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Finite Element Analysis of Thermomechanical Processes in Disk Clutch Systems

Zhuo Chen
University of Denver

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FINITE ELEMENT ANALYSIS OF THERMOMECHANICAL PROCESSES IN DISK CLUTCH SYSTEMS

A Thesis

Presented to

the Faculty of Engineering and Computer Science

University of Denver

In Partial Fulfillment

of the Requirements for the Degree

Master of Science

by

Zhuo Chen

March 2013

Advisor: Dr. Yun-Bo Yi
Abstract

The engagement of clutch system is a typical dynamic coupled temperature-displacement process which contains two or more sliding surfaces involved in frictional contact. This process influences evidently the thermal fatigue, noise and vibration of the clutch system. This effect can be more remarkable if the temperature distribution over the clutch disk is not uniform.

This thesis is organized into five chapters:

The first chapter is an introduction which reviews the background and previous work of thermoelastic instability and finite element analysis of clutch system and disk brake.

A parametric analysis of TEI (thermoelastic instability) is elaborated in the second chapter. A software package named “hotspotter” is used as a numerical tool. The objective of this chapter is to find the effect of geometrical dimension and the properties of material on TEI.

In the third and fourth chapters, several FEA (finite element analysis) models are developed by using ABAQUS to evaluate the thermomechanical processes in the frictional interface.

The fifth chapter is the summarization of the entire thesis.
Acknowledgements

First of all, I would like to give thanks to my advisor Dr. Yi for supporting me during my entire master’s study. Additionally I would like to thank my parents who were always there with me and devoted all they can to me.
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CHAPTER 1: INTRODUCTION

1.1 Brief Introduction to Clutches

Clutch is a widely used mechanical device which transfers power or motion from the driving component to the driven component when engaged. Usually, the clutch is installed in machines having two rotating shafts or axes. In these applications, one shaft or axis is connected with the driving member while the other shaft or axis is connected to the driven member. For example, the clutch connects the engine and the transmission of an automobile, for transmitting power and torques to the driveline.

Figure 1.1: Friction clutches

The most well-known type of clutch is frictional clutch as shown in Figure 1.1, in which the driven part is linked to the driving part by friction. In modern clutch system, the compound organic resin with copper wire facing or ceramic material serves as friction material. Usually, the ceramic friction material is equipped in heavy duty machines. By the cooling principle, frictional clutches can be classified as dry clutches and wet clutches.
The cooling liquid is used in a wet clutch to cool it down. The other types of clutch include belt clutch, dog clutch, hydraulic, electromagnetic clutch and so on.

1.2 Background and Motivation

1.2.1 Coupled Thermomechanical Phenomenon in Clutches

During the engagement process, the clutch system can be understood as an energy converter which transfers kinetic energy to heat energy. The frictional heat produced in the engagement process is absorbed by metal plates and friction plates. The thermal energy is diffused by means of heat conduction and dissipated to the environment by the convective heat transfer phenomenon. The heat generated by friction is uneven, because the sliding speeds at different points with different radii are not the same. The uneven heat generation produces non-uniform temperature distribution and hence non-uniform contact pressure distribution. Therefore, the engagement process of clutch system is a complex coupled thermal-mechanical problem. The thermal distortion, hot judder and thermalelastic instability are typical problems encountered in the engagement of clutch system.

The coning, SRO (surface run out) and DTV (disk thickness variation) are three typical classes of problem caused by thermal distortion. The primary reason of coning is the non-uniform distribution of thermal stress introduced by the non-uniform distribution of temperature. The SRO and DTV are resulted from thermal expansion. If the sliding speed is significantly high, the DTV phenomenon can be serious. Usually, we name the serious DTV as “hot judder”. A brief description of TEI will be introduced in the next section.
1.2.2 Brief Description of TEI

Figure 1.2: The mechanism of TEI

We always believe that the contact parts such as the metal plate and friction plate in clutches and the brake pad in disk brakes slide with respect to each other over a uniform area. In fact, there is always a disturbance in the environment or in the devices. Most often, the disturbance comes from the non-uniform temperature distribution or the imperfection left from manufacturing. The disturbance leads to the non-uniform distribution of contact pressure and hence the non-uniform heat generation during the sliding. The non-uniform heat generation in turn results in more serious thermal deformation and non-uniform contact pressure. If the sliding speed is sufficiently high, the thermal-mechanical feedback of the sliding system will be unstable. Here, the term “unstable” means that the contact area reduces to a small part of the nominal contact area, where the temperature rises rapidly. This mechanism is named as “thermalelastic
instability” or TEI in short. The schematic of TEI is shown in Figure 1.2. Through the Figure 1.2, we can conclude that the TEI is a positive feedback process. The TEI has been recognized as one of primary constraints in the designing of high performance sliding systems, such as aircraft disk brake and heavy duty clutch systems.

The TEI was first reported by Parker and Marshall (1948) in a railway brake test. What they found was the localization of heat generation during the train braking process. J.R. Barber (1968, 1969) studied the localization of contact and heat generation by using both experimental method and theoretical method. He found that thermal deformation results in the transformation of contact area from many small dots to several local areas (hot spots). If the thermal deformation weights out the effect of wear, the contact area changes will become unstable. Burton (1973) first proposed the frictional contact model based on the condition that there is an uneven contact pressure distribution in the components contacting with each other. Through his study, he created the concept of “critical speed”. He found that the TEI only happens when the sliding speed is higher than the critical speed. However, the predicted critical speed did not match the results observed experimentally. The other shortcoming of Burton’s theory is that there is no inherent length scale introduced in the model. For analyzing a specific system, special attentions must be paid to perturbations whose wavelengths below a certain value. Fec and Sehitoglu (1985) discovered that the TEI leads to the thermomechanical fatigue in the train brakes. Anderson and Knapp (1989, 1990) discovered that the locally distributed excessive temperature is the root cause of material degradation, cracks and unsatisfactory braking. They also proposed that there are four types of hot spots: asperity, focal,
distortional and regional. All kinds of hot spots, especially the focal hot spots, are primary reasons of braking fade and vibration. Barber and Lee (1993) developed a model in which a layer with finite thickness slides between two half planes on the basis of the previous work of Burton. Their conclusion shows that the antisymmetric mode is dominant and that the wavelength is proportional to the thickness of the layer. In other words, the hot spots are alternating on both sides of the disk. Their result explained the reason of discrepancy between the predicted critical speeds from Burton’s theory and the critical speeds observed experimentally. This model is widely used to analyze the clutch engagement process or the braking process. Vernersson (1999) tested both the cast iron brake blocks and composite brake blocks. He found that the root cause of uneven roughness in the brake disks and clutch plates: the disturbance during coupled thermomechanical braking process generates the hot spots where temperature is higher than the surrounding. Due to the phenomenon of thermal expansion, the hot spots come out more than the wheel tread, and therefore the wear effect is more obvious.

It is anticipated that the difference between the actual geometry of disk brake and clutch and Lee’s simplified two-dimensional model might bring some discrepancies in the stability behavior of system. Du combined the finite element analysis and Burton’s perturbation method to make this investigation possible. Du’s analysis assumed that the brake pad is completely rigid and thermally non-conducting and hence the growth rates of the perturbations are real. In other words, the perturbations do not migrate over the conducting interface. Yi used Du’s method to test the effect of geometric difference
towards the instability behavior. It was concluded that the critical speed is quite close to that of plain strain model when the hot spot number is large.

Du’s theory was based on the assumption that the friction material is completely rigid and thermally insulated. Based on this assumption, the migration speed of the perturbation is zero. However, we were unable to discover strong evidences to prove it. The perturbation migrates over the metal brake disk very slowly as the conductivity of pad material is remarkably lower than that of metal disk material. But this migration is of paramount importance on the stability behavior, because it decreases the thermal expansion and hence increases the lowest critical speed of system.

Yi used the eigenvalue method to implement Burton’s theory to evaluate the TEI phenomenon between two conductors by determining the exponential growth rate of the eigenmode. This analysis was integrated into the engineering software “hotspotter”. The hotspotter determines the exponential growth rate of eigenmodes for a given rotational speed by using the eigenvalue method. Then, the critical speed is computed. Here, the critical speed is defined as the lowest speed at which at least one mode is unstable.

1.2.3 Finite Element Analysis of Thermomechanical Phenomena in Clutches and Brakes

Zagrodzki (1985) analyzed the distribution of temperature and stress in the multidisk clutch system. This analysis was performed base on the assumption that the distribution of temperature does not effect the distribution of contact pressure and hence the distribution of heat flux generated by friction. The predicted thermal stress under this assumption is very low, even under the severe engagement condition. During this analysis, it was also found that the non-uniform heat flux caused by the different sliding
speeds at different locations along the radial direction results in severe increase in the circumferential stress.

It is natural to argue on the validity of the assumption. Usually, the initial distribution of contact pressure in the interface is not uniform. Due to the uneven heat flux and thermal distortion, the distribution of contact pressure changes. This phenomenon is so-called thermoelastic transition. Kennedy and Ling (1974) developed another model to analyze the thermomechanical problem in the interfaces between disks. In this analysis, they considered the effect of thermal distortion and wear on the distribution of contact pressure. But, the distribution of stress in the cross section of the disks was not examined. Their analysis was based on the assumption that the distributions of temperature, heat flux and stress are symmetric with respect to its midplane. In another word, all the phenomena taking place at the interfaces are local. It is obvious that there does not exist such a symmetrical plane in most cases. For example, the system is unsymmetrical in the first and last disks of the clutch systems because the frictional heat is only generated on one side of the disk at these locations.

