Hosting Capacity Optimization in Modern Distribution Grids

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HOSTING CAPACITY OPTIMIZATION IN MODERN DISTRIBUTION GRIDS

A Dissertation

Presented to

the Faculty of the Daniel Felix Ritchie School of Engineering and Computer Science

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Advisor: Dr. Amin Khodaei
Abstract

The availability of distributed renewable energy resources and the anticipated increase in new types of loads are changing the way electricity is being produced and supplied to consumers. This shift is moving away from a network delivering power solely from centralized power plants towards a decentralized network which supplements its power production by incorporating local distributed generators (DGs). However, the increased integration of DGs into existing distribution networks is impacting their behavior in terms of voltage profile, reliability, and power quality. To determine the maximum amount of DG that distribution grids can accommodate the concept of hosting capacity is introduced.

The distribution grid hosting capacity is defined as the amount of new production or consumption that can be added to the grid without adversely impacting the reliability or voltage quality for other customers. The study of the hosting capacity is commonly accomplished by simulating power flow for each potential placement of DG while enforcing operating limits (e.g. voltage limits and line thermal limits). Traditionally, power flow is simulated by solving full nonlinear AC power flow equations for each potential configuration. Existing methods for computing hosting capacity require extensive iterations, which can be computationally-expensive and lack solution optimality.
In this dissertation, several approaches for determining the optimal hosting capacity are introduced. First, an optimization-based method for determining the hosting capacity in distribution grids is proposed. The method is developed based on a set of linear power flow equations that enable linear programming formulation of the hosting capacity model. The optimization-based hosting capacity method is then extended to investigate further increasing hosting capacity by also optimizing network reconfiguration. The network reconfigurations use existing switches in the system to increase allowable hosting capacity without upgrading the network infrastructure. Finally, a sensitivity-based method is described which more efficiently obtains the optimal hosting capacity for larger distribution systems.

The proposed methods are examined on several test radial distribution grids to show their effectiveness and acceptable performance. Performance is further measured against existing iterative hosting capacity calculation methods. Results demonstrate that the proposed method outperforms traditional methods in terms of computation time while offering comparable results.
Acknowledgements

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<thead>
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<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>DER</td>
<td>Distributed Energy Resource.</td>
</tr>
<tr>
<td>DG</td>
<td>Distributed Generation.</td>
</tr>
<tr>
<td>EV</td>
<td>Electric Vehicle.</td>
</tr>
<tr>
<td>LP</td>
<td>Linear Programming.</td>
</tr>
<tr>
<td>LSF</td>
<td>Line Sensitivity Factor.</td>
</tr>
<tr>
<td>MIP</td>
<td>Mixed-Integer Programming.</td>
</tr>
<tr>
<td>POI</td>
<td>Point of Interconnection.</td>
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<tr>
<td>PV</td>
<td>Photovoltaic.</td>
</tr>
<tr>
<td>VSF</td>
<td>Voltage Sensitivity Factor.</td>
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Nomenclature

Indices

$c$  Index for points of interconnection.
$m, n, i$  Index for buses.
$\wedge$  Index for calculated variables.

Sets

$B$  Set of buses.
$C_m$  Set of points of interconnection connected to bus $m$.
$L$  Set of lines.
$L_m$  Set of lines connected to bus $m$.
$\Lambda$  Set of primal variables.
$U$  Set of uncertain parameters.
$\Gamma$  Set of Loops.

Parameters

$b_{mn}$  Susceptance of line $mn$.
$g_{mn}$  Conductance of line $mn$.
$M$  Large positive number.
$p_{m}^{D,min}$  Lower limit of active load at bus $m$.
$p_{m}^{D,max}$  Upper limit of active load at bus $m$.
$p_{c}^{M,max}$  Maximum active power exchange with upstream grid at point of interconnection $c$.
$p_{mn}^{L,max}$  Maximum active power flow of line $mn$.
$q_{m}^{D,min}$  Lower limit of reactive load at bus $m$.
$q_{m}^{D,max}$  Upper limit of reactive load at bus $m$. 
\( Q_{c}^{\text{M, max}} \text{ Maximum reactive power exchange with upstream grid at point of interconnection } c. \)

\( Q_{mn}^{\text{I, max}} \text{ Maximum reactive power flow of line } mn. \)

\( r_{mn} \text{ Resistance of line } mn. \)

\( x_{mn} \text{ Reactance of line } mn. \)

\( \Delta V_{m}^{\text{min}} \text{ Lower limit of voltage magnitude deviation in bus } m. \)

\( \Delta V_{m}^{\text{max}} \text{ Upper limit of voltage magnitude deviation in bus } m. \)

**Variables**

\( P_{m} \text{ Active power injection at bus } m. \)

\( P_{m}^{D} \text{ Active load at bus } m. \)

\( P_{m}^{DHC} \text{ Additional active power loading capacity at bus } m. \)

\( P_{m}^{G} \text{ Active power of distributed generation at bus } m. \)

\( P_{c}^{M} \text{ Active power exchange with upstream grid at point of interconnection } c. \)

\( P_{mn}^{L} \text{ Active power flow at line } mn. \)

\( P_{mn}^{loss} \text{ Active power loss of line } mn. \)

\( Q_{m} \text{ Reactive power injection at bus } m. \)

\( Q_{m}^{D} \text{ Reactive load at bus } m. \)

\( Q_{m}^{DHC} \text{ Additional reactive power loading capacity at bus } m. \)

\( Q_{m}^{G} \text{ Reactive power of distributed generation at bus } m. \)

\( Q_{c}^{M} \text{ Reactive power exchange with upstream grid at point of interconnection } c. \)

\( Q_{mn}^{L} \text{ Reactive power flow at line } mn. \)

\( Q_{mn}^{loss} \text{ Reactive power loss of line } mn. \)

\( V_{m} \text{ Voltage magnitude at bus } m. \)

\( z_{mn} \text{ Line status indicator. } \)
\[ \theta_m \quad \text{Voltage angle at bus } m. \]
\[ \Delta V_m \quad \text{Voltage magnitude deviation in bus } m. \]
\[ \Delta \theta_m \quad \text{Voltage angle deviation in bus } m. \]
\[ \lambda_m \quad \text{Marginal hosting capacity at bus } m. \]
\[ \alpha_m \quad \text{Reactive power ratio at bus } m. \]
Chapter One: Introduction

1.1 Background

In the past few decades, there has been a slow but consistent shift in the energy industry from centralized large-scale energy production to distributed localized generation. This is because incorporating distributed generation (DG) technologies provide a set of economic and environmental benefits by reducing power generation costs, supporting deployment of renewable energy sources, and increasing the systems’ overall energy efficiency [1]–[3]. However, to effectively incorporate additional consumption and production into an existing infrastructure, the impact on system operational performance must be analyzed. Integrating DGs into existing networks, for example, causes changes in voltages and currents throughout the distribution network and can potentially result in critical issues in system operation such as fluctuations in the voltage profile and reduced system stability [4]. Proper operation of a distribution network involves meeting design limits, technical standards, and trade regulations. In order to satisfy all these criteria, system planners must be able to forecast and control fluctuations due to changes in DGs and load. Additionally, distribution system planners can use their forecasting data to maximize the benefits provided by DG technologies. For instance, a competing beneficial impact can be achieved by adding DGs strategically close to end-consumers in a network, leading to a reduction in transmission losses during high load hours.
1.2 Changes in the Distribution of Electricity

Electricity distribution systems are entering a transition phase, from traditionally passive networks to highly active networks, as shown in Figure 1.1, requiring a correct understanding of the role of DG and unpredictable energy consumption. These changes are the result of new technologies including distributed energy resources (DERs), and the expansion of new types of dynamic loads, such as smart appliances and electric vehicles (EVs) [5]. With the increasing widespread use of renewable DGs, ensuring that load demands are met under changing network conditions has become a major concern.

Figure 1.1: A comparison between traditional and modern grid topologies.

Increasing deployment of DERs in distribution networks requires improved grid planning and operation strategies to accommodate the new changing behavior of customers as well as the two-way flow of electricity. DERs, such as wind generators, solar photovoltaic (PV) generators, energy storage systems, etc., are the cornerstone of future distribution grids. This is because incorporating such technologies can reduce greenhouse gas emissions, minimize energy costs, reduce dependence on fossil fuels, increase distribution efficiency, and meet growing energy demands [6], [7]. However, existing distribution networks were not designed with DG technology in mind.
Incorporating more DGs into the grid will change its operating conditions, with more DG leading to larger deviations. There is a limit to the amount of additional DG which a distribution grid can accommodate before the resulting deviations degrade the operational performance of the grid.

DGs are small units of generation that are directly connected to the distribution grid and are in close proximity to consumers. There is a growing proliferation of DGs in distribution grids, conceivably due to the falling cost of the technology as well as the promising benefits for end-use electricity customers such as payment reduction and potential load-point reliability improvement [8]–[10]. Once installed, the associated customers will be regarded as “prosumers”, meaning that they are consumers that also have the ability to produce electricity. Among available DG technologies, solar photovoltaic (PV) and small-scale wind turbines perceived to be the most adopted DG technologies for prosumers. At the end of 2017, the grid-connected solar PV installation in the United States reached a total capacity of 77 GW [11], up from 54.8 GW in 2016 and 39 GW in 2015.

Although interesting options for end-use customers and viable solutions for system operators to shift the generation from large-scale power plants to small-scale distributed resources, however, the DG installation can cause several negative impacts on distribution grids. Most notably, growing DG installations may put the grid at risk of having inefficient and/or low-reliability supply, as some of operational quantities can potentially hit their limits and result in power quality or reliability concerns at the system and customer levels [12], [13]. In this case, a variety of factors, such as the rise/drop in
nodal voltages and power flow in distribution branches (i.e., lines and transformers), need to be considered when adding DGs.

EV incorporation is anticipated to expand rapidly, and its incorporation into distribution grids requires the use of efficient and reliable analysis tools to ensure an efficient and cost-effective system operation. Primarily, this demand is triggered by the need to understand how local vehicle charging and renewable energy sources impact distribution networks. Additionally, these resources provide the potential of boosting system performance by introducing local means of electricity supply, distribution, and regulation. With such powerful potential benefits accessible to system planners, there is rising interest in facilitating the integration of larger quantities of EV charging and renewable energy resources [14]. However, caution needs to be warranted as large amounts of plugged-in EVs can potentially overload the network leading to significant voltage variations and overloading of network lines. If the integration of EVs increases peak demand beyond the network capabilities, system planners must either curb peak demand or increase peak supply. This can be accomplished by either purposefully controlling EV charging demand in real-time, introducing more DG to meet demand, or upgrading the distribution network. For an informed design and maintenance plan, the strategy chosen should depend upon the bottlenecks preventing further capacity upgrades.

A shift towards DG has raised two concerns: (i) how will introducing DG into existing transmission and distribution systems affect their operational performance and (ii) what is the best way to incorporate DG into existing distribution networks without jeopardizing system performance? Answering these questions requires an understanding of the impact of DG on an active distribution network. To determine how much new
generation/load can be added to a network without requiring additional investment and without jeopardizing system performance, the concept of hosting capacity is introduced. The hosting capacity is defined as the amount of new production or consumption which can be connected to the grid without adversely impacting the reliability or voltage quality for other customers [15]. The operational performance is measured using various factors, from voltage magnitudes to feeder power flows to power quality issues [16]. Protection can also be considered as a critical performance measure as the DG deployment will potentially result in a reverse power flow in distribution feeders. The hosting capacity calculation sheds light on the role and impacts of DGs within the distribution grids. It can further provide grid planners with the required insight on how to build and upgrade the grid in a cheaper, greener, and more sustainable way. Hosting capacity calculations can also determine the maximum amount of DG that can be deployed to support reducing peak demand and postponing required grid upgrades.

