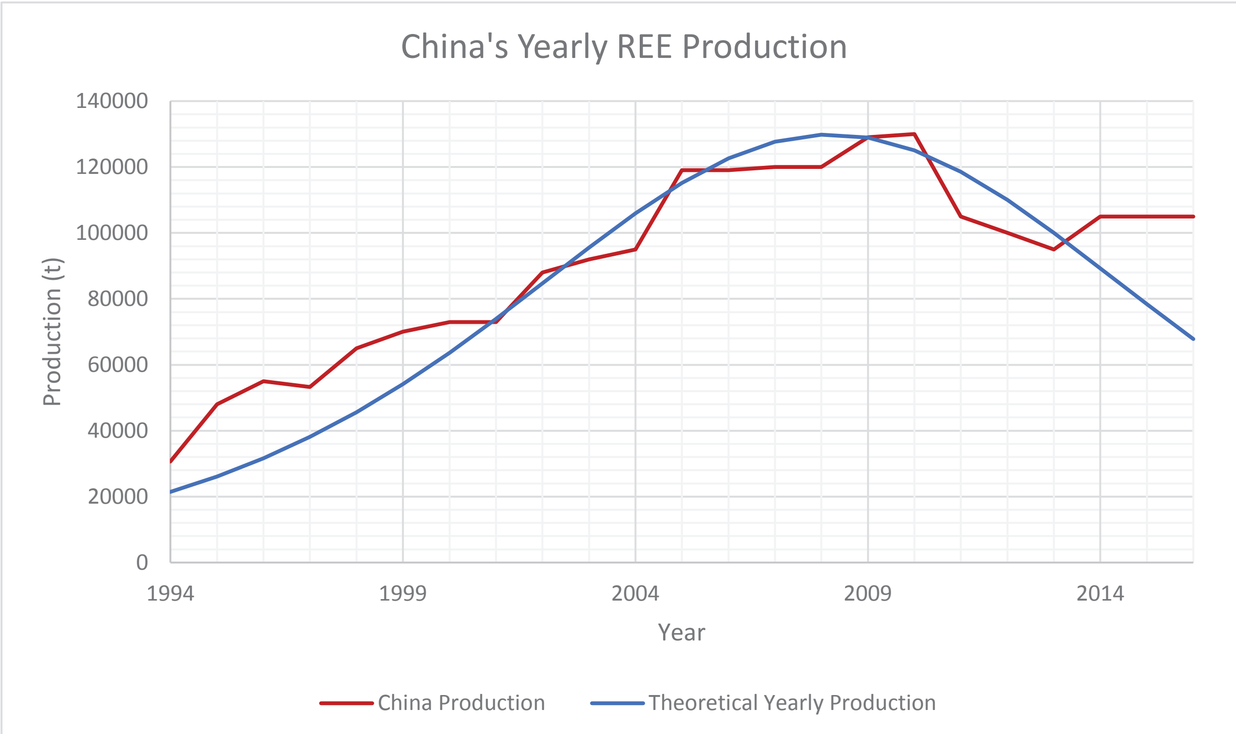
The data for China can also be found in the appendix. The data were of annual rare earth production from 1994 to 2016.The results of the regression for China can be found in the Table 2 above. The determined value for Q_{max} was 2,380,213 metric tons and the determined values for constants a and b were 24.89001and -.2185454 respectively. The R-squared and adjusted R-squared were higher than they were for the U.S. at 0.9994 and 0.9993.This again means that a lot of the variation in China's total REE production is explained by Hubbert's model. This is also

Figure 3



clearly visible in Figure 3 where China's cumulative production is graphed alongside their theoretical production across time. The theoretical cumulative production again closely follows the actual cumulative production. The same story is also repeated when the t-statistics and p-values of the variables are examined. The high t-statistics for Q_{max}, a and b illustrates that these values are significant at the 95% level. This again means that the Hubbert curve is effective at predicting cumulative production. These results also have a p-value of 0.000 for all of the tested variables which once again goes on to say that each variable is significant at the 99.99% level. Figure 4 shows both China's yearly production and its theoretical yearly production over time. Like with Figure 2, this graph illustrates that while theoretical cumulative production follows actual cumulative production, actual yearly production is much more volatile than the predicted