Choi and Lee simulated the thermomechanical behavior occurring on the interface between brake pad and brake disk under the repeated braking condition. The distributions of temperature, contact pressure and stress were obtained during their analysis. The TEI phenomenon was also investigated. For the purpose of design optimization, the effect of material properties on the thermomechanical behavior during the braking process was also investigated. They concluded that the hoop stress component is the most important stress component as it has the largest value. The elastic modulus and the thermal
expansion coefficient are among the dominant factors affecting the contact ratio of the contact pair.

All above analyses are based on two-dimensional models. If the model is three-dimensional, the conventional finite element method simulates the thermomechanical behavior very inefficiently as the frictional heat generation is transient and a very fine mesh is required to overcome the numerical difficulty introduced by the convective term in the governing equation. Cho and Ahn developed a method which combines the Fourier transformation and FEM to analyze the thermomechanical phenomenon during the braking process and the engagement of clutch system. By using this technique, they transferred the three-dimensional model to a two-dimensional model which is considerably more cost saving.

1.2.4 Disadvantages of Past Work

Through the lecture review, it has been found that there are some major disadvantages of previous researches:

1. Currently, the coupled thermomechanical analysis of sliding systems such as clutches and disk brakes are primarily based on the condition that the initial temperature is uniform. In other words, most researchers ignored the fact that there is always initial SRO and DTV.

2. Researches towards the SRO and DTV are still lacking.

3. To reduce the computational cost, some models are oversimplified. Some researchers evaluated the coupled thermal-mechanical interaction by using the sequential couple method. In this method, the temperature distribution
is obtained first, and then the stress distribution. In fact, the frictional interaction and coupled thermal-mechanical phenomenon between two sliding components are complicated. Therefore, the result can only be obtained by using the direct couple method. In the direct couple method, the temperature distribution and stress distribution are computed at the same time.

4. The thermal boundaries in the model are always simplified. Among all the related past researches, an assumption on the temperature distribution of the surfaces with a uniform convection heat rate was applied. In fact there does not exist such a surface where the distribution of temperature is even.

5. All the material properties were treated as constants.
CHAPTER 2: PARAMETRIC ANALYSIS OF TEI

2.1 Overview of Previous Work on TEI

During the engagement of clutch system and the disk brake, the frictional heat is generated. If the sliding speed exceeds a certain value, the thermoelastic feedback of the system will be unstable. The term of “unstable” here means the appearance of localized pressure and temperature distribution in the sliding interface. The localization of temperature and pressure is called as “hot spots”. In the automotive community, there are four types of hot spots reported: asperity, focal, distortion and regional. The focal hot spots is the main reason of the failure and durability problems of disk brakes. The material damage and thickness variation which results in low frequency vibration also can be caused by the localization of contact pressure and temperature introduced by the TEI.

The thermoelastic instability phenomenon was first discovered and studied theoretically and experimentally by Barber in 1969. Dow and Burton found that if the sliding speed exceeds a certain critical value which is proportional to the wavelength of the perturbation, the sinusoidal perturbation is unstable. Burton analyzed the stability of sliding between two half planes by using perturbation method and found that the perturbation grows exponentially with time. Burton also built up the expressions of the critical speed. However, the predicted critical speed did not agree with the values observed experimentally. The other shortcoming of Burton’s theory is that there is no
inherent length scale introduced in the model. In analyzing a specific system, researchers and engineers are typically more interested in perturbations whose wavelengths below a certain value.

In contrast to Burton’s model, the actual brake disks or clutch plates have finite thicknesses. It might be anticipated that the geometrical difference which affects the perturbations with long wavelengths governing the stability of system can be an important factor. Hence the finite thickness of disks should be chosen as the length scale of the TEI evaluation of the automotive disk brake and clutch system. The first step towards modeling the real geometry of disk brake and clutches was provided by Lee and Barber in 1993. In their model, two stationary half planes representing brake pads are pressed against the layer representing the disk by a uniform pressure. Because the temperature perturbation only penetrates the half planes by a limited thickness, representing the brake pads by two half planes does not affect the stability of the frictional system. Their conclusion showed that the antisymmetric mode is dominant and that the wavelength is proportional to the thickness of the layer. Although Lee’s model is greatly simplified, it predicts the critical speed pretty well in some situations. As a result it is widely used in automotive industry for analyzing the TEI problem in disk brake and clutch systems.

It is argued that the difference in geometry between actual disk brake and clutch and simplification in Lee’s model might affect the stability behavior. Du combined the finite element analysis and Burton’s perturbation method to make this investigation possible. In Du’s analysis, the brake pad is assumed to be rigid and thermally non-
conducting. In this case, the growth rates of the perturbations are real, i.e. the perturbations do not migrate over the conducting interface. Yi used Du’s method to investigate the effect of geometric difference especially the finite width and axisymmetric configuration of the real brake disk and the clutch plate on the instability behavior. It was concluded that the critical speed approaches the solution of the plane-strain model when the hot spot number is large.

It must be pointed out that Du’s theory was based on the assumption that the friction material is completely rigid and thermally insulated. In this circumstance, the migration speed of perturbation is equal to zero. However, we were unable to justify this assumption. Burton’s analysis concluded that the perturbation moves over the metal brake disk very slowly as the conductivity of pad material is at least two orders of magnitude lower than that of disk material.

Yi used the eigenvalue method to implement Burton’s theory for evaluating the TEI phenomenon between two conductors by determining the exponential growth rate of the eigenmode. This analysis was integrated into the engineering software “hotspotter”. In the following section a parametric analysis of real clutch system used in industry will be performed by using this software.

2.2 Background of Hotspotter

All the background of Yi’s theory was elaborated in the literature. Only a short description will be presented here. As observed by Burton, the distributions of temperature and contact pressure grow exponentially with time. It might be assumed that the distributions of temperature, displacement and stress can be expressed as a product of
a function of spatial coordinates and a term increasing exponentially with time. In order to avoid the numerical difficulty introduced by convective terms, the Fourier reduction must be employed, i.e. the Fourier forms of the perturbation are expressed in the circumferential direction. For example, the temperature distribution can be written in following form.

\[ T(r, \phi, z, t) = \Re \left\{ e^{j \omega t + j m \phi} \Theta(r, z) \right\} \]

In the Cartesian coordinates system, the expression of temperature distribution will have the following form.

\[ T(x, y, z, t) = \Re \left\{ e^{j \omega t + j m \phi} \Theta(y, z) \right\} \]

The governing equation for body \( \beta \) is

\[ K_\beta \nabla^2 T - \rho_\beta c_\beta \left( \frac{\partial T}{\partial t} + V_\beta \frac{\partial T}{\partial x} \right) = 0 \]

where \( K, \rho \) and \( c \) are thermal conductivity, density and specific heat of a specific material respectively. After a substitution of the field expression and boundary conditions along with some mathematical manipulations, the following equation that defines the eigenvalue problem of the growth rate \( b \) is obtained.

\[ \left[ (K + C + j \nu \Phi) + b H \right] \Theta = 0 \]

\[ K = \int_{\Omega} K_\beta \left( \frac{\partial \Omega}{\partial \eta} \frac{\partial \Omega^T}{\partial \eta} + \frac{\partial \Omega}{\partial \zeta} \frac{\partial \Omega^T}{\partial \zeta} \right) \delta \Omega \]

\[ H = \int_{\Omega} \rho_\beta c_\beta WW^T d\Omega \]

\[ C = \int_{\Omega} \left( K_\beta m^2 + j m \rho_\beta c_\beta V_\beta \right) WW^T d\Omega \]
K and H are the conductivity matrix and the mass matrix. W consists of the shape functions. The term of “fVΦA” represents the nodal heat source generated by friction. The matrix A is a general thermoelastic matrix. The Φ is the matrix relating the local heat flux generated in the contact nodes to the heat flux at every node in the entire model. The relation can be written in the following form.

\[ Q = ΦQ_{con} \]

In the polar coordinates system, the matrix K, H and C will have the following forms.

\[
K = \int_Ω K_β \left( \frac{∂W}{∂r} \frac{∂W^T}{∂r} - \frac{W}{r} \frac{∂W^T}{∂r} + \frac{∂W}{∂z} \frac{∂W^T}{∂z} \right) dΩ
\]

\[
H = \int_Ω ρ_β C_β W W^T dΩ
\]

\[
C = \int_Ω \left( \frac{K_β n^2}{r^2} + jnρ_β C_β ω_β \right) W W^T dΩ
\]

In these expressions, ω is the relative rotational speed between the driving component and driven component.

In summary, the growth rate at a sliding speed is determined by using the eigenvalue method. The critical speed of a specific mode is evaluated by searching the lowest speed at which the growth rate is positive.

### 2.3 Problem Statement and Model Description

#### 2.3.1 Problem Statement

During a set of experiments, TEI-related hot spots were found in a clutch system as shown in Figure 2.1. It can be seen that the hot spots are distributed circumferentially.
However, some hot spots are distributed both circumferentially and radially as shown in Figure 2.2.