1.3 Hosting Capacity Literature Review

The hosting capacity studies in the literature can be categorized into two main groups: (i) studies that propose hosting capacity calculation methods based on a variety of grid performance measures and system characteristics, and (ii) studies that focus on grid upgrades or operational practices to increase grid hosting capacity. Former studies further investigate the impact of DGs on selected operational performance measures as elaborated in [17]–[20]. These performance measures can be bus overvoltage, line overload, or power quality. The locational sensitivity analysis method of distribution feeders introduced in [21] estimates the grid hosting capacity by demonstrating the effect of DG distance on voltage deviations at feeder nodes. Similar studies are performed in
but with a focus on PV integration into distribution grids. Authors conclude that analyzing each feeder individually is faster than a simultaneous analysis of all feeders. However, the individual analysis method would not guarantee optimal, and in many cases, accurate solutions. Power quality as a performance measure for hosting capacity calculations, commonly studied in terms of harmonic distortion, is investigated in [23] and [24]. The model proposed in [23] explores the effects of harmonic distortion limits on hosting capacity under various active network management schemes, and authors in [24] investigate the impact of nondispatchable DGs on the harmonic distortion, and accordingly on grid hosting capacity. Optimal installation of DGs is derived in this work while preventing accumulated $h$ order harmonic current from driving the harmonic voltage past acceptable limits.

Among the methods proposed to increase grid hosting capacity, active power management, power curtailment, and voltage control can be pointed out. The study in [25] utilizes an active management strategy with the use of different voltage control strategies (i.e. on-load tap changer and reactive power control) to determine the optimal hosting capacity at the worst-case operation of medium voltage system. A profit maximization strategy is developed in [26] for distribution utilities specializing in providing network access for third party DGs. The strategy informs infrastructure investment decisions by optimizing the profit from the acceptable hosting capacity. In addition, The active/reactive power curtailment strategy, specifically for voltage rise mitigation, has been demonstrated to produce beneficial results in the hosting capacity optimization problems in [27]. In [28], an active and reactive power control of the solar PV inverter to increase overall hosting capacity is explored. The studies in this work,
however, are limited to only a few snapshots of demand and generation rather than a longer time horizon analysis. The impact of solar PV reactive power absorption on excessive voltage rise is inspected in [29] to assess DG performance. The study in [30] investigates the combination of DC links with power electronic converter interfaces to extend the installation of PV systems in distribution feeders. Multiple feed-in management strategies in order to increase the hosting capacity in a synthetic distribution system are studied in [31], benefiting from Monte Carlo simulations to derive general trends and to analyze specific grid, load, and DG architectures. A decentralized power control strategy is used in [32] to optimize grid hosting capacity by regulating the feeder voltage profiles. In a related study in [33], a hosting capacity optimization method is proposed to determine the optimal size and location of DGs using on-load tap changers (OLTC) and static Var compensators (SVC). This model is extended to a multi-objective optimization problem in [34], in which a cuckoo search method is used to improve voltage profiles and reduce losses by optimizing DG allocation. The authors indicate two indices to measure quality improvement: voltage deviations from a reference value (which should be minimized) and voltage differences before and after DG integration (which should be maximized). The cuckoo search method is reported to outperform competing algorithms in efficiency in this particular problem. In [35], the impact of increasing solar PV units in residential neighborhoods is investigated and the hosting capacity is obtained in systems ranging from low voltage to medium voltage through a stochastic analysis framework. A C-type passive filter is used to optimize the hosting capacity while reducing harmonic distortions from DGs in [36]. In a related study, a variety of PV inverters are tested in [37] to find out how efficient the use of active and
reactive power control strategies would be in increasing hosting capacity. However, it is concluded that the slow response time and switching restrictions of typical compensators prevent a fast and reliable control, which accordingly underscores the need for efficient voltage and reactive power control to achieve acceptable results when solving this problem. An optimization strategy for stabilizing nodal voltages and reducing system losses is employed in [38]. Bifurcation analysis is used to rank the nodal voltage stability. It is discussed that poorly-ranked buses benefit more from voltage stabilization via DG power injection, therefore the associated locations are weighed as preferred candidates for DG installation. It is further shown that DG reactive power limits directly impact the optimization results, highlighting their importance in voltage stability.

Network reconfiguration is perceived as another viable method in increasing grid hosting capacity as explored in [39]–[41]. In [42], static and dynamic reconfigurations are used to determine the optimal hosting capacity, further benefiting from a multi-period optimal power flow approach. In [43], a simultaneous optimization of reconfiguration and DG placement are performed to reduce system losses and improve voltage profiles. This is accomplished using a multi-objective optimization algorithm called modified plant growth simulation algorithm. In [44], a Harmony Search Algorithm is utilized to simultaneously reconfigure and identify the optimal DG size and locations. The study in [45] investigates the effect of PV incorporation on optimal reconfigurations for reducing line losses. Radial network configurations have also been performed which maximize PV capacity. This was accomplished by converting a mixed-integer nonconvex optimization into a convex optimization by the second order cone programming method [46]. In [47], a genetic algorithm method is introduced to evaluate the optimal hosting capacity based on
the optimal network reconfiguration. However, the optimization problem does not obtain the global optimal solution as the variation in DG output cannot be considered. In [48], power losses were reduced by reconfiguring the network and optimizing DG size and location using a Tabu search optimization, which however suffers from a long computation time. Improved Tabu search algorithms were also developed based on meta-heuristic methods to minimize energy losses [49]. The study in [50] uses an improved rolling horizon algorithm instead to optimize the mixed-integer nonlinear programming problem more quickly with the objective of minimizing DG curtailment. The study in [51] proposes the use of a heuristic constructive algorithm to optimize reconfiguration as well as size and locations of DGs. Sensitivity analysis was used to compute the sensitivity factors of candidate DG installation locations. In [52], soft open points (SOPs) are used to reconfigure radial distribution networks. Hosting capacity is increased by optimizing the size and location of SOPs while maintaining the network radiality. The study does not consider the optimal DG sizes and locations; instead uses a scenario generation method to find DGs operational characteristics. The authors report that SOPs can significantly reduce the operating costs of active distribution grids.

In addition to system-oriented approaches in increasing grid hosting capacity, there exists some methods with a focus on technology-oriented approaches, i.e., to increase hosting capacity by integrating other distribution grid-integrated technologies. The study in [53] proposes the control of EVs charging load to increase the acceptable hosting capacity of distribution network under various pricing schemes. The study in [54] investigates the impact of slow charging of EVs to harmonic distortions introduced by AC/DC inverters in distribution system hosting capacity. The study in [55] introduces a
time-dependent hosting capacity to increase the maximum acceptable EV loads and investigates the impact of EVs charging time on distribution girds. A decentralized approach based on multi-agent analysis is proposed in [56] to increase distribution grid hosting capacity for expected electric vehicle loads. A combination method based on storage system incorporation and day-ahead projection is used in [57] to optimize hosting capacity. The addition of a battery storage system increases the grid hosting capacity while also improving voltage profiles. Study in [58] uses hosting capacity calculation methods to compare the improvements from different types of DGs and energy storage systems on operating costs during power outages. However, this paper does not consider the impact of these technologies during normal operation. In [59], the use of residential storage systems and reactive power controls to increase the installation of PV in distribution systems was proposed. In [60], the authors optimize the system configuration while incorporating DG and energy storage systems. Their proposed multi-stage optimization has a fivefold minimization of costs as the objective function, including the investment, maintenance, energy, unserved power, and system emission costs. Results show a marginal impact of network switching on DG integration and grid hosting capacity.

1.4 Research Motivation

There are extensive discussions on the hosting capacity problem in the literature. However, a closer look at the aforementioned studies reveals that the majority of the existing methods, both on hosting capacity calculation and maximization, rely on an iterative approach to determine the distribution grid hosting capacity. That is, one initial value for DG capacity at a specific bus in the system is considered and that capacity is
incrementally increased to the point that the desired performance measure leaves the acceptable region. This common practice has two major drawbacks: first, the spatial interdependency of DG deployments is ignored, i.e., these methods do not offer the capability to study and analyze the impact of DG deployment in one bus to other buses, nor the impact of simultaneous DG deployments in several buses to the grid hosting capacity results. This is a major shortcoming as it can potentially prevent finding the optimal, or even a near-optimal, hosting capacity solution. Second, these methods are time-consuming as increasing the DG capacity after each increment should be followed by solving a complete power flow problem. For example, each bus that can accommodate DG will contribute an independent variable to the search space. Sampling such a search space requires performing AC power flow analysis for each potential DG profile. Since the number of DG profiles grows rapidly with the number of active buses, optimization of larger systems becomes impractical. In some cases, it may cause thousands of iterations to find the grid hosting capacity, which is proven to be ineffective for large distribution grids. The computation time and solution accuracy in this case further depend on the DG size increments. If large increments are considered, the model will find the solution faster but at the expense of losing accuracy. If small increments are considered, the solution will be potentially accurate, but it will need a long computation time to reach the final solution. Large and small increments, of course, are relative terms in these cases, depending on the distribution grid size.

For this reason, the aim of this dissertation is to address these two shortcomings by proposing an optimization-based hosting capacity calculation method that removes the need for iterations by linearizing the AC power flow equations and then finding the
optimal hosting capacity through. Moreover, the advantage of using a linearized method over more conventional iterative methods is twofold. First, computation time is dramatically reduced, especially when a large search space is considered. Second, it makes the optimization across all possible DG deployments possible to be considered. These advantages are combined to allow for a fast scanning of a huge search space, exact solutions, and rapid computation. Additionally, linear analysis does not require iterations, eliminating concerns about proper convergence.

1.5 Dissertation Outline

This dissertation is organized as follows:

Chapter 2 details the proposed techniques for deriving linearized optimal hosting capacity methods used throughout the dissertation. The method in this chapter provides the mathematical foundation for the rest of the dissertation. Linearizing the power flow equations lead to a dramatic reduction in the optimization problem’s complexity.

Chapter 3 discusses in detail the proposed optimization-based hosting capacity method. A model for calculating the optimal loading capacity is also proposed in this chapter to provide system planners with the required insight needed to build and upgrade distribution systems in a costly, efficient, and sustainable way. Moreover, this chapter applies the linearized hosting capacity optimization method to determine the marginal hosting capacity of distribution nodes. That is, to determine the maximum possible change in generation/consumption without violating operational requirements.

Chapter 4 presents a linearized network reconfiguration and voltage control method for maximizing distribution grid hosting capacity by taking advantage of accessible network reconfigurations. The problem is formulated as a mixed-integer linear
programming problem with appropriate radiality constraints. Network reconfiguration allows to increase grid hosting capacity by using existing switches without the need to upgrade the system’s infrastructure.

Chapter 5 presents the sensitivity-based hosting capacity calculation method that can be utilized to determine the optimal DG capacity for large-scale distribution networks. The sensitivity-based hosting capacity calculation method is developed based on the sensitivity analysis of line power flow and voltage magnitudes with respect to nodal active and reactive injection power.

Chapter 6 validates the linear power flow approach by comparing its results with those obtained from nonlinear full AC power flow analysis. Moreover, different case studies for a variety of scenarios are performed to showcase the accuracy of these methods in a variety of diverse scenarios. Results from two radial distribution test systems, i.e., the IEEE 33-bus distribution test system and the IEEE 123-bus distribution test system, are collected. Traditional iterative hosting capacity methods are compared with the proposed methods. These comparisons demonstrate the superior performance of the proposed methods as well as the tradeoff between speed and accuracy that occurs.
Chapter Two: AC Power Flow Linearization

As the amount of nodal generation in the distribution grid changes, as a result of DG integration, the network power flow will accordingly change. It is important in this case to closely monitor grid performance to ensure that it is not negatively impacted. To study this impact, a full AC power flow should be solved to determine changes in line flows and bus voltage magnitudes and angles. A majority of existing distribution power flow methods are nonlinear and should be solved in an iterative manner (either through successive linearization around the current operating point or through successive update of network quantities based on calculated increments). There exist some linear models as well [61]–[63] however, these models are mostly based on ZIP load models which are not useful in modeling DG generation. To address this issue, a linear power flow model is developed.

Let’s start with generic line flow equations. Equations (2.1) and (2.2) represent the active and reactive flow of line \( mn \), which is assumed to be between buses \( m \) and \( n \) as shown in Figure 2.1, respectively:

\[
P_L_{mn} = g_{mn} V_m^2 - g_{mn} V_m V_n \cos(\theta_m - \theta_n) - b_{mn} V_m V_n \sin(\theta_m - \theta_n) \quad \forall mn \in L \tag{2.1}
\]

\[
Q_L_{mn} = -b_{mn} V_m^2 - b_{mn} V_m V_n \cos(\theta_m - \theta_n) - g_{mn} V_m V_n \sin(\theta_m - \theta_n) \quad \forall mn \in L \tag{2.2}
\]
Figure 2.1: Representation of radial distribution lines.