Figure 4



yearly production. According to Hubbert's equations, maximum annual production should have been roughly 130,046.15 metric tons of REE and it should have occurred between 2007 and 2008. Actual maximum annual production was 130,000 metric tons and occurred in 2010. While the estimated value for maximum production is accurate, the estimated time period is off. Though the estimated annual production for China more closely follows the actual production of REE than it did for the U.S., it still is off in its prediction. China's large yearly production can be attributed to China aggressively promoting its REE industry and not being hindered by standard market factors (Castor, 2008). This led to the overproduction of rare earth elements which in turn lead China announcing it would assert control over the industry and restrict exports of rare earth elements (Castor, 2008). Once again, the Hubbert curve does not have enough parameters to have been able to predict this would happen. This is also supported by recent announcements by

China. On October 18, 2016, China's Ministry of Industry and Information Technology announced that they would limit the yearly production of rare earth elements to 140,000 metric tons by 2020 (Aspa, 2016). This illustrates that China expects there yearly REE production to still be around 140,000 metric tons in 2020. According to the model, yearly REE production in 2020 should be around 34,188.73594 metric tons which is well below the expected yearly production. This again leads to the conclusion that the estimated annual REE production calculated by the Hubbert curve is not suitable to accurately predict the actual annual REE production, unlike its ability to predict the total production.

Table 3

Source	SS	df	MS		Number of obs	116
Model	1.160e+14	3	3.8657e+13		**R-squared**	0.9995
Residual	5.727e+10	113	506802933		**Adj R-squared**	0.9995
Total	1.160e+14	116	1.0002e+12		**Root MSE**	22512.28
					Res. Dev.	2651.216

Constant	Coef.	Std. Err.	t	P>t	[95% Conf.	Interval]
Q_{max}	8438208	377526.9	22.35	0.000	7690260	9186157
a	4607.965	170.8803	26.97	0.000	4269.42	4946.51
b	-.0690139	.0008164	-84.54	0.000	-.0706313	-.0673966

Now it is time to apply the Hubbert curve to global rare earth element production. The data for world rare earth element production can also be found in the appendix. The data were of annual rare earth production from 1900 to 2015. The results of the regression for global production can be found in the Table 3 above. The determined value for Q_{max} was 8,438,208 metric tons and the determined values for constants a and b were 4,607.965 and -.0690139 respectively. The R-squared and adjusted R-squared were higher than they were for both the U.S. and China at 0.9995 and 0.9995 respectively. Like with the other regressions, this means that a lot of the variation in China's total REE production is explained by Hubbert's model. This is also clearly visible in Figure 5 where world cumulative production is graphed alongside their theoretical production across time. The theoretical cumulative production again closely follows the actual cumulative production. The same story is also repeated when the t-statistics and p-values of the variables are examined. The high t-statistics for Q_{max}, a and b illustrates that these values are significant at the 95% level. This once again indicates that the Hubbert curve is effective at predicting cumulative production. Like with the other tests, this regression has a p-value of 0.000 for all of the tested variables which goes further on to say that each variable is significant at the 99.99% level.