![Figure 2.1: Hot spots around circumference](image1)

![Figure 2.2: Hot spots in both radius/circumference](image2)

As we know scar-like hot spots can also be generated due to other reasons, such as the imperfection during manufacturing and misalignment of the system. There are three primary objectives for the current research:

1) To find out the mechanism of hot spot generation. If the engagement sliding speed is considerably higher than the calculated lowest critical speed, it can be concluded that the focal hot spots are indeed generated by TEI.
2) To test the susceptibility of the clutch system to TEI by evaluating the critical speed using *hotspotter*. The system whose lowest critical speed is relatively higher is less susceptible.

3) To propose an optimization method by performing parametric analyses of the system.

### 2.3.2 Model Description

The mechanical structure of the clutch is shown in Figure 2.3. According to the mechanical drawing and other technical data provided by the manufacturer, a back pressure of 5 psi is applied on the surface of one copper plate by inflating the air tube. The two friction plates and the iron plate are bonded together to serve as the driven component which is constrained along the axial direction at the inner radius. The two copper plates are driving components.

![Figure 2.3: The structure of clutch](image)
Although the mechanical structure is complex, only the dimensions of driving and driven components are needed for TEI analysis. In the numerical model of the clutch, the detailed structure of the clutch is not incorporated in the model: the spline sections, teeth and holes are ignored. We assume that all the components are manufactured and assembled perfectly. To minimize the computational cost, the symmetric boundary condition can be applied to the midplane of iron plate. This simplification greatly reduces the computational cost and does not affect the stability boundaries significantly.

The dimensions of all the parts are tabulated into Table 2.1. The idealized computational model is shown in Figure 2.4. All the material properties that will be used in the analysis are provided by the manufacturer and are tabulated in Table 2.2. The coefficients of friction are 0.22 and 0.34 for the standard lining and hico lining.

<table>
<thead>
<tr>
<th></th>
<th>Inner diameter</th>
<th>Outer diameter</th>
<th>Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two copper plates</td>
<td>419mm</td>
<td>979mm</td>
<td>9.53mm</td>
</tr>
<tr>
<td>Two friction plates</td>
<td>489mm</td>
<td>914mm</td>
<td>30.2mm</td>
</tr>
<tr>
<td>One iron plate</td>
<td>371mm</td>
<td>914mm</td>
<td>25.4mm</td>
</tr>
</tbody>
</table>

Table 2.1: Dimensions of the clutch parts
<table>
<thead>
<tr>
<th></th>
<th>Copper alloy</th>
<th>Cast iron</th>
<th>Standard lining</th>
<th>Hico lining</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus</td>
<td>117.2GPa</td>
<td>200GPa</td>
<td>0.11GPa</td>
<td>0.0873GPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.33</td>
<td>0.29</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Thermal expansion coefficient</td>
<td>17.7×10⁻⁶K⁻¹</td>
<td>12.2×10⁻⁶K⁻¹</td>
<td>14×10⁻⁶K⁻¹</td>
<td>14×10⁻⁶K⁻¹</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>347.5W/m·K</td>
<td>76.2W/m·K</td>
<td>0.2W/m·K</td>
<td>0.2W/m·K</td>
</tr>
<tr>
<td>Specific heat</td>
<td>385.2m²/s</td>
<td>447m²/s</td>
<td>1600m²/s</td>
<td>1600m²/s</td>
</tr>
<tr>
<td>Density</td>
<td>8910kg/m³</td>
<td>7870kg/m³</td>
<td>1900kg/m³</td>
<td>1900kg/m³</td>
</tr>
</tbody>
</table>

Table 2.2: The material properties

### 2.4 Results and Discussion

The software “Hotspotter” is very user-friendly and its usage is elaborated in the User’s Manual. Therefore, it will not be explained here. User input to the software includes the dimensions of all the components, material properties, the coefficient of friction and the boundary conditions. Outputs from the program are the growth rates of the eigenmodes, the critical speed as a function of hot spot number and the spatial forms of the dominant eigenmodes as shown in Figure 2.5 and Figure 2.6.

![Figure 2.5: Parameter editing interface](image-url)
We start the parametric analysis for standard lining using the fine mesh. The result is tabulated into Table 2.3. Next the analysis was performed to evaluate the convergence speed of the analysis. The critical speed is chosen as the key result in the convergence study due to its importance. The result obtained by using a very fine mesh is tabulated in Table 2.4. It is found that the largest discrepancy is 9.63 percent corresponding to mode 1 and that the critical speeds around the threshold (n=5,6,7,8,9) agree with the previous results very well. So, the method using a fine mesh is suitable for the parametric analysis.

<table>
<thead>
<tr>
<th>Mode</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical speed</td>
<td>62.50rad/s</td>
<td>81.05rad/s</td>
<td>85.94rad/s</td>
<td>63.48rad/s</td>
<td>53.7 rad/s</td>
<td>49.80rad/s</td>
</tr>
<tr>
<td>Mode</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>Critical speed</td>
<td>44.43rad/s</td>
<td>42.48rad/s</td>
<td>43.95rad/s</td>
<td>47.85rad/s</td>
<td>53.71rad/s</td>
<td>61.52rad/s</td>
</tr>
<tr>
<td>Mode</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Critical speed</td>
<td>71.29rad/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.3: The critical speeds calculated by using the fine mesh
<table>
<thead>
<tr>
<th>Mode</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Mode</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Critical speed</td>
<td>57.62rad/s</td>
<td>73.24rad/s</td>
<td>79.10rad/s</td>
<td>61.52rad/s</td>
<td>52.73rad/s</td>
<td>48.83rad/s</td>
</tr>
<tr>
<td>Critical speed</td>
<td>43.95rad/s</td>
<td>42.48rad/s</td>
<td>43.46rad/s</td>
<td>47.85rad/s</td>
<td>53.71rad/s</td>
<td>61.52rad/s</td>
</tr>
<tr>
<td>Critical speed</td>
<td>71.29rad/s</td>
<td>71.29rad/s</td>
<td>71.29rad/s</td>
<td>71.29rad/s</td>
<td>71.29rad/s</td>
<td>71.29rad/s</td>
</tr>
</tbody>
</table>

Table 2.4: The critical speeds calculated by using the very fine mesh

Figure 2.7: Typical temperature profiles
In Figure 2.7, several typical temperature profiles on the surfaces are shown. The profile shown in Figure 2.7(a) corresponds to mode 0. Usually, this model is called as a banding mode in which there are no focal hot spots generated. This mode usually does not affect the clutch system detrimentally. The focal hot spots distributed circumferentially are shown in Figure 2.7(b). The red regions correspond to relatively higher temperatures whereas the blue regions represent relatively lower temperatures. This distribution of hot spots is most frequently seen. It has to be pointed out that the mode is antisymmetric, which means the hot spots alternate on both sides of the disk. In Figure 2.7(c), the focal hot spots are distributed both circumferentially and radially. Usually, this type of temperature profile corresponds to a significantly high critical speed. If the clutch system operates at a sliding speed considerably higher than the critical speed, according this type of mode, there will be a number of small size hot spots generated on the interface.

![Graphs showing critical speed as function of hot spot number](image)

(a) Standard lining  
(b) Hico lining

Figure 2.8: The critical speed as function of hot spot number
The critical speeds from mode 0 to mode 12 for both friction materials are shown in Figure 2.8. The result corresponding to Figure 2.8 is also tabulated in Table 2.5. Through the figure and table, we can conclude that mode 7 (i.e. 7 focal hot spots distributed on the interface) is the dominant mode for both materials as the critical speeds are the lowest ones. It is clearly seen that the lowest critical speed (corresponding to mode 7) calculated from Hico lining is about 12.21 rad/s lower than that of the standard lining. This decrease is probably caused by the relatively lower elastic modulus or the higher coefficient of friction. If the sliding speed is slightly higher than the critical speed, the hot spots will not be generated. This is because the computed critical speed corresponds to a growth rate slightly above zero. Only when the sliding speed is considerably higher than the critical speed, the temperature perturbation will grow rapidly, leading to hot spot development. The critical speeds of the neighboring modes (mode 6 and mode 8) are close to that of the dominant mode. Therefore the actual hot spot pattern is determined by the detailed structure of the system and the boundary conditions as well.

<table>
<thead>
<tr>
<th>Mode</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical speed</td>
<td>43.43rad/s</td>
<td>56.64rad/s</td>
<td>57.62rad/s</td>
<td>42.48rad/s</td>
<td>37.11rad/s</td>
<td>34.18rad/s</td>
</tr>
<tr>
<td>Mode</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>Critical speed</td>
<td>30.76rad/s</td>
<td>30.27rad/s</td>
<td>32.23rad/s</td>
<td>35.64rad/s</td>
<td>40.53rad/s</td>
<td>46.88rad/s</td>
</tr>
<tr>
<td>Mode</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Critical speed</td>
<td>54.69rad/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.5: The computed critical speeds by using Hico lining
The temperature distribution in cross section of mode 7 is shown in Figure 2.9. As the thermal conductivity of copper is higher than the thermal conductivity of the standard lining by two orders of magnitude, the temperature barely penetrates into the friction plate. The red region on the interface represents the hot spot where the temperature is relatively higher. Because the mode is antisymmetric, the temperature on the opposite side of disk is relatively low.

The mesh distribution in the iron plate is uniform because of the mild temperature change in the interior. However, the finite element mesh distribution in the friction plate is biased towards the interface. The reason is that the thermal conductivity of friction lining is low and consequently the temperature variation through the friction lining thickness is sharp.
To optimize the clutch design against TEI, a parametric analysis is performed. The parametric analysis on the effect of the material properties will be presented first. All the results are tabulated into the following tables. For convenience, only the standard lining will be used in the parametric analysis.