These line flow equations are nonlinear, as they include second order terms, the multiplication of variables, and trigonometric terms. The conductance \( g_{mn} \) and the susceptance \( b_{mn} \) represent the real and imaginary components of the line admittance, respectively. They are calculated as follow:

\[
\begin{align*}
g_{mn} &= \frac{r_{mn}}{(r_{mn}^2 + x_{mn}^2)} \\
b_{mn} &= -\frac{x_{mn}}{(r_{mn}^2 + x_{mn}^2)}
\end{align*}
\]  
\( \forall \, mn \in L \quad (2.3) \)

\( \forall \, mn \in L \quad (2.4) \)

2.1 AC Power Flow Linear Approximation and Problem Formulation

When performing a steady state analysis of the distribution grid, it can be assumed that the voltage magnitude and angle at the Point of Interconnection (POI), i.e., where the distribution grid is connected to the upstream subtransmission system, are known and fixed. This is a valid assumption as the upstream grid acts as an infinite bus with a constant voltage. Assuming that the voltage at the POI is \( 1 \angle 0^\circ \) p.u., all downstream bus voltages and angles can be represented as deviations from this value as in (2.5) and (2.6). In other words, the voltage magnitude in each bus is defined as 1.0 p.u. plus the deviation from the POI voltage magnitude, and the phase angle of each bus is defined as \( 0^\circ \) plus the deviation from the POI voltage angle.
It is important to note that (2.5) and (2.6) add no approximations to line flow equations; rather, they simply redefine $V_m$ and $\theta_m$ using the POI as a reference. Any other constant values can be considered for reference voltage magnitude and angle at the POI without loss of generality. After this initial change in problem variables, two assumptions are made to simplify line flow equations:

(i) The difference in voltage angles of adjacent buses $m$ and $n$ is considered to be small, thus trigonometric terms can be approximated as follows:

\[
\sin(\theta_m - \theta_n) \approx \theta_m - \theta_n = \Delta \theta_m - \Delta \theta_n \quad \forall mn \in L \tag{2.7}
\]

\[
\cos(\theta_m - \theta_n) \approx 1 \quad \forall mn \in L \tag{2.8}
\]

By using (2.5)-(2.8), the line flow equations can be reformulated as:

\[
PL_{mn} = g_{mn}(1 + \Delta V_m)^2 - g_{mn}(1 + \Delta V_m)(1 + \Delta V_n) \quad \forall mn \in L \tag{2.9}
\]

\[
QL_{mn} = -b_{mn}(1 + \Delta V_m)^2 - b_{mn}(1 + \Delta V_m)(1 + \Delta V_n) - g_{mn}(1 + \Delta V_m)(\Delta \theta_m - \Delta \theta_n) \quad \forall mn \in L \tag{2.10}
\]

(ii) Terms including the multiplication of $\Delta V$ and $\Delta \theta$ are very small and can be ignored. In other words, it is assumed that $\Delta V_m\Delta \theta_m = \Delta V_m\Delta \theta_n = \Delta V_n\Delta \theta_m = \Delta V_n\Delta \theta_n \approx 0$. This a reasonable assumption because both voltage magnitude and angle deviations from the POI values are small. Based on this assumption, the real and reactive line flows in (2.9) and (2.10) can be simplified, and then
by rearranging the terms, can be reformulated as in (2.11) and (2.12), respectively.

\[ PL_{mn} = g_{mn} (\Delta V_m - \Delta V_n) + g_{mn} \Delta \hat{V}_m (\Delta V_m - \Delta V_n) - b_{mn} (\Delta \theta_m - \Delta \theta_n) \quad \forall mn \in L \]  

\[ QL_{mn} = -b_{mn} (\Delta V_m - \Delta V_n) - b_{mn} \Delta \hat{V}_m (\Delta V_m - \Delta V_n) - g_{mn} (\Delta \theta_m - \Delta \theta_n) \quad \forall mn \in L \]  

These two equations represent real and reactive line flows, not based on actual bus voltage magnitudes and angles but based on voltage magnitude and angle deviations from the POI values. In both equations, the first and third terms are linear, however the second terms are nonlinear.

This nonlinearity can be taken care of in two easy successive steps: In the first step, the nonlinear terms are simply removed from the equations and the resultant linear line flow equations are used to find the power flow solution. The power flow solution in this case will ensure that \( PL_{mn} + PL_{nm} = 0 \) and \( QL_{mn} + QL_{nm} = 0 \) (can be seen from the equations), so line losses would be zero, hence it can be called a “lossless power flow”. In the second step, \( \Delta V_m \) values obtained from the lossless power flow solution can be considered as constants in the nonlinear terms in line flow equations, i.e., \( \Delta V \) with \( ^\wedge \) is, where \( ^\wedge \) represents the already-calculated variable obtained from the lossless power flow solution. The nonlinear terms are now converted into linear ones, further ensuring that the approximation is much smaller than the lossless power flow. In this case, \( PL_{mn} + PL_{nm} \neq 0 \) and \( QL_{mn} + QL_{nm} \neq 0 \), so these equations consider line losses as well.

It can be discussed that if \( \Delta V_m \) value is calculated again and plugged back into the line flow equations, a more accurate solution will be achieved. This is a valid discussion and the results will definitely improve. However, it can be shown with simple
calculations that the amount of change in voltage magnitudes and angles after the second step will be minimal, thus eliminating the need to perform additional steps beyond the second step.
Chapter Three: Hosting Capacity Optimization

3.1 Optimization-based Distribution Grid Hosting Capacity Calculations

The linearized active and reactive power flow model, as discussed previously, is used to model the proposed optimization-based hosting capacity calculation method [64]. The objective will be to maximize allowable DG capacity that can be hosted in the distribution grid without negatively impacting grid performance. Two performance measures, namely bus overvoltage and line overload, are used for this purpose. Unlike the existing methods, the proposed method can effectively consider the spatial interdependencies and also find the solution in one instance instead of using many iterations. The proposed method uses a linear model for power flow analysis and formulates the problem based on linear programming. This would allow for dynamic changes to the model to account for installed generation and to update the hosting capacity results as new DG is integrated to the grid. The major contributions of this work are listed as follows:

- The distribution grid power flow is linearized based on a few approximations obtained from practical assumptions. Unlike traditional nonlinear power flow models, the linearized model does not require iterations to find the final feasible solution and can be efficiently integrated into an optimization framework.

- An optimization-based hosting capacity calculation method is developed based on the linear power flow model. This method is capable of finding a near-optimal
hosting capacity solution in a short amount of time, eliminating the need for extensive iterations as in traditional methods.

- The spatial interdependency of DG deployments is effectively considered within the developed method, in which all buses in the distribution grid are simultaneously analyzed for their hosting capacity (both individual and in aggregate).

- Load variations are accounted for based on robust optimization. A worst-case solution is obtained which encompasses all possible realizations of loads in all buses. Accordingly, the seasonal load variations are captured in hosting capacity calculations, removing the need for repeating studies when load values are changed.

### 3.1.1 Hosting Capacity Problem Formulation

The objective of the hosting capacity calculation is to maximize the total amount of DG capacity that can be installed in the distribution grid (3.1). The total installed DG capacity is considered as the summation of installed DG capacity in all buses.

\[
\min_U \max_\Lambda \sum_{m,B} \rho^G_m
\]

(3.1)

The objective function is maximized over the set of “primal variables” shown with Λ, and is further minimized over the set of “uncertain parameters” denoted by U. Primal variables include DG capacities, i.e., the primary variable to be determined via this problem, along with bus voltage magnitudes and angles, real and reactive line flows, and real and reactive power exchange with the upstream grid. The uncertain parameters include real and reactive loads in each bus. The distribution grid hosting capacity is
highly dependent on bus load values. If load values change, in one or more buses, the hosting capacity solution may accordingly change. Therefore, either all possible load variations should be considered when calculating the grid hosting capacity and then the minimum obtained solution can be considered as the final solution, or a worst-case analysis should be performed using a robust optimization. The latter is employed in this study, in which the maximum hosting capacity value is minimized over a set of uncertain parameters, here loads. Loads are further assumed to change within a polyhedral uncertainty set. This way, the worst-case solution is obtained without the need for considering all possible load variation scenarios. This solution is robust against all realizations of load variations, i.e., if loads obtain any other values within their identified bounds, the hosting capacity solution will not change.

This objective is subject to operational constraints (3.2)-(3.14):

\[
\sum_{c \in C_m} P_c^M + \sum_{m \in L_m} P_{mn}^L + P_m^G = P_m^D \quad \forall m \in B \tag{3.2}
\]

\[
\sum_{c \in C_m} Q_c^M + \sum_{m \in L_m} Q_{mn}^L + Q_m^G = Q_m^D \quad \forall m \in B \tag{3.3}
\]

\[
PL_{mn} = g_{mn} (1 + \Delta \hat{V}_m)(\Delta V_m - \Delta V_n) - b_{mn} (\Delta \theta_m - \Delta \theta_n) \quad \forall mn \in L \tag{3.4}
\]

\[
QL_{mn} = -b_{mn} (1 + \Delta \hat{V}_m)(\Delta V_m - \Delta V_n) - g_{mn} (\Delta \theta_m - \Delta \theta_n) \quad \forall mn \in L \tag{3.5}
\]

\[
-P_c^{M,\text{max}} \leq P_c^M \leq P_c^{M,\text{max}} \quad \forall c \in C_m \tag{3.6}
\]

\[
-Q_c^{M,\text{max}} \leq Q_c^M \leq Q_c^{M,\text{max}} \quad \forall c \in C_m \tag{3.7}
\]

\[
P_m^{D,\text{min}} \leq P_m^D \leq P_m^{D,\text{max}} \quad \forall m \in B \tag{3.8}
\]

\[
Q_m^{D,\text{min}} \leq Q_m^D \leq Q_m^{D,\text{max}} \quad \forall m \in B \tag{3.9}
\]
The active power balance equation (3.2) ensures that the generation from local installed DGs plus the line flows in each bus \( m \) will be equal to the load at that bus. The installed DG generation is considered a free positive variable in all buses. If that bus is the POI, the utility power exchange is further considered in the load balance equation. In a similar manner, the reactive power balance equation (3.3) ensures that a balance is met for the reactive power at each bus. Equations (3.4) and (3.5) represent the active and reactive line flows as developed in the previous chapter. Constraints (3.6) and (3.7) impose limits on the active and reactive power exchange with the upstream grid. It should be noted that this power exchange is another free variable that can be positive (importing power from the upstream grid) or negative (exporting power to the upstream grid), or zero (no power exchange). Constraints (3.8) and (3.9) show nodal load variations which are limited by a lower bound and an upper bound. These bounds can be obtained based on historical load data. Loads can freely change within their associated bounds, and at the end would select the values that will result the worst-case hosting capacity solution under load variations. Performance measures are considered as line overload and bus overvoltage. To prevent such violations, real and reactive line flows are constrained by

\[-P_{L_{mn}} \leq P_{L_{mn}} \leq P_{L_{mn}}^{\text{max}} \quad \forall mn \in L \quad (3.10)\]

\[-Q_{L_{mn}} \leq Q_{L_{mn}} \leq Q_{L_{mn}}^{\text{max}} \quad \forall mn \in L \quad (3.11)\]

\[\Delta V_{m}^{\text{min}} \leq \Delta V_{m} \leq \Delta V_{m}^{\text{max}} \quad \forall m \in B \quad (3.12)\]

\[\Delta V_{m}^{\text{min}} = V_{m}^{\text{min}} - 1 \quad \forall m \in B \quad (3.13)\]

\[\Delta V_{m}^{\text{max}} = V_{m}^{\text{max}} - 1 \quad \forall m \in B \quad (3.14)\]
(3.10) and (3.11), respectively, and bus voltage magnitude is limited by (3.12). With these three constraints, it is ensured that DG injections do not cause a deterioration in grid performance measures by violating associated operational limits. The lower and upper bounds on the voltage magnitude deviation from the POI at each bus are defined by (3.13) and (3.14), respectively.