Figure 5

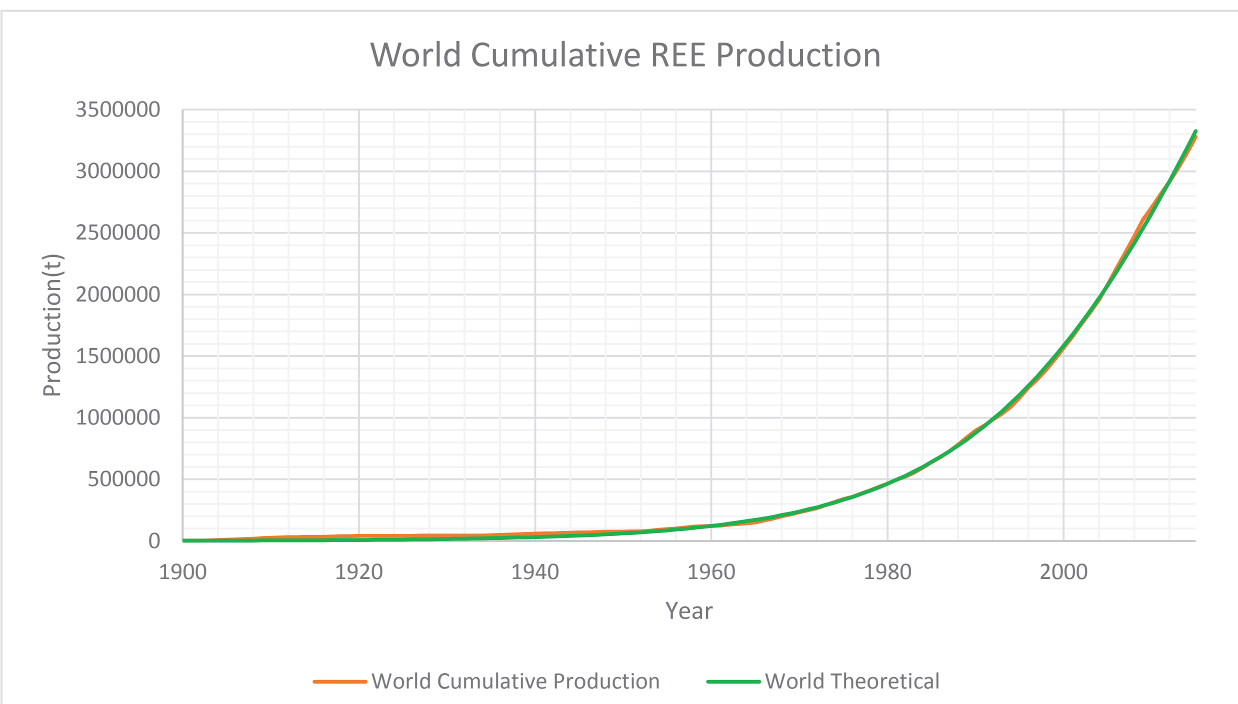


Figure 6

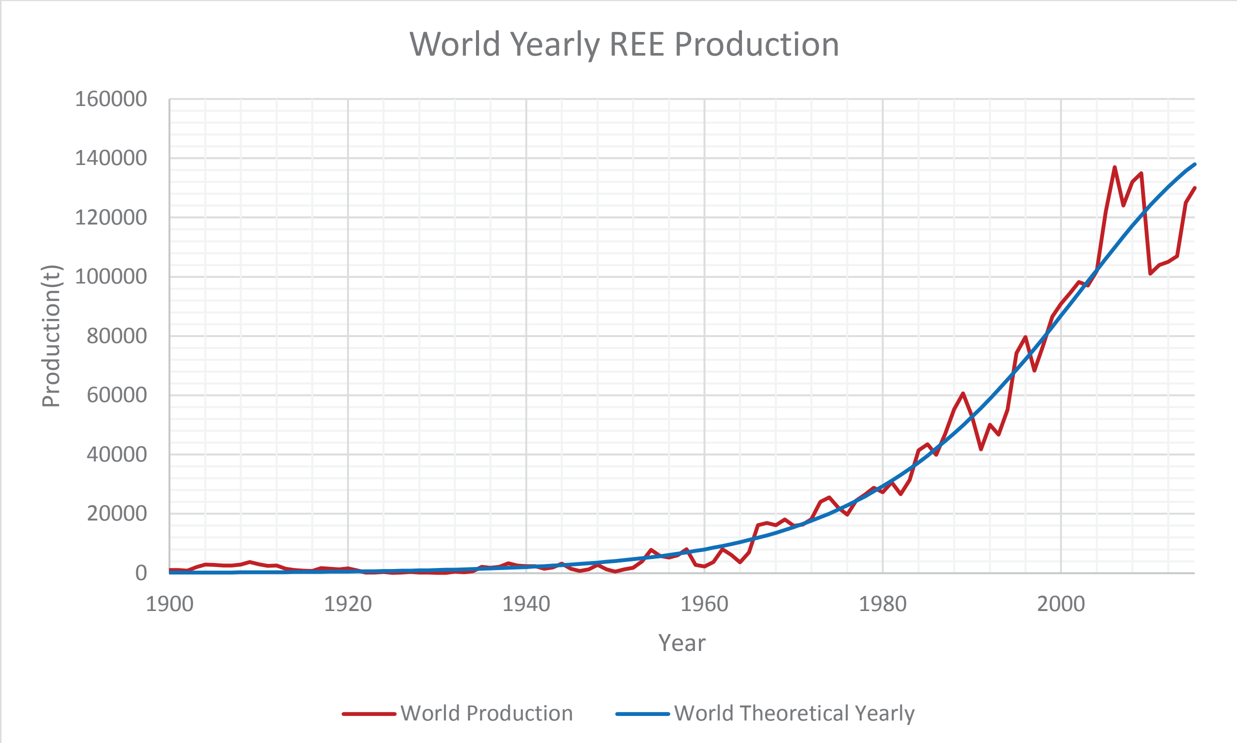


Figure 6 shows both global yearly production and global theoretical yearly production over time. Like with the previous figures 2 and 4, this graph demonstrates that while theoretical cumulative production follows actual cumulative production, actual yearly production is much more volatile than the predicted yearly production. Though the variation is not as pronounced as it was for U.S. yearly production, it is still varying across the theoretical yearly production. According to Hubbert's equations, maximum annual production should have occurred between 2021 and 2022 at a value of 145,588.4108 metric tons. The ability of this Hubbert curve to create accurate predictions of future is doubtful once again due to the simplicity of the model. It was shown previously shown that accurate predictions for China's REE production were not created which is problematic for global predictions due to China composing over 90% of global rare earth element production in recent years. This once again confirms that the Hubbert curve cannot be used to create accurate predictions of future rare earth element production.

Section 4:

This paper sought to apply the Hubbert curve to rare earth element production for the United States, China and global production. The goal was to see if observed REE production followed the predicted REE production created by the Hubbert Curve and to see if the Hubbert Curve could be used to created accurate predictions of future REE production. To test this, Hubbert curves were created for the United States, China and global production. In all three cases, the trend of cumulative prediction and