Figure 2.11: The critical speeds when the conductivity of friction material is increased by 10 times

<table>
<thead>
<tr>
<th>Mode</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical speed</td>
<td>57.62rad/s</td>
<td>96.68rad/s</td>
<td>119.1rad/s</td>
<td>95.70rad/s</td>
<td>85.94rad/s</td>
<td>83.98rad/s</td>
</tr>
<tr>
<td>Mode</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>Critical speed</td>
<td>77.15rad/s</td>
<td>76.17rad/s</td>
<td>82.03rad/s</td>
<td>95.70rad/s</td>
<td>117.2rad/s</td>
<td>148.4rad/s</td>
</tr>
<tr>
<td>Mode</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Critical speed</td>
<td>189.5rad/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.6: The critical speeds when the conductivity of friction material is increased by 10 times

Through the above figure and table, we can find that the critical speeds of all the modes are increased, but mode 0 is an exception whose critical speed stays at 57.62 rad/s. The critical speeds of the higher order modes increase more significantly than those of
the low order modes. The dominant mode is still mode 7 whose critical speed increases to 76.17 rad/s.

![Figure 2.12: The critical speeds when the elastic modulus of friction material is halved](image)

<table>
<thead>
<tr>
<th>Mode</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>101.6rad/s</td>
<td>136.7rad/s</td>
<td>113.3rad/s</td>
<td>86.91rad/s</td>
<td>78.13rad/s</td>
<td>68.36rad/s</td>
</tr>
<tr>
<td>Mode</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>65.43rad/s</td>
<td>69.34rad/s</td>
<td>78.13rad/s</td>
<td>91.80rad/s</td>
<td>109.4rad/s</td>
<td>130.9rad/s</td>
</tr>
<tr>
<td>Mode</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Critical speed</td>
<td>156.3rad/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.7: The critical speeds when the elastic modulus of friction material is halved

By reducing the elastic modulus of the friction material, the critical speed of each mode increases by 50% to 100%. The dominant mode has switched to mode 6.

![Figure 2.13: The critical speeds when the expansion coefficient of friction material increases by 2 times](image)
<table>
<thead>
<tr>
<th>Mode</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical speed</td>
<td>51.76rad/s</td>
<td>73.24rad/s</td>
<td>79.10rad/s</td>
<td>61.52rad/s</td>
<td>52.73rad/s</td>
<td>48.83rad/s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical speed</td>
<td>43.95rad/s</td>
<td>42.48rad/s</td>
<td>43.46rad/s</td>
<td>47.85rad/s</td>
<td>53.71rad/s</td>
<td>61.52rad/s</td>
</tr>
</tbody>
</table>

| Mode | 12  |
|------|
| Critical speed | 71.29rad/s |

Table 2.8: The critical speeds when the expansion coefficient of friction material increases by 2 times

It is obvious that the expansion coefficient barely affects the critical speeds.

![Figure 2.14: The critical speeds when the thickness of copper plate is halved](image)

<table>
<thead>
<tr>
<th>Mode</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical speed</td>
<td>171.9rad/s</td>
<td>187.5rad/s</td>
<td>187.5rad/s</td>
<td>185.5rad/s</td>
<td>175.8rad/s</td>
<td>164.1rad/s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical speed</td>
<td>150.4rad/s</td>
<td>134.8rad/s</td>
<td>127.0rad/s</td>
<td>117.2rad/s</td>
<td>109.4rad/s</td>
<td>105.5rad/s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical speed</td>
<td>105.5rad/s</td>
<td>107.3rad/s</td>
<td>111.3rad/s</td>
<td>117.2rad/s</td>
<td>125.0rad/s</td>
</tr>
</tbody>
</table>

Table 2.9: The critical speeds when the thickness of copper plate is halved

Apparently, the outline of the curve is changed as the critical speeds are remarkably increased by 200% to 300%. It has been found that Mode 11 is dominant;
whose critical speed is 105.5 rad/s. The critical speed of mode 12 is almost the same as that of mode 11. Therefore the actual number of hotspots is dependent on the structural details and the material properties of the system.

![Graph showing critical speeds](image)

**Figure 2.15: The critical speeds when the thickness of friction plate is halved**

<table>
<thead>
<tr>
<th>Mode</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical speed</td>
<td>38.09 rad/s</td>
<td>44.43 rad/s</td>
<td>44.92 rad/s</td>
<td>44.92 rad/s</td>
<td>37.60 rad/s</td>
<td>33.69 rad/s</td>
</tr>
<tr>
<td>Mode</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>Critical speed</td>
<td>32.23 rad/s</td>
<td>29.30 rad/s</td>
<td>27.83 rad/s</td>
<td>27.83 rad/s</td>
<td>29.30 rad/s</td>
<td>32.23 rad/s</td>
</tr>
<tr>
<td>Mode</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Critical speed</td>
<td>35.64 rad/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 2.10: The critical speeds when the thickness of friction plate is halved**

The critical speeds are reduced by around 50% after the thickness of friction plate is halved. The dominant mode is mode 8, whose critical speed is 27.83 rad/s. In contrast, increasing the thickness of friction plate will increase the critical speed.
Table 2.11: The critical speeds when the thickness of iron plate is halved

<table>
<thead>
<tr>
<th>Mode</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>57.62rad/s</td>
<td>73.24rad/s</td>
<td>79.10rad/s</td>
<td>61.52rad/s</td>
<td>52.73rad/s</td>
<td>48.83rad/s</td>
</tr>
<tr>
<td>Mode</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>43.95rad/s</td>
<td>42.48rad/s</td>
<td>43.46rad/s</td>
<td>47.85rad/s</td>
<td>53.71rad/s</td>
<td>61.52rad/s</td>
</tr>
<tr>
<td>Mode</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>71.29rad/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The thickness of iron plate dose not effect the critical speed.

Figure 2.17: The critical speeds when the outer radius of copper plate is the same as that of friction plate
<table>
<thead>
<tr>
<th>Mode</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical speed</td>
<td>56.64rad/s</td>
<td>71.29rad/s</td>
<td>78.13rad/s</td>
<td>62.50rad/s</td>
<td>53.71rad/s</td>
<td>50.78rad/s</td>
</tr>
<tr>
<td>Mode</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>Critical speed</td>
<td>47.36rad/s</td>
<td>45.41rad/s</td>
<td>45.90rad/s</td>
<td>48.83rad/s</td>
<td>53.71rad/s</td>
<td>60.55rad/s</td>
</tr>
<tr>
<td>Mode</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Critical speed</td>
<td>68.36rad/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.12: The critical speeds when the outer radius of copper plate is the same as that of friction plate

The radial width of the copper plate does not affect the critical speed considerably. The lowest critical speed is increased by 2.93 rad/s. The decreases in the critical speeds of other modes are not significant.

2.5 Conclusions

1) The hot spots observed in the tests are caused by the TEI phenomenon as the operating sliding speed is higher than the predicted critical speed.

2) The critical speed of the dominant mode (mode 7) is 42.48 rad/s when the standard lining is used. The critical speed of the dominant mode (mode 7) is 30.27 rad/s when the Hico lining is used. Obviously, the Hico lining is less resistant to the TEI problem as the minimum critical speed is lower.

3) Only when the sliding speed is considerably higher than the critical speed, the temperature perturbation can grow rapidly and hence the hot spots start to develop, because it takes time for the hot spots to develop on the disk surfaces.
4) The critical speeds of the neighboring modes (mode 6, 8 and 9) are very close to that of mode 7. Therefore, the actual scars left by the TEI phenomenon might be superimposed by these neighboring modes.

5) According to the parametric analysis, there are several potential methods to increase the critical speed of the system. (i) reducing the elastic modulus of friction material, (ii) increasing the conductivity of the friction material, (iii) increasing the thickness of the friction plate, (iv) decreasing the thickness of copper plate.

6) It is found that the critical speed of every mode significantly increases when the thermal conductivity is increased by 10 times. The critical speed of model 0 is the only exception. But it is the thermally buckling mode and does not develop the hot spots.

7) It is shown that the thermal expansion coefficient almost does not effect the susceptibility of system to the TEI problem.

8) If the elastic modulus of friction material is halved, the critical speed of every modes increases and the dominant mode switches to mode 6. Because the application of the friction makes the distribution of contact pressure more uniform. This argument was proved by Anderson and Knapp in the past.

9) We can conclude that reducing the thicknesses of copper plate will increase the critical speed considerably, but reducing the thicknesses of friction plate will decrease the critical speed. The thickness of iron plate does not effect the critical speed for all the modes.
10) Suppose that we simultaneously change the material properties in the following way: (1) the thermal conductivity of friction material is increased by 10 times; (2) the elastic modulus of friction material is reduced by half; (3) the thickness of friction plate is doubled; and (4) the thickness of copper plate is halved, the lowest critical speed of system will be increased considerably. The dominant mode will switch to mode 8 in that case.

Figure 2.18: The critical speeds when the following parameters are changed simultaneously: the thermal conductivity of friction material is increased by 10 times, the elastic modulus of friction material is reduced by a half, the thickness of friction plate is doubled and the thickness of copper plate is halved
Table 2.13: The critical speeds when the following parameters are changed simultaneously:

- the thermal conductivity of friction material is increased by 10 times,
- the elastic modulus of friction material is reduced by a half,
- the thickness of friction plate is doubled and the thickness of copper plate is halved.