The objective and all the constraints in the formulated problem are linear, except line flow equations (3.4) and (3.5). To convert this problem to a linear problem, and accordingly enable a faster and better solution, the two-step process explained in Chapter Two will be used. Figure 3.1 depicts the flowchart of this optimization-based hosting capacity calculation method. The method starts by identifying grid topology and characteristics, as well as a set of selected grid performance measures. In the first step, the grid hosting capacity is calculated by ignoring losses, i.e., based on the lossless power flow model. In the second step, the full power flow equations are solved by using the results for $\Delta V_m$ obtained from the lossless model as a constant introduced to linearize the nonlinear terms.
With the proposed model, it becomes possible to obtain the hosting capacity solution quickly, efficiently, and with access to active and reactive power flow information at the solution operating point. The solution is further robust against all realizations of load variations. One last issue to consider is how to solve the min-max problem. The complex objective function is not tractable in its current form which makes it challenging to solve the problem. To address this issue, the dual problem of the inner maximization problem is obtained and then combined with the outer minimization problem. This is doable as the problem is linear. The final form will be min-min problem which can be written in terms of a single-objective minimization problem. This solution method is generic and applicable to any radial distribution grid.

### 3.2 Optimal Loading Capacity in Distribution Grids

Any of the methods focused on maximizing DG production could have potentially been used in obtaining loading capacity instead. This can be accomplished by maximizing loading capacity (instead of DG production) while enforcing system
performance limits as constraints. Indeed, it is often more advantageous to look at the loading capacity as meeting load demands is the primary responsibility of a distribution network. With this in mind, techniques in DG optimization models are used to design a loading capacity approach.

When loads increase in the distribution grid, voltages and power flows will change throughout the system in response. Network lines and buses have design and operational limits that need to be met when operating the system. As more loads are integrated into a network, it is possible for the system operating conditions to perturb beyond the specified limits, and thus, negatively impact the operational performance of the system. In this study, a mathematical algorithm is proposed which obtains the optimal loading capacity of a radial distribution network and the performance constraint which bottlenecks the loading capacity [65]. This method is able to efficiently calculate the hosting capacity by using the linearized power flow analysis, with the ability to resolve both real and reactive power flow in a single step (as opposed to an iterative approach). Hosting capacity for each performance constraint can be obtained, which can quantify the effectiveness of upgrades to the system by highlighting which constraints are limiting the maximum loading of the system. Moreover, the optimal locations of the additional load capacities are provided, which can evaluate the network based on how well it matches the additional changes in load demands.

3.2.1 Optimal Loading Capacity Problem Formulation

To find the optimal network loading capacity, in terms of the maximum load that can be added to the system, the model in (3.15)-(3.26) is proposed:
The objective of the proposed optimization model is to find the maximum additional active load capacity that can be added to the system. This objective is subject to a set of operational constraints. Equations (3.16) and (3.17) respectively represent the active and reactive power balance equations in each system bus. Using these equations, it is ensured that the power exchanged with the utility grid plus the injected power from all the lines connected to each specific bus, equals bus load. The bus load includes the forecasted load, which is predicted and is thus constant, plus a hosting capacity load, \( P_{m}^{DHC} \).
which is variable and represents the maximum load that can be added to that bus. This variable load, only the active one, appears in the objective, summed over all buses. The exchanged power with the utility is limited by respective active and reactive limits at the POI to the upstream network, as in (3.18) and (3.19), respectively. Set C includes all the buses that are considered as a POI to the upstream network. Active and reactive line flows are calculated as in (3.20) and (3.21), based on line characteristics, i.e., conductance and susceptance, as well as nodal voltage magnitudes and angles. These line flows are limited by respective active and reactive line limits, as in (3.22) and (3.23), respectively. Furthermore, the change in bus voltage magnitudes are limited by the specified minimum and maximum limits (3.24). The minimum and maximum change in voltage magnitude limits are defined as in (3.25) and (3.26), respectively.

3.3 Marginal Hosting Capacity Calculation for Electric Vehicle Integration in Active Distribution Networks

This section focuses on calculating the marginal hosting capacity of a distribution network in order to evaluate the future charging and discharging capacities associated with EV integration into a radial distribution grid [66]. The methodology is capable of feeding valuable information rapidly, potentially allowing real-time control over the system with implications for future smart grid execution. The marginal hosting capacity is defined as the additional, i.e., the next increment, injected/withdrawn power at a specific node while ensuring that the system’s operating conditions are within acceptable limits. To further clarify, the objective of the optimal hosting capacity is to determine optimal DG size and locations, while marginal hosting capacity investigates the impact of
adding and/or removing locational power injection on the optimal hosting capacity value. In both cases the limit is reached when additional power, whether additional load or generation, will cause the system to move beyond acceptable operational limits. To evaluate the marginal hosting capacity, it is imperative to model and develop the system-aggregated optimal hosting capacity. Note that based on the definition, the hosting capacity includes both additional generation and consumption, i.e., in terms of EV integration, so both charging and discharging will be accounted for in calculations.

The work presented in this study uses the linearized AC power flow model presented in Chapter Two to model the optimal hosting capacity model and to further determine the targeted marginal hosting capacities. The obtained solutions from this model are ensured to be achieved quickly and with a high degree of accuracy. A more detailed procedure for the method is outlined in Figure 3.2. To begin, information about the network and its design specifications are fed into the model. Relevant performance indices are defined, as quantities of interest for assessing the performance of a system, and their performance limits are detailed. Network and performance limit information are used to construct the power balance equations, line flow equations, and voltage deviation equations. These equations act as the basis for the hosting capacity optimization. The linearization is performed in two steps, similar to what discussed in Chapter Two. Step one employs a lossless power flow model, while step two reintroduces losses in a linearized framework. Results from the first step are used to estimate system parameters which are required for the full loss model in the second step. Next, the marginal hosting capacity is evaluated based on the proposed optimal hosting capacity model. The
marginal hosting capacity determines the effects of variation in power consumption or production (e.g., due to EV integration in specific locations) on the optimal hosting capacity solutions.

![Diagram of marginal hosting capacity calculation procedure]

Figure 3.2: Proposed marginal hosting capacity calculation procedure.

While the marginal hosting capacity is general enough to be used in multiple scenarios, one of its most natural applications is in the successful integration of EVs into a distribution grid. With this in mind, two different EV scenarios can be considered using the marginal hosting capacity analysis, representing the impacts of EVs in distribution grids:

- **EV Charging**: The worst-case scenario, in which EVs are simultaneously charging during peak hours, is considered. This can be viewed as a loading problem, such that a stochastic load distribution is introduced to the network. The assessment strategy is to determine how much virtual additional load from EVs can be accommodated without negatively impacting system performance.
- **EV Discharging:** Considers the case in which EVs are supplying power during off-peak load hours by discharging their batteries. In this scenario, EVs are assumed to be plugged into optimal locations. This is not a common situation to occur in practice but it represents the worst-case scenario and is necessary to consider for the marginal hosting capacity calculations.

A range of optimal EV charging/discharging may be described by ensuring that an EV charging station’s power flow be within the capacity limits for each location. This definition guarantees that integrated production and loads from EVs do not adversely affect the grid performance. Details of this method will be discussed in the following subsection.

### 3.3.1 Marginal Hosting Capacity Problem Formulation

The optimal hosting capacity model is proposed as follows:

\[
\min_{PD} \max_{PH} \sum_{m \in B} P_m^G + P_m^{DHC} \tag{3.27}
\]

\[
\sum_{c \in C_m} P_c^M + \sum_{mn \in L_m} P_{mn}^L + P_m^G = P_m^D + P_m^{DHC} \Leftrightarrow \lambda_m \quad \forall m \in B \tag{3.28}
\]

\[
\sum_{c \in C_m} Q_c^M + \sum_{mn \in L_m} Q_{mn}^L + Q_m^G = Q_m^D + Q_m^{DHC} \quad \forall m \in B \tag{3.29}
\]

\[
PL_{mn} = g_{mn}(1 + \Delta \hat{V}_m)(\Delta V_m - \Delta V_n) - b_{mn}(\Delta \theta_m - \Delta \theta_n) \quad \forall mn \in L \tag{3.30}
\]

\[
QL_{mn} = -b_{mn}(1 + \Delta \hat{V}_m)(\Delta V_m - \Delta V_n) - g_{mn}(\Delta \theta_m - \Delta \theta_n) \quad \forall mn \in L \tag{3.31}
\]

\[-P_{mn}^{max} \leq PL_{mn} \leq P_{mn}^{max} \quad \forall mn \in L \tag{3.32}\]

\[-Q_{mn}^{max} \leq QL_{mn} \leq Q_{mn}^{max} \quad \forall mn \in L \tag{3.33}\]

\[-P_{c}^{max} \leq P_c \leq P_{c}^{max} \quad \forall c \in C_m \tag{3.34}\]
The objective function to be optimized, i.e., the additional generation/load that can be added to the network, is defined in (3.27). This objective is maximized over the additional active power that can be added to the network, in form of either generation or load, and further minimized over the network available load demand, i.e., a robust optimization. Equations (3.28) and (3.29) represent the active and reactive power balance equations in the system. The power balance equations ensure that power exchanged with the utility grid plus injected power from connected lines equals the load at each node. A forecasted load is considered, which represents the maximum additional capacity that may be added to that bus. Equations (3.30) and (3.31) are the linear active and reactive power flows previously developed. Limits on active and reactive power flows through distribution lines connecting adjacent buses are represented in (3.32) and (3.33). Limits to the total productions of active and reactive power through buses which include POI to the upstream network are represented in (3.34) and (3.35). In these equations, the $C_m$ includes all buses which are connected to the upstream network. Equations (3.36) and (3.37)
restrict the additional reactive power generated/consumed based on the additional active power and the associated power factor. Assuming a constant power factor, parameter $\alpha$ can be determined and used in (3.36) and (3.37). A polyhedral uncertainty set is considered for the load that can change between a lower and upper bound (3.38). Equation (3.39) imposes performance limits on voltage deviations. Definitions for voltage deviations are represented by (3.40) and (3.41). These conditions, taken together, fully define the optimization problem which computes the marginal hosting capacity. The marginal hosting capacity value $\lambda_m$ is the dual variable of the active power balance equation (3.28).
Chapter Four: Increasing Distribution Grid Hosting Capacity

Most of the existing distribution grids are designed as passive networks with a radial or weakly meshed topology. This design is preferred due to the ease and low cost of operation and maintenance. However, the implementation of DGs was not anticipated and transmission lines close to consumers were not expected to handle high level generation. To maximize the potential of these grids in light of the trend toward DG integration, this chapter focuses on network reconfiguration strategies for increasing the hosting capacity of distribution grids [67]. Reconfiguration is commonly performed for emergency operation, reduction of power loss, system load balancing, voltage profile improvements, reliability improvements, and service restoration. These uses can be imposed against technical problems arising from the high penetration of DGs into a network, such as reverse power flow and voltage rise. However, assessment of the effect of system reconfiguration on DG hosting capacity must be done to determine the most beneficial system configuration. Such assessment must also be fast and reliable to be incorporated into smart grid technologies for distribution automation.

System reconfiguration can be realized by opening and closing already installed switches. Physical infrastructure changes are sometimes necessary to reconfigure a system, but the timescales required to implement this reconfiguration prevents its use in short timescale responses, i.e., the fluctuations in power generation and local energy demands. For this reason, this work focuses on network reconfiguration via switching.
Network reconfiguration is defined as the process of changing the status of the network switches to obtain different configurations of a distribution grid without changing the system’s infrastructure [68]. In distribution grids, switches are classified into two types of sectionalizing switches, which are normally closed, and tie switches, which are normally open.

Network reconfiguration is introduced into the optimization by formulating it as a mixed-integer programming (MIP) problem. The advantage of using a linearized method over more conventional iterative methods is twofold. First, computation time is dramatically reduced, especially when a large search space is considered. Second, it makes the optimization across all possible reconfiguration topologies and DG deployment profiles possible to be considered. These advantages combine to allow for a rapid and robust computation of optimal reconfiguration since nonlinear analysis is avoided.

4.1 Radiality Constraint on Network Reconfiguration

The reconfiguration of a distribution grid is executed by changing the state of switches in the grid. The purpose of the distribution network reconfiguration is to obtain the configuration which would maximize the grid hosting capacity. Since we are optimizing radial networks, reconfiguring the network should not affect its radiality structure. The term “radial” here refers to a configuration which connects all nodes but does not contain connected loops. The radiality condition is enforced by verifying that in all potential loops the number of closed lines is less than the total number of lines.
comprising the loop. Therefore, there should be at least one open line in each potential loop. The radially constraint can be defined as follows:

\[
\sum_{mn \in \Gamma} z_{mn} \leq L_{mn} - 1 \quad \forall mn \in L
\]  

Here, \( \Gamma \) is the set of all possible loops, \( L_{mn} \) is the total number of lines in each loop, and \( z_{mn} \) is the binary variable that represents the status of the line connecting buses \( m \) and \( n \). \( z_{mn} \) is 0 when the line switches are opened (line is switched out) and 1 when the switches are closed (line is switched in).