annual production followed the predicted path created by the Hubbert curve; however, it was also determined that the Hubbert curve could not be used to create accurate predictions of future REE production. This is due to the simplicity of the model which does not have enough parameters for all the political and technological factors that determine REE production for a country. Though this result illustrates its inability to create accurate predictions, the Hubbert curve clearly has a use in examining the trends in the production of rare earth elements.

Appendix 1

Year	China Yearly Production	Time	China Cumulative Production	Theoretical Cumulative Production	Theoretical Yearly Production
1994	30600	1	30600	113323.0929	21387.5162
1995	48000	2	78600	139383.0465	26059.95359
1996	55000	3	133600	170983.7718	31600.7253
1997	53300	4	186900	209080.4879	38096.7161
1998	65000	5	251900	254686.9367	45606.44885
1999	70000	6	321900	308826.4248	54139.48807
2000	73000	7	394900	372457.8314	63631.40666
2001	73000	8	467900	446374.7041	73916.87271
2002	88000	9	555900	531080.6487	84705.9446
2003	92000	10	647900	626651.9888	95571.34007
2004	95000	11	742900	732608.0552	105956.0664
2005	119000	12	861900	847817.6953	115209.6401
2006	119000	13	980900	970473.3848	122655.6896
2007	120000	14	1100900	1098157.474	127684.089
2008	120000	15	1220900	1228007.189	129849.715
2009	129000	16	1349900	1356959.852	128952.6633
2010	130000	17	1479900	1482036.189	125076.3371
2011	105000	18	1584900	1600607.595	118571.4055
2012	100000	19	1684900	1710598.34	109990.7451
2013	95000	20	1779900	1810593.545	99995.20556
2014	105000	21	1884900	1899849.515	89255.96965
2015	105000	22	1989900	1978224.754	78375.23846
2016	105000	23	2094900	2046061.546	67836.79245