<table>
<thead>
<tr>
<th>Mode</th>
<th>0</th>
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CHAPTER 3: THERMOMECHANICAL PROCESSES IN CLUTCH SYSTEM

UNDER EVENLY DISTRIBUTED TEMPERATURE FIELD

In this chapter, several ABAQUS models are created. With these models, the feasibility of temperature-displacement simulation for clutch system by using ABAQUS is explored. A two-dimensional model is constructed first, followed by a three-dimensional model. The results from these two differently models are compared.

3.1 Background and Literature Review

3.1.1 Determination of Local Convective Heat Transfer Coefficient in Disk Model

There are three modes of heat transfer: conduction, convection and radiation. The convection mode occurs when the temperature gradient exists between a solid surface and a moving fluid (liquid or gas). The convection mode transfers the heat energy by two mechanisms: the random molecular motion and the macroscopic motion of the fluid. There are two types of heat convection: the forced convection and free (or natural) convection. In the forced convection, the motivation of fluid flow is provided by external force, such as fan and wind. In the free (or natural) convection, the motion of fluid is caused by the difference in density induced by the difference in temperature. The rate equation of convection is Newton’s law of cooling.

\[ q'' = h(T_s - T_\infty) \]

where \( q'' \), \( h \), \( T_s \) and \( T_\infty \) are the convective heat flux, convective heat transfer coefficient, temperature of surface and environmental temperature, respectively.
In the interface between fluid and surface, the velocity of fluid, the chemical concentration and temperature may change. The layer with finite thickness over the interface can be defined as velocity boundary layer, concentration boundary layer and thermal boundary layer. Furthermore, the heat convection and chemical species convection taking place in the interface can be represented by the following dimensionless equations.

\[
Nu = C_1Re^{m1} Pr^{n1} \\
Sh = C_2Re^{m2} Sc^{n2}
\]

These two processes are analogous as the dimensionless governing equations have the same form. The concept of analogy is of great importance. This is because when the dimensionless parameters for mass convection are known, the dimensionless parameters for heat convection can be obtained by analogy.

As a number of engineering products such as clutch plates and automotive tires can be simplified as rotating disks, the heat and mass transfer characteristics on rotating disks have been widely studied. Von Karman (1921) found that the boundary layer thickness of the laminar flow induced by the rotation of a disk in still air is constant over the entire disk. Wagner (1948) assessed the convective heat transfer coefficient over a rotating disk in the laminar regime based on Von Karman’s result. The following relation was established.

\[
Nu_r = a\sqrt{Re_r}
\]

where \(a\) is a function of the Prandtl number. When the Prandtl number equals 0.74; \(a\) is 0.335; The \(Nu_r\) and \(Re_r\) in the above equation are the local Nusselt number and Reynolds
number. Millsaps and Pohlhauson (1952) solved the governing equation and concluded that when the Prandtl number is equal to 0.71, the constant $a$ approaches 0.325. Cobb and Sanders (1956) performed an experimental investigation on the rotating disk system in still air. In their conclusion, the transition from laminar flow to turbulent flow starts when the Reynolds number equals about 240,000. By using the Reynolds analogy, they detected that when the boundary layer is turbulent the relationship between the local Nusselt number and the local Reynolds number is

$$Nu_r = 0.0193 \frac{Re}{Pr}$$

Popiel and Boguslawski (1975) reported a similar relation for the rotating disk system in still air:

$$Nu_r = 0.0188 \frac{Re}{Pr}$$

in which they considered the free convection effect. Cardone, Astarita and Carlomagno (1997) published their own relation for the rotating disk system in still air. When the boundary layer is turbulent the relationship between the local Nusselt number and the local Reynolds number is

$$Nu_r = 0.0163 \frac{Re}{Pr}$$

The reason of the obvious discrepancy between these results is that the difference between the adiabatic wall temperature and the ambient temperature was ignored in the previous work. The analytical and experimental investigations about the heat and mass transfer characteristics of rotating disks in still air will not be elaborated here.

Some researchers evaluated the convective heat and mass transfer characteristics over rotating disks in nonstatic air flow. Evans, Greif (1988) and Yen, Wang (1992)
analyzed the convective heat and mass transfer coefficients from a rotating disk to the uniform air flow which is parallel to the axis of the disk. Several researchers such as Popiel, Boguslaeski (1986) and Brodersen Metzger (1992) performed investigations on the convective heat transfer characteristics on rotating disks in non-centered impinging jet. Astarita and Cardone (2007) evaluated the convective heat transfer coefficient over a rotating disk under a centered impingement. He, Ma and Huang (2005) analyzed the convective heat transfer coefficient from a rotating disk to the uniform air flow parallel to the surface of the disk. As the relations between the local Nusselt number and the local Reynolds number were established explicitly in the last two articles, their research findings will be explained in more detail in the current work.

Astarita and Cardone (2007) assumed that by taking account the disk rotation only, the boundary layer over the entire disk is laminar. A new similitude parameter $\xi$ was introduced to represent the ratio of the momentum caused by jet and the momentum caused by rotation of disk.

$$\xi^2 = \text{Re}^2 \left( \frac{V}{\omega \delta} \right)^2$$

It is expected that the largest $h$ occurs at the center of the disk and $h$ decreases monotonically as moving outward. Actually, due to the interaction between the jet flow and the flow caused by disk rotation, the boundary layer could transfer from laminar to turbulent. A transition Reynolds number is defined as follows:

$$Re_{tr} = 11200 \text{Re}_{j}^{0.24}$$
All the results are summarized into the following equations along with Figure 3.1 and Figure 3.2.

\[
\frac{h_0}{k} \sqrt{\frac{v}{\omega}} = 0.33 + 1.57 \xi
\]

\[
\frac{h_0}{k} \sqrt{\frac{v}{\omega}} = 1.81 \xi^{0.597}
\]

The \( h_0 \) here is the convective heat transfer coefficient measured at stagnation point. By using the following two figures, the convective heat transfer coefficient at any position over the disk can be estimated. Here, \( z \) and \( r \) are the nozzle-to-surface distance and local radius, respectively.

He, Ma and Huang (2005) analyzed the convective heat transfer coefficient from a rotating disk to the uniform air flow parallel to the surface of the disk by using the naphthalene sublimation technique. The Sherwood number and convective mass transfer coefficient are measured first. By using the analogy method, the Nusselt number and convective heat transfer coefficient can be determined. Their conclusion is summarized in the following equations.

\[
Nu = 0.00588 Re_\xi^{0.925}
\]

\[
Re_\xi = \frac{(\omega r + V)r}{v}
\]
3.1.2 Element Selection in Contact Problem

The Finite Element Method is an efficient and powerful tool to solve complicated engineering problems, including the contact problem. The element selection plays a very important role in the simulation of contact problem. The inappropriate element selection can result in inaccuracy results, even the convergence problem. For the relatively high accuracy, the high order elements are usually preferred in the simulations of engineering problems without contact.
In isoparametric elements, the displacement field (or other field variable) and the geometry of the element are interpolated by the same shape function. Usually, the higher order interpolation function produces more complicated element geometry and hence relatively high accuracy. In the global coordinates, any hexahedral elements with curved edges or straight edges can be represented by cube elements in the auxiliary coordinates system.

When the density of the surface force on the top surface of the element is \( \{q\}^e = (q_x, q_y, q_z)^{-1} \), the equivalent nodal force is

\[
\iint [N]^T \{q\}^e ds = \int_1^1 \int_1^1 [N]^T \{q\}^e \sqrt{EG - F^2} \, d\xi \, d\eta
\]

\[
[N] = (IN_1, IN_2, \ldots, IN_n) \quad n = 8 \text{ or } 20
\]

where \( N_n \) is shape function of node \( n \).

If the \( \{q\}^e = (0,0,-1)^{-1} \), the summation of equivalent nodal loads in quadratic element is equal to -1. It can be concluded that the quadratic hexahedral elements can not be used in contact problems.

There is a defect which is called shear locking about the fully integrated linear solid element. The reason of this defect is that the mesh consisting of fully integrated linear elements displays spurious transverse shear strain which absorbs energy. There are
two potential solutions to eliminate this defect: usage of quadratic elements and usage of reduced integrated linear elements. The other advantage of the reduced integrated element is that the reduced integrated elements save a lot of CPU time as less integration points are needed. The disadvantage of reduced integrated elements is that the “hourglassing” phenomenon in which strains calculated at integration nodes are zero and hence the mesh deforms excessively. The solution to the phenomenon is to add the artificial stiffness to elements.