### 4.2 Increasing Grid Hosting Capacity Problem Formulation

The objective of the proposed optimization method is to find the optimal configuration which simultaneously maximizes the total DG capacity that can be deployed in the distribution grid (4.2).

\[
\max \sum_{m \in B} P_{m}^G
\]  

This objective is subject to the operational constraints (4.3)-(4.12):

\[
\sum_{c \in C_m} P_c^M + \sum_{mn \in L_m} P_{mn}^L + P_{m}^G = P_{m}^D \quad \forall m \in B
\]  

\[
\sum_{c \in C_m} Q_c^M + \sum_{mn \in L_m} Q_{mn}^L + Q_{m}^G = Q_{m}^D \quad \forall m \in B
\]  

\[
\sum_{mn \in \Gamma} z_{mn} \leq L_{mn} - 1 \quad \forall mn \in L
\]  

\[
\left| PL_{mn} - \left( g_{mn} (1 + \Delta V_m) (\Delta V_m - \Delta V_n) \right) - b_{mn} (\Delta \theta_m - \Delta \theta_n) \right| \leq M (1 - z_{mn}) \quad \forall mn \in L
\]  

\[
\left| QL_{mn} - \left( -b_{mn} (1 + \Delta V_m) (\Delta V_m - \Delta V_n) \right) - g_{mn} (\Delta \theta_m - \Delta \theta_n) \right| \leq M (1 - z_{mn}) \quad \forall mn \in L
\]  

35
The active power balance equation (4.3) and reactive power balance equation (4.4) must be satisfied for each bus $m$. It is added as a constraint to ensure that the total active and reactive power supplied from the upstream grid and installed DG matches the load at the associated bus. Constraint (4.5) enforces the radiality of the optimal configuration. Constraints (4.6) and (4.7) are the linearized active and reactive AC power flow equations discussed in Chapter Two. The active and reactive power exchanged with the upstream grid is limited by (4.8) and (4.9). Constraints (4.10) and (4.11) impose the active and reactive power flow limits in the distribution lines. Constraint (4.12) imposes voltage deviation constraints relative to the POI voltage. In (4.6), (4.7), (4.10), and (4.11), $z_{mn}$ is a binary variable used to define the status of the line connecting buses $m$ and $n$. If $z_{mn}$ is zero, the line connecting buses $m$ and $n$ is open. Mathematically, setting $z_{mn}=0$ in (4.10) and (4.11) will force both $PL_{mn}$ and $QL_{mn}$ to be zero ensuring no power flow. Also, an open line should not enforce the power flow equation through the line connecting buses $m$ and $n$. To completely relax the power flow equations, the constant $M$ is set to a large positive number. If, on the other hand, the line is closed and $z_{mn}=1$, then the power flow limits (4.10) and (4.11) are allowed to be nonzero and the power flow equations

\[
-P_c^{M,\text{max}} \leq P_c^{M} \leq P_c^{M,\text{max}} \quad \forall c \in C \tag{4.8}
\]

\[
-Q_c^{M,\text{max}} \leq Q_c^{M} \leq Q_c^{M,\text{max}} \quad \forall c \in C \tag{4.9}
\]

\[
-z_{mn} \cdot PL_{mn}^{\text{max}} \leq PL_{mn} \leq z_{mn} \cdot PL_{mn}^{\text{max}} \quad \forall mn \in L \tag{4.10}
\]

\[
-z_{mn} \cdot QL_{mn}^{\text{max}} \leq QL_{mn} \leq z_{mn} \cdot QL_{mn}^{\text{max}} \quad \forall mn \in L \tag{4.11}
\]

\[
\Delta V_m^{\text{min}} \leq \Delta V_m \leq \Delta V_m^{\text{max}} \quad \forall m \in B \tag{4.12}
\]
(4.6) and (4.7) would be enforced in the optimization problem. Connectivity is further guaranteed in the optimal solution as a consequence of the power balance equations.
Chapter Five: A Fast Hosting Capacity Calculation Method for Large Distribution Grids

This chapter presents the sensitivity-based hosting capacity calculation method that can be utilized to determine the optimal DG hosting capacity for large-scale distribution networks [69]. The developed method was proposed based on the sensitivity analysis of line power flow and voltage magnitudes with respect to nodal active and reactive injections. The method uses an optimization approach that reduces the number of variables in the search space, avoids extensive iterations, and significantly reduces the runtime while providing results comparable with traditional methods. The smaller computation time allows distribution system operators to scale up the optimization to larger systems without losing the robustness. For additional robustness, load uncertainties are considered to obtain a conservative grid hosting capacity solution.

Similar to the previously discussed optimization-based hosting capacity method, the objective of the proposed method is to maximize the amount of DG deployment in an active distribution network without negatively affecting the operational performance of the network. A comprehensive optimization should explore the effects of injecting varying amounts of DG generation into various locations simultaneously. However, each possible location for DG installation introduces another variable to be optimized, causing an exponential increase in the computation required as more buses are considered. Sensitivity analysis can overcome this problem by simplifying the optimization problem.
This is conducted by considering the effect of variations of DG power in each location on the system’s steady-state bus voltages and line flows.

Sensitivity analysis relies on the linearization assumption of AC power flow equations presented in Chapter Two. This permits a reduction of the number of solutions required for DG injection. Figure 5.1 illustrates the flowchart of the proposed method. In step one, operational performance indices are defined which measure whether the performance of the system is within the acceptable limits. For each performance index, operational upper and lower bounds are defined within which the system is operating properly (e.g. thermal limits and voltage limits). The sensitivity-based method used here may be generalized to any operational performance index defined based on the operational behavior of the distribution network (i.e., emphasizing different operational concerns). In step two, the sensitivity analysis of the line flow and voltage magnitude is performed while enforcing limits defined in step 1. The results obtained from step two will be the optimal DG hosting capacity.

Figure 5.1: The proposed sensitivity-based hosting capacity method.
5.1 Sensitivity-based Problem Formulation

5.1.2 Linear Sensitivity Factors

The objective function is to maximize the hosting capacity (i.e. the maximum installed amount of DG). To simulate network behavior, the objective function is subject to the active and reactive power injections at bus \( m \) and can be defined as follows:

\[
P_m = \sum_n (g_{mn} (1 + \Delta V_m \Delta V_m - \Delta V_n \Delta V_n - b_{mn} \Delta \theta_m \Delta \theta_n)) \quad \forall m, n \in B \quad (5.1)
\]

\[
Q_m = \sum_n (-b_{mn} (1 + \Delta V_m \Delta V_m - \Delta V_n \Delta V_n - g_{mn} \Delta \theta_m \Delta \theta_n)) \quad \forall m, n \in B \quad (5.2)
\]

where, \( g \) and \( b \) are the line conductance and susceptance, respectively, and \( B \) is the set of all buses. By assuming \( \Delta V_m \) (in the term \( 1+\Delta V_m \)) is zero, system losses will be ignored and thus these equations would convert to linear equations. Based on (5.1) and (5.2), the active and reactive injected power can be defined in matrix form as follows:

\[
\begin{bmatrix}
P \\
Q
\end{bmatrix} =
\begin{bmatrix}
G & -B \\
-B & -G
\end{bmatrix}
\begin{bmatrix}
\Delta V \\
\Delta \theta
\end{bmatrix} \quad (5.3)
\]

Here, \( P \) and \( Q \) are respectively the injected active and reactive power, \( G \) and \( B \) are respectively the conductance and susceptance matrices, and \( \Delta V \) and \( \Delta \theta \) are, respectively, the change in voltage magnitude and angle with respect to the POI. The voltage sensitivity factors (VSF) with respect to the active and reactive injected power can be easily calculated from (5.3) as:

\[
\text{VSF}^P = (B^{-1}G + G^{-1}B)^{-1}(B^{-1}) \quad (5.4)
\]

\[
\text{VSF}^Q = (B^{-1}G + G^{-1}B)^{-1}(-G^{-1}) \quad (5.5)
\]
Based on the injected power and the line flow equations, the active and reactive line sensitivity factors (LSF) can also be calculated as:

\[
\begin{align*}
\text{LSF}^p &= \mathbf{D}(g)\mathbf{A}(\mathbf{B}^{-1}\mathbf{G} + \mathbf{G}^{-1}\mathbf{B})^{-1}(\mathbf{B}^{-1}) + \mathbf{D}(b)\mathbf{A}(\mathbf{B}^{-1}\mathbf{G} + \mathbf{G}^{-1}\mathbf{B})^{-1}(-\mathbf{G}^{-1}) \\
\text{LSF}^q &= -\mathbf{D}(g)\mathbf{A}(\mathbf{B}^{-1}\mathbf{G} + \mathbf{G}^{-1}\mathbf{B})^{-1}(\mathbf{G}^{-1}) + \mathbf{D}(b)\mathbf{A}(\mathbf{B}^{-1}\mathbf{G} + \mathbf{G}^{-1}\mathbf{B})^{-1}(\mathbf{B}^{-1})
\end{align*}
\]  

(5.6)  
(5.7)

Here, \( \mathbf{D}(g) \) and \( \mathbf{D}(b) \) are diagonal matrices of the lines conductance and susceptance, respectively. \( \mathbf{A} \) represents the bus-line incidence matrix.

### 5.1.2 Line Limits

For the line connecting buses \( m \) and \( n \), the active power flow \( PL_{mn} \) is defined based on the line sensitivity factors (LSF) as follows:

\[
PL_{mn} = \sum_{i \in L_m} (\text{LSF}_{mn,i}^p P_i + \text{LSF}_{mn,i}^q Q_i) \quad \forall mn \in L, \forall i \in B \tag{5.8}
\]

In (5.8), \( \text{LSF}_{mn,i}^p \) and \( \text{LSF}_{mn,i}^q \) are the active and reactive line sensitivity factors of the line connecting buses \( m \) and \( n \) subject to the power injection at bus \( i \). \( P_i \) and \( Q_i \) are the net injected active and reactive power at bus \( i \), defined respectively as \( P_i = P_i^G - P_i^D \) and \( Q_i = Q_i^G - Q_i^D \). \( P_i^G \) and \( Q_i^G \) are the active and reactive power generated by DG at bus \( i \), and \( P_i^D \) and \( Q_i^D \) are the active and reactive load at bus \( i \). Assuming a constant power factor for injected DG, a constant parameter \( \alpha \) can be defined as the ratio of the reactive power to the active power. With this assumption, the net injected reactive power can be defined as \( Q_i = \alpha P_i^G - Q_i^D \).

To simulate the impact of load variations, the worst-case scenario should be taken into account. This step is taken to guarantee that the system performs within the
acceptable limits regardless of variations in load. The line limit is represented using LSFs as follows:

\[ P_{mn}^{adj,\min} \leq \sum_{n \in L_m} (LSF_{mn,i}^P P_i^G + LSF_{mn,i}^Q Q_i^G) \leq P_{mn}^{adj,\max} \quad \forall mn \in L, \forall i \in B \quad (5.9) \]

In (5.9), the bounds represent the adjusted active power flow limits of line \( mn \).

The lower and upper adjusted limits are calculated as:

\[ P_{mn}^{adj,\min} = -P_{mn}^{\max} + \min \left( \sum_{n \in L_m} (LSF_{mn,i}^P P_i^D + LSF_{mn,i}^Q Q_i^D) \right) \quad \forall mn \in L, \forall i \in B (5.10) \]

\[ P_{mn}^{adj,\max} = P_{mn}^{\max} + \min \left( \sum_{n \in L_m} (LSF_{mn,i}^P P_i^D + LSF_{mn,i}^Q Q_i^D) \right) \quad \forall mn \in L, \forall i \in B (5.11) \]

Here, \( P_{mn}^{\max} \) is the maximum line capacity limit, and ‘min’ ensures that the most restricting limit, i.e., the worst-case is applied.