Appendix 2

Year	Time	US Yearly Production	US Cumulative Production	US Theoretical Production	US Theoretical Yearly Production	World Yearly Production	World Cumulative Production	World Theoretical Production	World Theoretical Yearly
1900	1	227	227	3.238008729	3.238008729	1040	1040	1961.608974	130.7840487
1901	2	187	414	3.74050515	0.502496421	1090	2130	2101.733187	140.1242134
1902	3	200	614	4.320981823	0.580476673	863	2993	2251.864264	150.1310773
1903	4	215	829	4.991539969	0.670558146	2030	5023	2412.716443	160.8521785
1904	5	186	1015	5.76615865	0.774618681	2860	7883	2585.05488	172.3384369
1905	6	335	1350	6.660986142	0.894827492	2780	10663	2769.699274	184.6443941
1906	7	211	1561	7.694676518	1.033690377	2600	13263	2967.527744	197.8284696
1907	8	137	1698	8.888778447	1.194101928	2580	15843	3179.480978	211.9532349
1908	9	105	1803	10.26818429	1.379405848	2840	18683	3406.566686	227.0857072
1909	10	135	1938	11.8616489	1.593464606	3690	22373	3649.864349	243.2976629
1910	11	25	1963	13.70238881	1.840739909	3020	25393	3910.530322	260.6659738
1911	12		1963	15.82877444	2.126385631	2490	27883	4189.803289	279.2729662
1912	13		1963	18.2851296	2.456355157	2500	30383	4489.010093	299.2068044
1913	14		1963	21.12265494	2.837525346	1480	31863	4809.571995	320.5619024
1914	15		1963	24.40049465	3.277839711	992	32855	5153.011357	343.4393619
1915	16	9	1972	28.18696842	3.786473764	870	33725	5520.9588	367.9474426
1916	17	9	1981	32.56099437	4.374025955	731	34456	5915.160863	394.2020631
1917	18	25	2006	37.61373255	5.052738174	1730	36186	6337.4882	422.3273375
1918	19		2006	43.45048289	5.836750344	1470	37656	6789.944348	452.4561476
1919	20		2006	50.19287728	6.742394391	1210	38866	7274.675103	484.7307552
1920	21		2006	57.98141092	7.788533633	1590	40456	7793.978558	519.3034546
1921	22		2006	66.97836549	8.996954574	929	41385	8350.315829	556.3372708
1922	23		2006	77.37118464	10.39281915	189	41574	8946.322532	596.0067033
1923	24		2006	89.37637135	12.00518671	138	41712	9584.821053	638.498521
1924	25		2006	103.2439877	13.86761633	348	42060	10268.83366	684.0126089
1925	26	0.499	2006.499	119.2628495	16.01886186	12	42072	11001.59653	732.7628722
1926	27		2006.499	137.7665231	18.5036736	146	42218	11786.57473	784.9781993
1927	28		2006.499	159.1402462	21.37372307	352	42570	12627.47822	840.9034879
1928	29		2006.499	183.8289154	24.68866916	180	42750	13528.27896	900.8007389
1929	30		2006.499	212.3463026	28.51738724	197	42947	14493.22918	964.9502198
1930	31		2006.499	245.285688	32.93938543	17	42964	15526.88088	1033.651703
1931	32		2006.499	283.3321241	38.04643606	50	43014	16634.10667	1107.225785
1932	33		2006.499	327.2765782	43.94445406	530	43544	17820.12195	1186.015284
1933	34		2006.499	378.0322367	50.75565854	302	43846	19090.50868	1270.386729
1934	35		2006.499	436.6532955	58.62105884	564	44410	20451.24062	1360.73194
1935	36		2006.499	504.3566072	67.70331167	2130	46540	21908.71032	1457.469703
1936	37		2006.499	582.5466092	78.19000198	1840	48380	23469.75787	1561.047544
1937	38		2006.499	672.8440162	90.29740698	2150	50530	25141.70148	1671.943613
1938	39		2006.499	777.1188257	104.2748096	3310	53840	26932.37016	1790.668675
1939	40		2006.499	897.5282607	120.409435	2510	56350	28850.13837	1917.76821
1940	41		2006.499	1036.560353	139.0320919	2370	58720	30903.96301	2053.82464