3.1.3 Solution of Contact Problem

If there are two three-dimensional elastic bodies in contact with each other, the FEA governing equations for both bodies can be developed in the global coordinate system.

\[
\begin{align*}
K_A \delta_A &= F_A + R_A \\
K_B \delta_B &= F_B + R_B
\end{align*}
\]

where \( K_A, K_B, \delta_A, \delta_B, F_A, F_B, R_A \) and \( R_B \) are the global stiffness matrix of two bodies, nodal displacement vectors of two bodies, external load vectors and unknown contact force applied on both bodies. After the assembling, the above two equations can be combined into the global matrix equation as follows.

\[
K' \delta' = F' + R'
\]

The global governing FEA equation cannot be solved directly as the number of unknowns is more than that of the equations. Therefore, additional relations which indicate the contacting conditions are needed to solve the above equations iteratively.
3.1.4 Brief Introduction of ABAQUS

The ABAQUS is an engineering software package which is capable of simulating a wide range of engineering problems based on the finite element method. ABAQUS was first released in 1978 by Hibbitt, Karlsson & Sorensen, Inc. ABAQUS is able to solve a great quantity of engineering problems ranging from relatively easy linear problems to complex nonlinear problems. It provides a widely ranging element library which brings the ability to depict any complicated geometry. It also provides an extensive material library which guarantees the accuracy of material behavior in simulations. Besides the traditional structural analysis, it is also able to process the heat transfer, mass diffusion, acoustics, soil mechanisms and so on. In the ABAQUS, there are two main analysis products: Standard and Explicit. The Abaqus/Standard is a general-purpose analysis product which is able to simulate both linear and nonlinear problems. The Abaqus/Standard employs the Newton’s method or the Newton Raphson method to solve equations iteratively. Therefore the solution may possibly diverge. In Abaqus/Explicit, the explicit dynamic finite element formulation is used to simulate problems in every time increment. So, no convergence problem will occur in this product. In Abaqus/standard, the equations are solved iteratively, and therefore the cost of solving in each increment is more expensive. On the other hand, the iteration is not needed in the Explicit package, and therefore the cost of solving in each increment is less expensive. However, more increments are required. Overall, Abaqus/Standard requires less computational time.
3.2 Two-Dimensional Basic Model

3.2.1 Model Description

The dimensions of parts, material properties, mechanical load and boundary conditions are the same as those mentioned in Chapter 2. So, they will not be repeated here. The development of the two-dimensional model is based on the following assumptions:

1) The initial temperature distribution is even. The entire clutch is at the temperature of 20 degrees centigrade. The ambient temperature is 20 degrees centigrade.

2) The distribution of temperature does not affect the distribution of contact pressure and hence the distribution of heat flux generated by friction is not affected by the distribution of temperature. In another word, there is no thermoelastic transition. Usually, this assumption is valid when the initial temperature distribution is even and the sliding speed is small.

3) The convective heat transfer occurs at the surface of copper plate. The boundary layer condition over this surface is laminar and hence the convective coefficient over the copper plate is constant. The sliding speed and convective coefficient are computed as follows.

4) The relative rotation between the driving component and the driven component lasts for two seconds at a constant angular speed.
The local Reynolds number of the rotating disk system in still air is defined as

\[ Re = \frac{\omega r^2}{\nu} \]

where \( \omega \), \( \nu \) and \( r \) are the angular speed, kinematic viscosity and local radius, respectively. When the ambient temperature is 20 degrees centigrade, and the kinematic viscosity of air is \( 15.11 \times 10^{-11} \, \text{m}^2/\text{s} \). 240,000 is recognized as the value of the critical Reynolds number for the rotating disk in still air. In the still air, the highest angular speed at which the boundary layer over the whole rotating disk remains laminar can be determined by the following equation.

\[ \frac{\omega_l R_o^2}{\nu} = 240000 \]

\[ \omega_l = 240000 \frac{\nu}{R_o^2} \]

\[ = 24000 \times 15.11 \times 10^{-6} + 0.489 \times 15.13 \]

\[ = 15.13 \, \text{rad/s} \]

According to the literature review, the convective heat transfer coefficient can be determined by the following equation.

\[ h = a k \sqrt{\frac{\omega}{\nu}} \]

\[ = 0.33 \times 0.0257 \times \frac{15.13}{15.11 \times 10^{-6}} \]

\[ = 8.49 \, \text{W/(m}^2\cdot\text{C)} \]

3.2.2 Development of Two-Dimensional Heat Transfer Model

The steps needed to build the model are elaborated here.

1) Double-click the Parts container to create an axisymmetric, deformable part. In the Create Part dialog, copper is an input name of the part. The approximate size is 2.
Then click the continue bottom. As the clutch cannot be idealized as the plane strain or plane stress condition, we are unable to use two-dimensional planar elements to model it.

2) Click the Create lines: rectangle button in toolbox to create a rectangle. As shown in Figure 3.3, (0.2095, 0), (0.4895, 0), (0.4895, 0.00953), (0.2095, 0.00953) are input successively in the prompt area as the coordinates of points in four corners of the rectangle. And the “click done” bottom in the prompt area to exist the sketcher.

![Figure 3.3: The copper plate](image)

3) By using the same procedures, the other two parts whose names are friction and iron are created to represent the friction plate and iron plate.

4) Double-click the material container in the model tree to create the material copper. 8910 is the input as the density under the General menu. In the Elasticity submenu of Mechanical menu, 117.2×10⁹ and 0.33 are the inputs as the values of Young’s modulus and Poisson’s ratio, respectively. In the expansion option under the Mechanical menu, 1.77×10⁻⁵ is the input for the expansion coefficient. In the conductivity option under the Thermal menu, 347.5 is the input for the value of conductivity coefficient. Under the same menu, 385.2 is the input as the value of
specific heat. Then, click OK to complete the definition of material copper. By suing the same procedure, the other two kinds of material: friction and iron can be defined.

5) Double-click sections bottom to enter the Edit Section dialog. In this dialog, copper is input as the name of the section. Solid and Homogeneous are selected as category and type respectively. Select the material copper for this section. This step is showed in Figure 3.4.

![Create Section dialog](image.png)

**Figure 3.4: Create Section dialog**

6) By using the same procedures, another two sections are also created.

7) Double-click the Instances bottom in the model tree to create instances for all three parts. Independent is selected as the type for all instances.

8) We assume that there is no heat loss in the interface between friction plate and iron plate. So, we just need to merge the part friction and the part iron together. For merging them together, we click the Merge/Cut Instances in the toolbox first. In the Merge/Cut Instances dialog, bottom is input as the name of new instance. For assigning different section properties to the original parts friction and iron, the
Intersecting boundaries option is selected as Retain. The merged part is showed in Figure 3.5.

![Figure 3.5: The merged part](image)

9) If one face of a contact pair is larger than the other, Abaqus may deny the simulation. For defining the contact between Part copper and Part friction correctly, Part copper is partitioned. The next step is to click the Tools option in the menu bar. In the Create Partition dialog, Face and Sketch are chosen as the Type and Method. In the viewpoint, connect point 1, 2 and 3, 4 by two line segments as showed in Figure 3.6. After clicking “Done” in the prompt area, Part copper is divided into three pieces.

![Figure 3.6: Partition the part copper by two line segments](image)

10) Assign the section properties to the parts by double-clicking the Section Assignments bottom under the Parts container.

11) After the initial step, a new heat transfer step named “heat” is created. The response is set to “transient” and the time period is set to 2 seconds. In the increment tab of
Edit Step dialog, 1,000,000 is an input to the Maximum number of increments. If the simulation cannot be completed within the maximum number of increments, Abaqus will terminate the simulation. Therefore a sufficiently large number should be chosen as the input to the Maximum number of increments.

12) Double click the interaction properties container in the model tree. In the Mechanical tab, all the options of Normal Behavior are chosen as default. For defining a contact condition without heat dissipation, a large value of thermal conductance is specified.

![Edit Interaction dialog]

**Figure 3.7: Definition of convection**

13) Create two interactions whose names are defined as *convection* and *copper-friction*. The top surface of Part copper is chosen as the region of interaction convection. As shown in Figure 3.7, 8.49 and 20 are the Film Coefficient and Sink Temperature. The top surface of Part bottom is chosen as the master surface of the interaction copper-
friction. The surface of Part copper overlapping with the master surface is chosen as the slave surface.

14) By using the predefined field function, 20 is defined as the initial temperature over the entire assembly as showed in Figure 3.8.

![Figure 3.8: Predefined temperature](image)

15) In the Load module, the surface heat flux which is a function of the applied pressure and the radius is created.

16) Mesh the entire assembly and submit the job.

![Figure 3.9: Temperature distribution of two-dimensional heat transfer model](image)

The simulated temperature distribution in the clutch system is shown in Figure 3.9. The temperature of the blue region is relatively lower, while the red region has
relatively higher temperature. The highest temperature is found at the rightmost of the interface as the linear sliding speed, and hence the frictional heat generation, is the maximum at that location. For a very low thermal conductivity, the temperature in the most part of the friction plate stays at 20 °C.

3.2.3 Coupled Temperature-Displacement Model

Because the simulation steps of the coupled temperature-displacement model are almost the same as the heat transfer model. Here listed are the differences only:

- Run Abaqus.
- Create three parts: Copper, Friction and Iron.
- Create three materials.
- Create three sections: Copper, Friction and Iron.
- Create three instances: Copper, Friction and Iron.
- Merge the instance Friction and Iron to a new instance Bottom. A new part Bottom is also generated at the same time.
- Assign the section properties to parts.
- Delete the Heat Transfer step and create a new General, Static step pressure which Time period is 1 second. After the step pressure, a coupled temperature-displacement step heat whose time period is 2 seconds is created.
- Partition the part copper in a proper way.
- Create interaction property.
- Define the interaction.
In addition to the heat flux, another load pressure is applied to the bottom of the entire assembly as showed in figure 3.10.

Figure 3.10: Distribution of load pressure

- The leftmost two lines are fixed in the x-direction. The top line is fixed in the y-direction.
- Submit the job. The simulated results are shown in Figure 3.11 and Figure 3.12.