### 5.1.3 Voltage limits

The voltage constraint, in terms of the VSF with respect to the active and reactive injected power, can be expressed as:

\[ \Delta V_{m}^{adj,\min} \leq \sum_{m} (VSF_{m,i}^P P_i^D + VSF_{m,i}^Q Q_i^D) \leq \Delta V_{m}^{adj,\max} \quad \forall m, i \in B \quad (5.12) \]

The bounds represent the adjusted voltage magnitude limits are defined as:

\[ \Delta V_{m}^{adj,\min} = \Delta V_{m}^{\min} + \min \left( \sum_{m} (VSF_{m,i}^P P_i^D + VSF_{m,i}^Q Q_i^D) \right) \quad \forall m, i \in B \quad (5.13) \]

\[ \Delta V_{m}^{adj,\max} = \Delta V_{m}^{\max} + \min \left( \sum_{m} (VSF_{m,i}^P P_i^D + VSF_{m,i}^Q Q_i^D) \right) \quad \forall m, i \in B \quad (5.14) \]

\( V_{m}^{\min} \) and \( V_{m}^{\max} \) are respectively the lower and upper voltage limits in bus \( m \).
5.1.4 Hosting capacity calculation

Based on the calculated sensitivity factors, the hosting capacity calculation model can be developed as follows:

$$\text{max} \sum_i P^G_i$$  \hspace{1cm} (5.15)

Subject to (5.9) and (5.12).

The objective function of the proposed model is to maximize the total network DG hosting capacity (5.15) that is subject to the line capacity (5.9) and voltage magnitude (5.12) limits. The uncertainties in load are integrated into the model through adjusted line and voltage limits. This model has only one variable, i.e., $P^G$, so it can be solved in a very short amount of time even for very large-scale problems.
Chapter Six: Numerical Simulation

The IEEE 33-bus distribution test system is studied to show the performance of the proposed hosting capacity methods. This system contains 33 buses, 32 sectionalizing switches, and 5 tie switches, and no existing DG as shown in Figure 6.1. The detailed data of the system is given in [70]. Bus 1 is considered as the POI where no DG can be installed. During the analysis, all loads are initially set to be a constant value, called the base load. When accounting for the inherent uncertainty of loads within the system, an uncertainty range, i.e., lower and upper bounds, is defined. For each candidate DG, the maximum power output is assumed to be equal to its installed capacity and the minimum power output is assumed to be zero. Voltage at POI is assumed to be 1 p.u. with an angle of 0°. Considering respective minimum and maximum bus voltage limits of 0.9 p.u. and 1.1 p.u., the lower and upper voltage deviation limits are obtained as -0.1 p.u. and 0.1 p.u., respectively. Active power exchanged with the upstream grid is capped at 4.6 MW. The problems are formulated as linear programming (LP) problems and solved using CPLEX 12.4.
6.1 Validating the Accuracy of the Linear Power Flow Solution.

This case validates the linear power flow solution by providing comparisons with solutions from nonlinear full AC power flow. This comparison is needed to show the accuracy of the developed model and furthermore allow integration with the hosting capacity calculation method.

The linear power flow is applied to the radial distribution test system shown in Figure 6.1 to find power flow solution and compare it with those of nonlinear AC power flow analysis. Results obtained from the linearized method compared with the nonlinear method show an average percent error for voltage magnitudes, voltage angles, line flows, and total line losses of 0.002 %, 16.2 %, 0.21 %, and 9.4 %, respectively. The results advocate a very high accuracy in determining voltage magnitude and line flows. This accuracy is less for voltage angles, however, it should be considered that voltage angles are less important factors in distribution network power flow studies when compared to
voltage magnitudes. Their impact on line flows can be clearly seen from line flow equations. The average values of the percent error are found by first calculating the percent error for each individual bus/line, and then averaging across all buses/lines. Figure 6.2 displays voltage magnitude results in each bus for both methods. The results advocate solution accuracy of the proposed linear method compared to the nonlinear AC power flow method. As discussed, the results can be improved by incorporating additional steps in finding voltage magnitudes and angles, which is however not required as the obtained results are already close to actual values.

![Figure 6.2: Voltage magnitude comparison between linear and nonlinear power flow methods.](image)

### 6.2 Validating the Accuracy of the Optimization-based Hosting Capacity Model

The effectiveness of the proposed optimization-based hosting capacity model is showcased on the IEEE 33-bus test system shown in Figure 6.1. The results, including runtime, of the linearized algorithm are compared with the results of the iterative nonlinear hosting capacity algorithm. Both algorithms are initialized with the same parameters (i.e. nodal loads, line flow limits, and voltage limits) to enable a direct comparison.
The traditional hosting capacity optimization approach is restricted by computational requirements. To demonstrate one manifestation of this restriction, the resolution of the hosting capacity was increased, and the runtime was measured. Hosting capacity resolution can be increased in the iterative approach by reducing the DG step size during each iteration. With reduced step size, more values of DG injection power are sampled in a given range at the cost of requiring more iterations. Four DG step sizes were chosen for this demonstration, 1 kW, 10 kW, 100 kW, and 1 MW. To avoid impractical computation times, DG generation was only swept in one location at a time. Figure 6.3 shows the relationship between accuracy and time. A trade-off emerges in which decreasing error causes an increase in computation time and decreasing computation time causes an increase in error. For DG step sizes of 1 kW, 10 kW, 100 kW and 1 MW the computation time is 472 s, 49 s, 6 s, and 2 s, respectively.

![Figure 6.3: Tradeoff between speed and accuracy for each DG step size based on the iterative method.](image)

While it is feasible to find a balance between accuracy and time in this case, it becomes infeasible to do so when trying to expand the analysis, e.g., to optimize DG placement to multiple buses simultaneously. The linearized hosting capacity optimization was performed on the same system and results are compared to the highest fidelity
iterative optimization executed (DG step size of 1 kW). For the first comparative study, individual hosting capacities are determined for each bus assuming there will be no DG installation at other buses (i.e., ignoring spatial interdependency). Each bus’s individual hosting capacity is optimized for the uncertain load profile. Figure 6.4 represents the grid hosting capacity results for each individual bus using the proposed method and the traditional method. As can be seen, the results of the two methods are very similar. The time required to solve the problem based on the proposed method is 2 s, while the computation time in traditional method is 359 s. The average percent error of the proposed method is 1.08 % compared to the traditional method. These results demonstrate the significantly-improved computation speed and the acceptable accuracy that the proposed method provides over the traditional hosting capacity method when analyzing single-bus hosting capacities.

![Figure 6.4: Hosting capacity for each individual bus (accuracy comparison).](image)

In the second comparative study, the proposed method and the traditional method are used to compute the grid-level hosting capacity when all buses are considered. The traditional hosting capacity method takes 84 hours to find the final solution, while the proposed method finds the solution in 21 s. The average percent difference in final
solution is 1.2 %. The obtained results advocate that the proposed method is accurate and extremely fast.

6.3 Optimal Hosting Capacity

The effectiveness of the proposed optimization-based hosting capacity method is showcased on the IEEE 33-bus test system shown in Figure 6.1. The following cases are studied:

Case 6.3.1: Grid-level hosting capacity calculation for base loads.

Case 6.3.2: Grid-level hosting capacity calculation with uncertain loads.

Case 6.3.3: Sensitivity analysis with respect to line flow limits.

Case 6.3.4: Sensitivity analysis with respect to voltage limits.

Cases 6.3.1 and 6.3.2 use the developed optimization-based method to calculate hosting capacity at grid level, i.e., considering all buses at the same time. Case 6.3.1 focuses on base load, i.e., one single load snapshot, while Case 6.3.2 captures load uncertainty. The comparison of results between these two cases will show a tradeoff that will occur when uncertainties are considered. Cases 6.3.3 and 6.3.4 further elaborate results of Case 6.3.2 by analyzing the sensitivity of the hosting capacity results on performance measures, here line overload and bus overvoltage, respectively. Cases 6.3.1, 6.3.2, and 6.3.3 are studied under three scenarios:

Scenario 1: All buses are considered for DG installation.

Scenario 2: DG installations is allowed at buses 2 and 3 only, as these two buses are directly connected to the highest-capacity lines in the system.

Scenario 3: DG installation is allowed at end buses only (buses 18, 22, 25, and 33).
**Case 6.3.1**: In this case, the grid hosting capacity is determined using the base case load for the three considered scenarios. The grid hosting capacity in scenario 1 is when DGs are installed at buses 2, 19, and 20 with capacities of 7624 kW, 90 kW, and 770 kW, respectively, resulting in a grid hosting capacity of 8484 kW. This result is shown in Table 6.1. The hosting capacity is limited by the maximum acceptable active power flow through the lines connected to bus 1. The hosting capacity in scenario 2, for which DGs may be placed only at buses 2 and 3, is calculated at the same value of 8484 kW, with the difference that it is fully installed at bus 2. The point of this scenario is to explore the variation for which the influence of line capacity limits is the weakest (i.e. limiting the optimal placement of DGs). This highlights the bottlenecking role that a line may play in the grid. In scenario 3, where DGs may only be installed at end buses, the grid hosting capacity results in installations at buses 18, 22, 25 and 33 with capacities of 190 kW, 200 kW, 920 kW, and 160 kW, respectively, for a total capacity of 1470 kW. Line losses are decreased by 34.7 % in this case, but the overall hosting capacity is decreased by 82.7% when compared with the first two scenarios. This result could be foreseen as the end-buses are connected to lines that have considerably smaller capacities compared to other buses.

<table>
<thead>
<tr>
<th>Bus #</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7624</td>
<td>8484</td>
<td>0</td>
</tr>
<tr>
<td>18</td>
<td>0</td>
<td>0</td>
<td>190</td>
</tr>
<tr>
<td>19</td>
<td>90</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>770</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>22</td>
<td>0</td>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td>25</td>
<td>0</td>
<td>0</td>
<td>920</td>
</tr>
<tr>
<td>33</td>
<td>0</td>
<td>0</td>
<td>160</td>
</tr>
<tr>
<td><strong>Total DG (kW)</strong></td>
<td>8484</td>
<td>8484</td>
<td>1470</td>
</tr>
<tr>
<td><strong>Total loss (kW)</strong></td>
<td>175.12</td>
<td>175.15</td>
<td>114.43</td>
</tr>
</tbody>
</table>
Case 6.3.2: Using the uncertain load data, the grid hosting capacity is calculated for the same three scenarios used in Case 6.3.1 and the results are summarized in Table 6.2. The minimum/maximum load recorded over a year-long horizon is used as the lower/upper bound of uncertain load in each bus. Since the worst-case solution will be obtained based on this uncertain load profile, the grid hosting capacity will never result in unacceptable performance for other load profiles. The total real and reactive base load in Case 6.3.1 is 3715 kW and 2300 kVAR, respectively, while in this case the real and reactive load can change in the range of [1490 kW, 3715 kW] and [922.5 kVAR, 2300 kVAR], respectively.

For scenario 1, the total hosting capacity is calculated as 6116 kW with DGs being placed in buses 2, 19, and 20. Similar to Case 6.3.1, scenario 2 has all the DGs placed at bus 2 to optimize the hosting capacity with a total of 6116 kW. As in Case 6.3.1, the capacity of line connecting this system to the upstream grid is the limiting factor. The results here are considered more reliable, as they demonstrate the minimum expected hosting capacity when including load uncertainty into the model, i.e., the obtained result will still be valid for any other realizations of loads. Comparing the obtained solution in these two scenarios, the grid hosting capacity is reduced down to 63.15% of the hosting capacity in Case 6.3.1. For scenario 3, the grid hosting capacity is calculated as 1074 kW and the system losses are reduced by 63.4%. Power flow capacities at lines 17, 21, 24, and 32 are limiting the hosting capacity in this scenario. The obtained results in this case underscore that when handling the worst-case load profile, the system cannot accommodate more than 72% of the base load hosting capacity. In this
case, none of the scenarios have been limited by voltage magnitude constraints; rather, distribution line capacities limit the hosting capacity. Figure 6.5 shows the voltage magnitudes for studied scenarios. Note that the voltage never dips below 0.96 p.u. and thus falls within the acceptable range of 0.90 p.u. to 1.1 p.u.

Table 6.2: Uncertain load hosting capacity results (kW).