1941	42		2006.499	1197.08396	160.5236077	2380	61100	33103.42268	2199.459669
1942	43		2006.499	1382.406114	185.3221541	1500	62600	35458.75943	2355.336752
1943	44		2006.499	1596.337679	213.9315647	1900	64500	37980.92312	2522.163693
1944	45		2006.499	1843.268433	246.9307538	3200	67700	40681.61849	2700.695365
1945	46		2006.499	2128.252776	284.9843436	1440	69140	43573.35506	2891.736576
1946	47		2006.499	2457.107381	328.8546049	721	69861	46669.50011	3096.145051
1947	48		2006.499	2836.522192	379.4148106	1300	71161	49984.33467	3314.834555
1948	49	20	2026.499	3274.186268	437.6640765	2720	73881	53533.11281	3548.778142
1949	50		2026.499	3778.930005	504.7437366	1290	75171	57332.12435	3799.011534
1950	51	383	2409.499	4360.885255	581.9552499	470	75641	61398.76095	4066.636609
1951	52	747	3156.499	5031.664815	670.7795603	1240	76881	65751.58597	4352.825014
1952	53	1110	4266.499	5804.562544	772.8977291	1820	78701	70410.40784	4658.82187
1953	54	615	4881.499	6694.775054	890.2125094	3960	82661	75396.3574	4985.949567
1954	55	983	5864.499	7719.645392	1024.870338	7840	90501	80731.96903	5335.611629
1955	56	608	6472.499	8898.928342	1179.28295	5760	96261	86441.26566	5709.296623
1956	57		6472.499	10255.07581	1356.147473	5230	101491	92549.84775	6108.582095
1957	58	499	6971.499	11813.53923	1558.463418	5980	107471	99084.98624	6535.138487
1958	59	625	7596.499	13603.08364	1789.544411	8060	115531	106075.7192	6990.733009
1959	60	600	8196.499	15656.10547	2053.021825	2810	118341	113552.9527	7477.233411
1960	61	1050	9246.499	18008.94211	2352.836641	2270	120611	121549.5643	7996.611609
1961	62	1030	10276.499	20702.15702	2693.21491	3690	124301	130100.5114	8550.947091
1962	63		10276.499	23780.77814	3078.62112	8020	132321	139242.9414	9142.430043
1963	64	278	10554.499	27294.46085	3513.682708	6060	138381	149016.3055	9773.364096
1964	65	256	10810.499	31297.53874	4003.077894	3680	142061	159462.4741	10446.16861
1965	66	2900	13710.499	35848.91707	4551.378326	6960	149021	170625.8545	11163.38038
1966	67	12200	25910.499	41011.75484	5162.837776	16200	165221	182553.5091	11927.6546
1967	68	12900	38810.499	46852.87375	5841.118905	16900	182121	195295.2742	12741.76507
1968	69	10300	49110.499	53441.82596	6588.952216	16200	198321	208903.8774	13608.60325
1969	70	12500	61610.499	60849.55146	7407.725498	18100	216421	223435.0537	14531.17628
1970	71	9110	70720.499	69146.56021	8297.008745	15900	232321	238947.6572	15512.60347
1971	72	9820	80540.499	78400.5895	9254.029293	16400	248721	255503.7684	16556.11127
1972	73	10700	91240.499	88673.71432	10273.12482	18200	266921	273168.7948	17665.02639
1973	74	17500	108740.499	100018.9318	11345.21744	24000	290921	292011.5615	18842.76672
1974	75	19900	128640.499	112476.3006	12457.36881	25600	316521	312104.3915	20092.82993
1975	76	15000	143640.499	126068.7923	13592.49175	22100	338621	333523.1708	21418.77933
1976	77	13000	156640.499	140798.0958	14729.3035	19700	358321	356347.3975	22824.22672
1977	78	15400	172040.499	156640.7015	15842.60569	24500	382821	380660.2093	24312.81176
1978	79	14100	186140.499	173544.6619	16903.9604	26500	409321	406548.3871	25888.1778
1979	80	16500	202640.499	191427.4594	17882.79745	28800	438121	434102.3305	27553.94341
1980	81	16000	218640.499	210175.3946	18747.93524	27300	465421	463416.0001	29313.66957
1981	82	17100	235740.499	229644.8251	19469.43048	30600	496021	494586.8221	31170.82203
1982	83	17500	253240.499	249665.4284	20020.6033	26600	522621	527715.5506	33128.72847
1983	84	17100	270340.499	270045.4549	20380.02649	31400	554021	562906.0808	35190.53025
1984	85	25300	295640.499	290578.6947	20533.23979	41400	595421	600265.2092	37359.12841
1985	86	13400	309040.499	311052.6569	20473.9622	43500	638921	639902.333	39637.1238