Figure 3.11: Temperature distribution of 2-D coupled displacement-temperature model
As shown in Figure 3.11, the temperature distribution of the coupled temperature displacement model is exactly the same as that of the heat transfer model. This implies that both models are correct. Figure 3.12 shows the distribution of contact pressure along the interface at the end of analysis (the nodes are selected from the leftmost to rightmost of the interface). It is obvious that there is a plateau in the middle of the interface, showing a very slight variation.

### 3.3 Three-Dimensional Thermomechanical Model

It is not clear whether the assumption that the temperature distribution is independent of the distribution of contact pressure and hence the distribution of frictional heat flux is valid. So, the preliminary objective of this section is to verify the validity of the assumption.
3.4 Model Description and Development

The dimensions of parts, material properties, mechanical load and boundary conditions are the same as mentioned in the previous section. The three-dimensional models are developed by applying the following conditions.

1) The initial temperature distribution is even. The distribution of temperature over the entire clutch is shown in Figure 3.13.

2) The relative sliding velocity between the copper plate and the assembly of friction plate and iron plate is 15.13 rad/s.

3) The copper plate serves as the driving component of the system. Its rotational speed with respect to the axis is constant.

4) The convection heat transfer takes place at one surface of copper plates, which is not in contact with the friction plate. The distribution of the convective local heat transfer coefficient can be determined by the equations described in previous section.

All three parts were created as shown in Figure 3.14.
Figure 3.14: The parts in three-dimensional model

For controlling the rotation of the copper plate, the inner part of the copper plate is coupled with a reference point which is coincident with the origin of the global coordinate system of the model as shown in Figure 3.15.

By defining the velocity boundary condition of the reference point, the rotation speed of the copper plate is easily controlled. As shown in Figure 3.16, the boundary conditions from V1 to VR2 are set to 0. If their values were not zero, there would be a nonzero rigid body rotation in the copper plate. The boundary condition VR3 is the rotational speed with respect to the z-axis. In this model, its value is the relative rotational velocity between the copper plate and the friction plate.
Figure 3.15: The velocity of the inner annular ring is defined at the reference point

Figure 3.16: The angular velocity of the reference point
The mesh of the model is shown in Figure 3.17. As mentioned in the previous section, the three-dimensional coupled temperature-displacement reduced integration element type is used, and the hourglass control is set to *enhanced*.

### 3.5 Result and Discussion

As mentioned, the preliminary objective of this model is to verify the validity of the assumption that the distribution of temperature does not affect significantly the distribution of contact pressure and hence the distribution of heat flux generated by friction. The distributions of temperature in the cross section of both two models are displayed here.
Figure 3.18: The temperature distributions in both 2-D and 3-D models. “NT11” represents the nodal temperature (°C)

In Figure 3.18, the results from the three-dimensional model are placed before the results obtained from the two-dimensional model. Through the figure, we can find that the temperature distributions in the cross section of the two-dimensional model and those of the three-dimensional model are almost the same at the beginning (T≤ 1.2) of the relative rotation. As time goes on, the high temperature region (red region) of the three-dimensional model becomes narrower than that of the two-dimensional model. The difference of the highest temperature in the two models increases as the process continues.
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</tr>
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<tbody>
<tr>
<td>Temperature Difference (°C)</td>
<td>0.08</td>
<td>0.13</td>
<td>0.19</td>
<td>0.26</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Table 3.1: The temperature difference

As shown in Table 3.1, the temperature difference increases with time. From all above results, it can be found that the assumption that the distribution of temperature does not affect significantly the distribution of contact pressure and hence the distribution of heat flux is valid for the short term sliding only in the situations when the initial temperature distribution is even, the relative sliding velocity is low and the pressure is not high.

Figure 3.19: The nodes selection for exploring the temperature variation
We name the nodes selected in Figure 3.19 from rightmost to leftmost as node 1, 2, 3 and 4. In Figure 3.20, the red curve is the temperature variation at node 2, the blue curve is the temperature variation at node 3, the black curve is the temperature variation at node 4, the green curve is the temperature variation at node 1. From all these curves, it is found that the temperature of node with a larger radius increases more rapidly than that of node with a smaller radius. The exception is node 1, due to its low contact pressure. The temperatures of all these four nodes increase linearly at a constant sliding velocity.
Figure 3.21: The temperature distribution on the interface as a function of time

The temperature distribution on the interface is shown in Figure 3.21. In the temperature distribution of each contour plot, there is a red ring corresponding to the highest temperatures. Since the nodal velocity increases with the radius, more heat is generated by friction in the red annular region near the outer diameter. The temperature at
the outer diameter is relatively low, which is explained previously. As the contact between the copper plate and the friction plate is axisymmetric, the frictional heat flux is the same for all the nodes of the same radial positions at a specific time. This leads to an axisymmetric temperature distribution. Because of the constant sliding speed, the temperature growth rate does not change considerably. The inner blue ring is actually the iron plate. Due to the relatively low thermal conductivity in the friction material, the heat is transferred into the copper plate instead of the iron plate.

Figure 3.22: The temperature distribution along radial direction at different time

![Graph showing temperature distribution](image)
The temperature distributions along the radial direction at different times are shown in Figure 3.22. The highest temperature is found at the annular location where the radius is approximately 400 mm.

As shown in Figure 3.23, the highest contact pressure is found at the outer diameter and inner diameter at the beginning of rotation. Although the contact pressure at
the inner diameter is the highest, the frictional heat flux remains low as the sliding velocity is low. As the process continues, the thermal expansion becomes more and more significant, and the contact pressure at the outer diameter keeps decreasing.

The nodes shown in Figure 3.19 are selected to plot the variation in the contact pressure,

![Graph showing the contact pressure variation of selected nodes.](image)

Figure 3.24: The contact pressure variation of selected nodes
As shown in the above two figures, the contact pressure of node 1 increases rapidly at the beginning as the sliding speed is the highest and hence most thermal energy is generated at this node. However, the contact pressure decreases subsequently as the thermal expansion effect starts to develop. The contact pressures of other nodes increase at the beginning of rotation and oscillates after reaching certain levels.

Figure 3.25: The contact pressure variation of node 1 at the beginning of rotation
CHAPTER 4: THERMOMECHANICAL PROCESSES IN CLUTCH SYSTEM UNDER UNEVENLY DISTRIBUTED TEMPERATURE FIELD

In this chapter, several three-dimensional models will be created. First, a static model will be created to explore the feasibility of contact pressure simulation when there is an uneven temperature distribution on the interface. Second, four models at different rotational speed and different convective heat transfer coefficient distributions will be developed to evaluate the effect of convective heat transfer towards the thermomechanical processes on the frictional interface.

4.1 Contact Pressure Evaluation Under Uneven Temperature Distribution

If the non-uniformity of temperature distribution is significant, the thermal distortion will affect the contact pressure considerably. In this section, a static model will be developed to evaluate the effect of the non-uniformity of temperature distribution. Because the most steps for creating this model are the same as those steps for creating the previous model. Therefore, only the different steps will be elaborated.

1) To create a three-dimensional part, double-click the Parts container to create a 3-D, deformable part. In the Create Part dialog, copper is an input to the name of the part. Solid and Extrusion are chosen as Shape and Type respectively. The approximate size is 2. Then the “bottom” is clicked.

2) In the sketch region, draw two circles whose radii are 0.2095 and 0.4895 respectively. Then, the annular part is cut into a sector shaped part by using a vertical
construction and a horizontal construction. The completed part is showed in Figure 4.1. After clicking the done button in the prompt zone, the depth of extrusion is set to 0.00953. The reason of using a quarter part is that the symmetric boundary condition can be applied to the vertical and horizontal lines shown in the figure to save the computational time. The next step is to create the other parts using the same method.

3)

Figure 4.1: Sketch of 3-D part copper

Figure 4.2: Uneven distribution of temperature
The uneven temperature distribution is defined as the thermal boundary condition shown in Figure 4.2. The highest temperature in the red focal spot is 124 degrees centigrade, the lowest temperature in the blue focal spot is 52 degrees centigrade. The simulated contact pressure is shown in Figure 4.3. The blue region shown in this figure is of zero contact pressure. In other words, the seriously uneven temperature distribution can cause a separation between the two plates.

![Figure 4.3: Uneven distribution of contact pressure](image)

The nodes shown in Figure 4.4 are selected to plot the contact pressure as a function of the circumferential location.

![Figure 4.4: The selected path to plot the contact pressure](image)
Figure 4.5: The contact pressure along the selected circumferential direction

In Figure 4.5, node 0 is the leftmost node in the direction, node 40 is the rightmost node. From the figure, surface separation is confirmed at the beginning and end regions since the contact pressure is zero at these locations. The opening gap due to surface separation is shown in Figure 4.6.

Figure 4.6: The opening at the inter face

Figure 4.7: The contact pressure distribution when the temperature is 25% of the original value
If the temperature decreases to 25% of the original value, the opening disappears as shown in Figure 4.7.

4.2 Thermomechanical Processes Under Uneven Temperature Distribution

In this section, several three-dimensional FEA models will be developed by combining all the previous work.

4.2.1 Model Description

The three-dimensional models are developed by considering the following conditions.

1) The initial temperature distribution is uneven. The distribution of temperature over the entire clutch is shown in Figure 4.8. It is assumed that the nonuniformity of the temperature distribution is insignificant. This assumption corresponds to the situation where the system is placed in the room temperature. As shown in Figure 4.8, the highest temperature in the clutch is 20.70 °C; the lowest temperature in the clutch system is 19.53 °C. The temperature difference is 1.17 °C.