<table>
<thead>
<tr>
<th>Bus #</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5471</td>
<td>6116</td>
<td>0</td>
</tr>
<tr>
<td>18</td>
<td>0</td>
<td>0</td>
<td>136</td>
</tr>
<tr>
<td>19</td>
<td>37</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>608</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>22</td>
<td>0</td>
<td>0</td>
<td>146</td>
</tr>
<tr>
<td>25</td>
<td>0</td>
<td>0</td>
<td>668</td>
</tr>
<tr>
<td>33</td>
<td>0</td>
<td>0</td>
<td>124</td>
</tr>
<tr>
<td><strong>Total DG (kW)</strong></td>
<td><strong>6116</strong></td>
<td><strong>6116</strong></td>
<td><strong>1074</strong></td>
</tr>
<tr>
<td><strong>Total loss (kW)</strong></td>
<td><strong>35.12</strong></td>
<td><strong>34.1</strong></td>
<td><strong>12.87</strong></td>
</tr>
</tbody>
</table>

Figure 6.5: Voltage magnitude profile in Case 6.3.2.

**Case 6.3.3:** In this case, the sensitivity of the hosting capacity results with respect to line capacities is studied. Line flow limits are changed to reflect capacity upgrades to specific lines instead of upgrading the entire system. Similar to Case 6.3.2, this case
explores what the worst-case solutions for a given system parameter for the same scenarios are. This is reflected in the problem by increasing the line limits of the critical lines 1 and 2. Cases 6.3.1 and 6.3.2 highlighted the crucial role that these lines play in grid hosting capacity. The capacity limits of lines 1 and 2 are increased by 10 % increments up to 40 %. Figure 6.6 shows the grid hosting capacity as a function of the line capacity limit variations. For the first two scenarios, the grid hosting capacity is increased by 7.5 %, 15 %, 22.5 % and 30 % when the capacity limits of the lines are increased by 10 %, 20 %, 30 % and 40 % respectively. However, the hosting capacity in the last scenario does not improve as the adjusted line limits are not hit, and thus are not involved, in hosting capacity calculations. Figure 6.7 shows the change in total system losses due to increase in line capacity limits. As the grid hosting capacity is increased in the first two scenarios, the total losses also increase by 5.2 %, 10.8 %, 14.2 %, and 22 % for the line limit increases of 10 %, 20 %, 30 % and 40 % respectively. However, there is no change in the total losses of the last scenario as the solution does not change in response to the increase in line limits. A clear pattern emerges, wherein hosting capacity in scenarios 1 and 2, which were demonstrated to be limited by line 1 capacity limits, are positively affected by the increased capacity. The hosting capacity results in scenario 3, however, remain unaffected as their limitations are due to line capacities in multiple smaller lines elsewhere in the grid. Figure 6.8 re-expresses the data as the percent change in the grid hosting capacity and total losses as the critical line limits are changed. This figure highlights that in some cases local upgrades can increase the grid hosting capacity.
Figure 6.6: Hosting capacity results based on the change in capacity limits of lines 1 and 2.

Figure 6.7: Line loss based on the change in capacity limits of lines 1 and 2.

Figure 6.8: Change in hosting capacity and losses due to the change in capacity limits of lines 1 and 2.

**Case 6.3.4:** In this case, the change in hosting capacity with respect to voltage magnitude limits is considered by reducing voltage deviations limits to ±0.05 p.u. The grid hosting capacity results in this case are compared to those of Case 6.3.2 and shown in Figure 6.9 using the same uncertain load data. The comparison shows that there is no

54
change in the grid hosting capacity results. However, the hosting capacity location of scenario 1 is changed to bus 2 but with the same amount. This is because in this case some of the downstream buses reach their voltage limit, so the DG installation is moved to bus 2. Table 6.3 compares the solution for the two considered voltage deviation limits, i.e., ±0.1 p.u. and ±0.05 p.u. The tighter voltage limits in the base load analysis lead to a reduction in the grid hosting capacity results. The result is decreased from 8484 kW to 8400 kW, as voltage magnitude at buses 15-18, 32, and 33 has reached the limit of 0.95 p.u. (Figure 6.10). This is expected, since a reduction in the allowed voltage fluctuations means that a smaller DG capacity can be accepted.

Table 6.3: Grid hosting capacity results (kW) based on the change in voltage deviation limits.

<table>
<thead>
<tr>
<th>Bus #</th>
<th>Base load hosting capacity</th>
<th>Uncertain load hosting capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>±0.1 p.u. voltage deviation limit</td>
<td>±0.05 p.u. voltage deviation limit</td>
</tr>
<tr>
<td>2</td>
<td>7624</td>
<td>7541</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>71</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
<td>38</td>
</tr>
<tr>
<td>17</td>
<td>0</td>
<td>108</td>
</tr>
<tr>
<td>18</td>
<td>0</td>
<td>121</td>
</tr>
<tr>
<td>19</td>
<td>90</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>770</td>
<td>0</td>
</tr>
<tr>
<td>32</td>
<td>0</td>
<td>481</td>
</tr>
<tr>
<td>33</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td><strong>Total DG (kW)</strong></td>
<td>8484</td>
<td>8400</td>
</tr>
<tr>
<td><strong>Total loss (kW)</strong></td>
<td>175.12</td>
<td>93.5</td>
</tr>
</tbody>
</table>
Figure 6.9: Uncertain load hosting capacity results.

Figure 6.10: Voltage magnitudes based on hosting capacity results under base load conditions.

These cases show that the hosting capacity in a given system can be achieved through the proposed optimization-based method, and DGs could be incorporated while guaranteeing no detrimental impact on grid performance. Adding DGs to the grid has also been demonstrated to reduce system losses when properly placed, again possible to study through the proposed method. In addition, variety of scenarios, system setups, and objectives are possible to investigate via this method.

6.4 Loading Capacity

The proposed loading capacity model is used to find the optimal loading capacity in the IEEE 33-bus distribution test system shown in Figure 6.1. The total base active and
reactive load values of 3.715 MW and 2.3 MW are assumed at all buses of the system.

The following cases are studied:

**Case 6.4.1:** Evaluating optimal loading capacity for each individual bus.

**Case 6.4.2:** Evaluating optimal loading capacity when all buses are considered.

**Case 6.4.3:** Evaluating optimal loading capacity considering maximum permissible load increase.

**Case 6.4.4:** Comparison with iterative approach.

**Case 6.4.1:** In this case, the proposed optimal loading capacity problem is solved to determine the additional loading capacity in each bus individually. These values are represented in Figure 6.11. In the proposed problem, the fixed loads represent a lower bound on power supplied to each bus, then the additional loading capacity for each individual bus is calculated. The additional loads shown in Figure 6.11 represent the maximum additional capacity that may be added to each bus above which system performance will degrade. The results suggest that there are many buses in the system with high loading capacity, many times larger than the current load. For instance, loading capacity in buses 2-6 and 26-30 exceeds 500 kW. Bus 2 has the largest individual loading capacity at 698.8 kW of additional load. Bus 18 has the lowest loading capacity at 9.95 kW of additional capacity. For the loading capacity optimization in each bus, Figure 6.12 shows the increased percentage of system loss in all lines. The largest system losses are seen when loading buses 29 and 30 with losses of 245 kW and 250 kW respectively. It is important to note, however, that line loss data should be interpreted in combination with loading capacity results, as lower capacities imply lower flows which lead to lower
losses. Thus, lower losses often indicate lower loading capacity as well, which is true for all buses except bus 2. It should be further noted that bus 1 is not considered in this simulation as it is the POI where no load is/can be connected to.

Figure 6.11: Fixed load and Additional load capacity for each individual bus.

Figure 6.12: Total change in system losses when optimizing additional capacity in each bus.

**Case 6.4.2:** Optimal loading capacity is calculated in this case with a similar setup to the previous case, however, loading is allowed in all buses simultaneously. This case simply maximizes the overall loading capacity considering inter-spatial impacts of the loads. Figure 6.13 shows the optimal distribution for additional loads to maximize the loading capacity. Maximum loading capacity of 4.6 MW is achieved by placing all 698.8 kW of additional loading in bus 2. Of the 4.6 MW of loading, 3.715 MW is provided to
the fixed loads, 698.8 kW to the additional load in bus 2, and 186.2 kW is dissipated as network loss. The optimal result is constrained by the maximum limit of active power flowing through the line connecting buses 1 and 2.

![Figure 6.13: Fixed loading and optimal additional loading capacity when considering all buses.](image)

**Case 6.4.3:** It is often helpful to express additional loading capacity in proportion to existing loads at all buses. For this reason, a maximum total loading that may be added at each bus as a proportion of the fixed load is introduced and added to the problem as a new constraint. The effect of changing this limit is investigated here. In this case, the maximum limit is varied from 120% to 150% in 10% increments and the optimal loading capacity is determined, allowing the additional loads to be placed in any bus in the system. The results of the analysis are summarized in Table 6.4. Overall, the results show that changing this limit does not seem to impact the overall loading capacity, although it does effect where that additional loading is placed and how the load is distributed. For some buses, increasing this limit causes the load to increase (e.g. buses 2, 3, 19). Figure 6.14 shows the impacts to the system losses as this limit is increased. In all the studied scenarios in this case, the same power flow limits are constraining the maximum additional load (i.e. line flow limits at lines 1, 18, and 22). The ineffectiveness of this
maximum limit on loading capacity suggests that upgrades targeting redistribution of loads are not necessary.

Table 6.4: Optimal loading placement for different α values.

<table>
<thead>
<tr>
<th>Bus #</th>
<th>Increases in maximum capacity for each bus according to the actual load values</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>120%</td>
<td>130%</td>
<td>140%</td>
<td>150%</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>120.0</td>
<td>130.0</td>
<td>140.0</td>
<td>150.0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>108.0</td>
<td>117.0</td>
<td>126.0</td>
<td>135.0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>144.0</td>
<td>156.0</td>
<td>139.146</td>
<td>123.895</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>39.46</td>
<td>3.83</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>108.0</td>
<td>117.0</td>
<td>126.0</td>
<td>135.0</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>30.771</td>
<td>21.901</td>
<td>12.868</td>
<td>3.6316</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>108.0</td>
<td>110.413</td>
<td>110.432</td>
<td>110.434</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>2.35</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>Total additional load (kW)</td>
<td>660.58</td>
<td>656.14</td>
<td>654.44</td>
<td>657.96</td>
<td></td>
</tr>
</tbody>
</table>

Figure 6.14: Total system loss considering the additional capacity limits.

**Case 6.4.4:** In this case, the proposed model is compared with the iterative approach in terms of computation time and solution accuracy. In the iterative approach an initial load of zero is considered in each bus, starting from bus 2, and then the load is incrementally increased. Each load increment is followed by a full AC power flow to determine possible violations, in line and voltage limits, in the network. Once a violation is identified, the previous value is considered as the loading capacity for that bus. The
process is then continued for the next bus, and accordingly, the loading capacity for each bus is identified. This approach only finds the loading capacity at each individual bus, i.e., ignoring the possibility of the loading capacity happening at two or more buses. To consider more than one bus, all the combinations between various buses should be considered.

This described iterative approach, which is the common approach as mentioned in the literature review, is applied to the studied 33-bus test system. For the case of loading capacity in individual buses, and considering an average power flow solution time of 0.1 s and increments of 1 kW, it takes 747 s, i.e., more than 12 min, to find the optimal loading capacity. The obtained result is exactly similar to the result from the proposed model in this study, however, computation time is increased from 9 s to more than 12 min. If the loading capacity in two buses is considered, the number of combinations will be more than 55.8 million that would take about 64 days to find the optimal solution. The solution from the proposed model is still 9 s. To improve the computation time of the iterative approach, a larger increment, for example 10 kW, can be used. This new increment can reduce the computation time to 6 days, but the obtained solution is not accurate anymore as only increments of 10 kW are considered. The comparison between these results show the merits of the proposed model compared to commonly-used iterative approaches.
6.5 Marginal Hosting Capacity

The proposed marginal hosting capacity method is applied to the IEEE 33-bus distribution system shown in Figure 6.1. Similar to what has been considered in Section 6.4, the system total base load is 3.715 + j2.3 MVA. The following cases are studied:

Case 6.5.1: Individual marginal hosting capacity for each bus.

Case 6.5.2: Nodal analysis based on marginal hosting capacity.

Case 6.5.3: Comparison of results obtained from marginal and optimal hosting capacity.