1986	87	10900	319940.499	331257.289	20204.63211	39900	678821	681929.0841	42026.75108
1987	88	11100	331040.499	350993.4856	19736.19662	46900	725721	726458.891	44529.80687
1988	89	11500	342540.499	370080.6666	19087.18104	55300	781021	773606.4629	47147.5719
1989	90	20800	363340.499	388362.8381	18282.17148	60700	841721	823487.1905	49880.72765
1990	91	22700	386040.499	405712.7558	17349.91765	52900	894621	876216.4584	52729.26789
1991	92	16500	402540.499	422034.0509	16321.29518	41700	936321	931908.8644	55692.40598
1992	93	20700	423240.499	437261.4062	15227.35523	50100	986421	990677.3431	58768.47871
1993	94	17800	441040.499	451359.0494	14097.64324	46700	1033121	1052632.191	61954.8482
1994	95	20700	461740.499	464317.9518	12958.9024	55100	1088221	1117879.995	65247.80328
1995	96	22200	483940.499	476152.1598	11834.20801	74300	1162521	1186522.457	68642.46229
1996	97	20400	504340.499	486894.6787	10742.51892	79700	1242221	1258655.137	72132.67967
1997	98	20000	524340.499	496593.2681	9698.589357	68300	1310521	1334366.095	75710.95885
1998	99	10000	534340.499	505306.4292	8713.16107	77100	1387621	1413734.47	79368.37436
1999	100	5000	539340.499	513099.778	7793.348843	86600	1474221	1496828.976	83094.50641
2000	101	5000	544340.499	520042.9169	6943.138894	90900	1565121	1583706.368	86877.3914
2001	102	0	544340.499	526206.8497	6163.932848	94500	1659621	1674409.86	90703.49195
2002	103	0	544340.499	531661.9364	5455.086695	98200	1757821	1768967.55	94557.69008
2003	104	0	544340.499	536476.3469	4814.410459	97100	1854921	1867390.857	98423.30718
2004	105	0	544340.499	540714.9554	4238.608473	102000	1956921	1969673.011	102282.1541
2005	106	0	544340.499	544438.6067	3723.651364	122000	2078921	2075787.625	106114.6145
2006	107	0	544340.499	547703.6856	3265.078815	137000	2215921	2185687.389	109899.7633
2007	108	0	544340.499	550561.923	2858.237432	124000	2339921	2299302.912	113615.5229
2008	109	0	544340.499	553060.3839	2498.460945	132000	2471921	2416541.768	117238.8569
2009	110	0	544340.499	555241.5853	2181.201329	135000	2606921	2537287.769	120746.001
2010	111	0	544340.499	557143.7048	1902.119534	101000	2707921	2661400.499	124112.7299
2011	112	0	544340.499	558800.8488	1657.144014	104000	2811921	2788715.156	127314.6566
2012	113	3000	547340.499	560243.3531	1442.504287	105000	2916921	2919042.716	130327.5597
2013	114	5500	552840.499	561498.0988	1254.745713	107000	3023921	3052170.448	133127.7321
2014	115	5400	558240.499	562588.8294	1090.730576	125000	3148921	3187862.793	135692.3451
2015	116	5900	564140.499	563536.4589	947.6295428	130000	3278921	3325862.611	137999.8181

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