Figure 4.8: The initial distribution of temperature of three dimensional model
2) The relative sliding velocity between the copper plate and the assembly of friction plate and iron plate decreases monotonically.

3) The copper plate serves as the driving component of the system. Its rotational speed with respect to the z-axis is constant.

4) The convection heat transfer takes place at one surface of copper plates, which is separated from the friction plate. The distribution of the convective local heat transfer coefficient can be determined by the equations described in the previous chapter. The calculation will be elaborated in the following sections.

As shown in Figure 4.9, the displacement is uniform at the beginning. To generate an unevenly distributed displacement field, a short term coupled temperature-displacement simulation is performed before the pressure is applied. The nonuniform distribution of displacement starts to develop because of the thermal expansion in the material.

Figure 4.9: The initial displacement field in friction plate
Figure 4.10: The displacement field in friction plate after the short term thermal expansion

As shown in Figure 4.10, the unevenly distributed displacement field has been developed after the short term thermal expansion. The blue regions shown in the figure are physically depressed, as the elastic modulus and thermal expansion coefficient of friction material are lower than those of copper.

As calculated in previous section, in still air, the highest angular speed at which the boundary layer over the entire rotating disk remains laminar is 15.13 rad/s. Hereafter, the models in which the copper plate rotates at 15.13 rad/s are called the low rotational speed models. The relative rotation between the copper plate and the friction plate lasts for two seconds.

In still air, the lowest angular speed at which the boundary layer over the rotating disk remains turbulent can be calculated from the following equations.
\[
\frac{\omega_f R_i^2}{\nu} = 240000
\]
\[
\omega_f = 240000 \times 15.11 \times 10^{-6} / 0.2444 \times 2 = 60.66 \text{ rad/s}
\]

In the subsequent part of the thesis, the models in which the copper plate rotates at 60.66 rad/s are called the high rotational speed models. The relative rotation between the copper plate and the friction plate lasts for five seconds.

4.2.2 Thermomechanical Processes in Low Rotation Speed Model

The nodes shown in Figure 4.11 are selected. The left node is selected in the region with lower temperature, the right node is selected in the region with higher temperature. Due to the combined effect of conductive heat transfer and thermal expansion, the temperatures at these two nodes oscillate, but the amplitude of oscillation decreases. After about 1 second, the two curves almost overlap each other. As the relative sliding speed decreases, the heat generation rate decreases.

Figure 4.11: The nodes selected for plotting the temperature and contact pressure variations
The distribution of temperature at the end of rotation along the path shown in Figure 4.13 is plotted in Figure 4.14. It is found that the difference between the highest temperature and the lowest temperature is reduced to about 0.02 °C.
The contact pressure variations of nodes selected in Figure 4.11 are plotted in Figure 4.15. It is shown that the contact pressure variations oscillate and become stable near the end of the rotation.
A method to cool down the clutch system is to employ the centered impingement. We assume that the nozzle diameter, jet velocity and nozzle to surface distance are 0.25 m, 22 m/s and 5 m, respectively. As indicated in the previous section, the convection heat transfer coefficient can be calculated by the following equations.

\[
\text{Re}_j = \frac{VD}{\nu} = \frac{22 \times 0.25}{15.11 \times 10^{-6}} = 363997.35
\]

\[
\text{Re}_n = 11200 \text{ Re}_j^{0.24} = 242036.90
\]

\[
\xi = \text{Re}_j^{\frac{2}{3}} \left( \frac{V}{\nu^{2}} \right)^{\frac{1}{2}} = 1.058
\]

\[
h_0 = 1.81 \xi^{0.597} k \sqrt{\frac{\nu}{V}}
\]

\[
= 1.81 \times 1.058^{0.597} \times 0.0257 \times 100066
\]

\[
= 48.14 \text{W} / (\text{m}^2 \cdot \text{C})
\]

In Figure 3.2, the value of \( r/z \) is less than 0.1, and the relation between \( h \) and \( h_0 \) is linear. Under these conditions, the value of \( h \) can be approximated as

\[
h = -14.44 r + 48.14
\]

Figure 4.16: The comparison of temperature variations in Section 4.2.2
The temperature variation obtained from this model (at the node of the highest temperature) and that of the former model are presented together in Figure 4.16. As the two curves are almost identical, we conclude that the centered impingement is unable to cool down the interface significantly.

4.2.3 Thermomechanical Processes in High Rotation Speed Model

At a high rotational speed (60.66 rad/s), the convection heat transfer coefficient can be presented by the following equations.

\[
\frac{h r}{k} = 0.0163 \left( \frac{\omega r^2}{v} \right)^2 \\

h = 80.36 r^{0.6}
\]

The temperature variations of the two nodes selected in Figure 4.11 are presented in Figure 4.17.

Figure 4.17: The temperature variations of two nodes selected in Figure 4.11
Figure 4.18: The beginning part of temperature variations of two nodes selected in Figure 4.11

From the above two figures, we find that the oscillation of temperature just lasts for 0.15 second. As the relative sliding speed decreases, the heat generation rate also decreases.

Figure 4.19: The distribution of contact pressure

The distributions of contact pressure at different times are shown in Figure 4.19. As the sliding speed is high, the contact pressure is localized into two sectors.
Suppose that the clutch is cooled by an air flow parallel to the surface of the copper plate, the distribution of convective heat transfer coefficient can be determined by the following equation when the velocity of air flow is 1 m/s. The temperature variation of hot spots is plotted along with that obtained from the previous model in Figure 4.20. Since the two curves overlap each other, it can be concluded that the air flow parallel to the surface of copper plate does not affect the temperature at the interface significantly.

\[ h = 0.00015 \left[ \frac{60.66r^2 + r}{15.11 \times 10^{-6}} \right]^{0.925} / r \]

Figure 4.20: The temperature variation of hot spots in models presented in this section
CHAPTER 5: CONCLUSION AND FUTURE WORK

5.1 Conclusion

First of all, the focal hot spots observed in the heavy duty clutch system tested in industry is evaluated by using the engineering software “Hotspotter”. In this software, the growth rates of perturbation at certain prescribed rotational speeds are computed by using the eigenvalue method. The critical speed of a specific mode is determined by searching the lowest speed at which the growth rate is positive. For reducing the computational cost, not all of the structural details in the system are taken into consideration in the simulations. After the analysis, we conclude that the focal scars observed on the interface were caused by the TEI phenomenon as the operating rotational speed is significantly higher than the estimated critical speed of the dominant mode. It has been found that the standard lining is more resistant to the TEI phenomenon as its critical speed of the dominant mode is higher than that of the other frictional lining. By (1) reducing the elastic modulus of friction material, (2) increasing the thermal conductivity of friction material, (3) increasing the thickness of friction plate, and (4) decreasing the thickness of the copper plate, the critical speed of the dominant mode can be enhanced.

A two-dimensional FEM model and a three-dimension FEM model are also developed under an evenly distributed initial temperature field. In the two-dimensional model, it is assumed that there is no thermoelastic transition phenomenon. For verifying the validity of this assumption, a full three-dimensional model was developed. In both
models, the initially predefined temperature was 20 °C, the relative rotation lasted for 2 second at 15.13 rad/s and the convective heat transfer coefficient was 8.49 W/(m²·°C). The result showed that the temperature distribution in the cross section of the two-dimensional model and that of the three-dimensional model is quite close at the beginning of relative sliding. The discrepancy between the highest computed temperature in the two-dimensional model and that in the three-dimensional model is reduced with the operating time. It implied that the assumption that there is no thermoelastic transition phenomenon is valid for the short term sliding when the initial temperature distribution is uniform, the relative sliding velocity is slow and the pressure is not significantly high. According to the results obtained from the three-dimensional model, it was found that the nodal temperature at a larger radius increases more rapidly than that at a smaller radius. But the nodes located at the outer diameter are exceptions due to the low contact pressure there. It has also been found that the temperature distribution are axisymmetric, and that the highest temperature is located at an annular region whose radius is approximately 400 mm.

Lastly four models at different rotational speeds and different convective heat transfer coefficient distributions were developed to evaluate the effect of convective heat transfer on the thermomechanical processes in the frictional interface. The initial temperature distribution was nonuniform. But the nonuniformity of the temperature distribution was insignificant. The relative sliding velocity between the driving component and the driven component decreased monotonically. By comparing the results with those obtained from models assuming that the clutch system is placed in still air, it
can be concluded that the centered impingement and air flow parallel to the surface cannot reduce the temperature in the interface.

5.2 Future work

For reducing the computational cost, the friction plate of the clutch system was simplified as an annular ring geometry. The realistic part assembled in the clutch consists of eight sectors. To obtain more accurate results from the analysis, this difference in the geometry should be taken into consideration as this feature may affect the stability behavior of the system.

Due to the limitations in the capability of the computers available, the finite element meshes used in the models developed in the current work were relatively coarse. The first step of the future work on this topic is to analyze the same problem by increasing the degrees of freedom (DOFs) in the model. The effects of non-linearity of materials and the detailed structures on the thermomechanical processes will be another direction for the future work. Simulations using the conventional finite element method for contact problems are typically inefficient. A potential method to reduce the computational effort is to implement the Fourier elements in the model, and that will be one of the future directions.
REFERENCES


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