Case 6.5.1: Base loads are considered in each bus. The marginal hosting capacity is calculated for each bus, simulating addition of EV charging or discharging. The marginal hosting capacity for each bus in terms of consumption is shown in Figure 6.15. This represents the impact of the additional amount of EV load that can be safely connected at each bus based on the optimal hosting capacity solution. Maximum marginal loading capacity is at bus 18 with the largest impact on the optimal hosting capacity solution and minimum marginal hosting capacity is at bus 19 with the least impact. The same figure presents the marginal hosting capacity at each bus in terms of additional generation. This figure represents the impact of discharging EVs at each bus based on the optimal hosting capacity solution. The marginal hosting capacity represents the decrease on the optimal hosting capacity based on the expected EV discharging in each bus. Maximum marginal hosting capacity occurs at bus 18 and minimum marginal hosting capacity occurs at bus 19. However, changing the charging/discharging profiles in the grid impacts the overall optimal capacity results. This figure advocates that load and generation both could be limiting factors in hosting capacity, however it depends on
each specific bus to be more sensitive to changes in the load or in the generation. In other words, both EV charging and discharging are of importance when determining the grid hosting capacity.

**Figure 6.15:** Marginal hosting capacity results.

**Case 6.5.2:** The system is analyzed with base load values with additional installation of ±10 kW, ±20 kW and ±30 kW of power into selected buses. This is done to investigate the impact of additional charging/discharging capacity allocated in a specific site to the overall hosting capacity. Two buses are selected for the additional capacity allocation, namely buses 7 and 19. The purpose here is to compare the results of the marginal hosting capacity found in Case 6.5.1 with the result calculated from performing the optimal loading/hosting capacity method in each of these buses. Table 6.5 shows the acceptable hosting capacity results for both the optimal hosting capacity and marginal hosting capacity analysis. Comparing the results shows negligible differences between the use of the optimal and marginal hosting capacity results.
Table 6.5: Hosting capacity results.

<table>
<thead>
<tr>
<th>Bus #</th>
<th>Change in actual load (kW)</th>
<th>Hosting capacity - consumption (kW)</th>
<th>Hosting capacity - generation (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Based on marginal values</td>
<td>Based on LP solution</td>
</tr>
<tr>
<td>7</td>
<td>-30</td>
<td>730.839</td>
<td>8452.837</td>
</tr>
<tr>
<td></td>
<td>-20</td>
<td>720.151</td>
<td>8463.521</td>
</tr>
<tr>
<td></td>
<td>-10</td>
<td>709.463</td>
<td>8474.205</td>
</tr>
<tr>
<td></td>
<td>+10</td>
<td>688.087</td>
<td>8495.573</td>
</tr>
<tr>
<td></td>
<td>+20</td>
<td>677.399</td>
<td>8506.257</td>
</tr>
<tr>
<td></td>
<td>+30</td>
<td>666.711</td>
<td>8516.941</td>
</tr>
<tr>
<td>19</td>
<td>-30</td>
<td>728.790</td>
<td>8454.874</td>
</tr>
<tr>
<td></td>
<td>-20</td>
<td>718.785</td>
<td>8464.879</td>
</tr>
<tr>
<td></td>
<td>-10</td>
<td>708.780</td>
<td>8474.884</td>
</tr>
<tr>
<td></td>
<td>+10</td>
<td>688.770</td>
<td>8494.894</td>
</tr>
<tr>
<td></td>
<td>+20</td>
<td>678.765</td>
<td>8504.899</td>
</tr>
<tr>
<td></td>
<td>+30</td>
<td>668.760</td>
<td>8514.904</td>
</tr>
</tbody>
</table>

**Case 6.5.3:** Here an analysis similar to Case 6.5.2 is performed to highlight a different system behavior based on marginal hosting capacity values. The additional capacities of ±10 kW, ±20 kW, and ±30 kW are installed in two new locations at a time, here in buses 25 and 33. These buses are chosen to study the effects of additional capacities into two end buses of the network. This allows a validation of the proposed marginal hosting capacity model to simultaneously investigate the impact of more than one location. The individual marginal hosting capacity of these buses are used to evaluate the impact of the installed power on the optimal hosting capacity. Comparing the results of two cases, i.e., using marginal values and the optimal hosting capacity solution, shows a very marginal difference (less than 1%) which can be neglected. In general, the choice of which bus hosts the additional power does not influence the results. This case demonstrates the use of the marginal hosting capacity method to illustrate the effect of changing the EV charging and discharging locations on the optimal grid hosting capacity.
### 6.6 Increasing Grid Hosting Capacity

The proposed network reconfiguration method is applied to the IEEE 33-bus distribution test system to demonstrate the performance of the developed method. A schematic of this system is shown in Figure 6.1. The system consists of 33 buses, 32 sectionalizing switches, and 5 tie switches. Closing any one of the five tie switches causes a corresponding loop to form (as depicted in Table 6.6). Base load values are used in each bus. The total base load is 3.715+j2.3 MVA. The maximum power exchanged with the main grid is set to 4.6 MW. The voltage at the POI (bus 1 in this system) is set to 1∠0° p.u. and all other buses are defined based on this value.

<table>
<thead>
<tr>
<th>Loop #</th>
<th>Lines making the loop</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9, 10, 11, 12, 13, 14, 34</td>
</tr>
<tr>
<td>2</td>
<td>6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 25, 26, 27, 28, 29, 30, 31, 32, 36</td>
</tr>
<tr>
<td>3</td>
<td>3, 4, 5, 22, 23, 24, 25, 26, 27, 28, 37</td>
</tr>
<tr>
<td>4</td>
<td>2, 3, 4, 5, 6, 7, 18, 19, 20, 33</td>
</tr>
<tr>
<td>5</td>
<td>2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 18, 19, 20, 21, 35</td>
</tr>
</tbody>
</table>

The objective is to increase the allowable hosting capacity through network reconfiguration while enforcing operational performance and radiality constraints. The problem is formulated as a MIP problem and developed in GAMS (General Algebraic Modeling System) using CPLEX solver on a personal computer with an Intel Core E7-4870 2.30 GHz CPU and 96 GB of RAM. The following cases are studied.

**Case 6.6.1:** Optimal hosting capacity with network reconfiguration when all buses are considered.

**Case 6.6.2:** Optimal hosting capacity and network reconfiguration when DG injection is allowed at end buses only (buses 18, 22, 25 and 33).
Case 6.6.3: Comparison with other existing methods.

The results of case 6.3.1 will be used to calculate the increase in optimal hosting capacity through network reconfiguration. Case 6.6.1 calculates the maximum hosting capacity when optimizing the grid’s configuration. This displays the fundamental feature of the proposed method: the ability to maximize hosting capacity through reconfiguration. Case 6.6.2 repeats the same analysis but when DG placement is considered to be at end buses only. This demonstrates the flexibility of the proposed method which can easily be applied to a scenario in which only a subset of buses is chosen to accommodate DGs. Case 6.6.3 compares the results of the proposed method to the other existing methods. This evaluates the accuracy of the proposed method and justifies assumptions used to linearize the AC power flow equations.

Case 6.6.1: In this case, the hosting capacity for the test distribution system is maximized by reconfiguration. All buses are considered as potential sites for DG injection. Table 6.7 summarizes the results, allowing direct comparison between the optimal hosting capacities with and without network reconfiguration. The optimal hosting capacity without network reconfiguration (in Case 6.3.1, Scenario 1) is 8.484 MW. The DG profile was optimal when injecting 7624 kW, 90 kW, and 770 kW into buses 2, 19, and 20, respectively. When allowing network reconfiguration, the optimal hosting capacity increases by 55 kW to 8.539 MW. In this case, DGs are deployed at 8509 kW, 10 kW, and 20 kW at buses 2, 8, and 16, respectively. The optimal configuration is obtained by closing all tie switches (lines 33, 34, 35, 36, and 37) and at the same time opening lines 14, 20, 21, 24 and 32. This configuration minimizes the power flow
distance between all buses and maximizes the available line capacities that allow for increasing grid hosting capacity without exceeding operational limits or requiring additional upgrades. Moreover, the voltage profile is improved in all buses when allowing reconfiguration, with the minimum voltage magnitude being 0.939 p.u. compared to 0.919 p.u. without network reconfiguration. Overall, this case demonstrates that reconfiguration will increase the optimal hosting capacity while also affecting the optimal DG locations.

Table 6.7: Optimal hosting capacity results with and without network reconfiguration.

<table>
<thead>
<tr>
<th>Case #</th>
<th>Optimal hosting capacity</th>
<th>Without network reconfiguration</th>
<th>Without network reconfiguration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total DG (kW)</td>
<td>8484</td>
<td>8539</td>
</tr>
<tr>
<td>Case 1</td>
<td>Line opened</td>
<td>33, 34, 35, 36, 37</td>
<td>14, 20, 21, 24, 32</td>
</tr>
<tr>
<td></td>
<td>Min. Voltage magnitude (p.u.)</td>
<td>0.919</td>
<td>0.939</td>
</tr>
<tr>
<td>Case 2</td>
<td>Total DG (kW)</td>
<td>1470</td>
<td>3160</td>
</tr>
<tr>
<td></td>
<td>Line opened</td>
<td>33, 34, 35, 36, 37</td>
<td>20, 21, 24, 34, 36</td>
</tr>
<tr>
<td></td>
<td>Min. Voltage magnitude (p.u.)</td>
<td>0.9377</td>
<td>0.976</td>
</tr>
</tbody>
</table>

**Case 6.6.2:** Similar to Case 6.6.1, the hosting capacity is maximized in this case through network reconfiguration. The difference, however, is that DGs installation is restricted to end buses, i.e. buses 18, 22, 25, and 33. The purpose of this case is to demonstrate the ability to selectively inject DG and to show the influence of the network reconfiguration when just considering locations that have smaller capacities compared to other buses. Table 6.7 compares the results of the analyses. Without network reconfiguration (Case 6.3.1, Scenario 3), the optimal hosting capacity is 1470 kW.
However, the optimal hosting capacity with the optimal network reconfiguration is 3160 kW, nearly double the other scenario. In this case, the optimal configuration is obtained by closing lines 33, 35, 37 and at the same time opening lines 20, 21 and 24. This configuration maximizes the benefits provided by DGs and consequently maximizes the hosting capacity. In other words, this optimal configuration aimed to minimize the power flow distance between end buses and the substation by using the largest available line capacities. Figure 6.16 compares the voltage profiles for optimal solutions with and without considering network reconfiguration. Voltage profiles show significant improvements in all buses when considering network reconfiguration.

![Figure 6.16: Comparison of the voltage profile with and without network reconfiguration.](image)

**Case 6.6.3:** Multiple methods are examined in order to compare the performance of the proposed method with other existing methods. The objective of these methods is to increase the grid hosting capacity via network reconfiguration in selected locations. Table 6.8 summarizes results from these methods, including the modified plant growth simulation algorithm (MPGSA) [43], the Harmony Search Algorithm (HSA) [44], and the genetic algorithm (GA) [44]. Note that the objective function of the GA method is to determine the optimal configuration first, and then to optimize DG capacity. The
proposed method achieves similar, if not better, solutions compared with the other methods. While the choice of open lines in the proposed method differs from other methods, the optimal hosting capacity results are almost similar. This indicates that the choice of which lines to open can in certain situations be flexible and can produce comparable results in a few similar configurations. Additionally, the proposed method has a significantly reduced computation time compared to the other methods which required more computation time.

<table>
<thead>
<tr>
<th>Method</th>
<th>Total DG (MW)</th>
<th>% of loss reduction</th>
<th>Min. voltage magnitude (p.u)</th>
<th>Line opened</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPGSA [43]</td>
<td>1.786</td>
<td>64.36</td>
<td>0.9724</td>
<td>7,10, 14, 28, 31</td>
</tr>
<tr>
<td>HSA [44]</td>
<td>1.668</td>
<td>63.95</td>
<td>0.9701</td>
<td>7, 10, 14, 28, 32</td>
</tr>
<tr>
<td>GA [45]</td>
<td>1.448</td>
<td>51.5</td>
<td>0.9691</td>
<td>7, 9, 12, 27, 32</td>
</tr>
<tr>
<td>Proposed method</td>
<td>1.986</td>
<td>66.24</td>
<td>0.9710</td>
<td>10, 21, 24, 33, 36</td>
</tr>
</tbody>
</table>

### 6.7 Hosting Capacity Optimization using Sensitivity-based Method

The modified IEEE 123-bus distribution test system shown in Figure 6.17 is used to show the performance of the proposed method. This distribution test system contains 123 buses and 122 lines and is structured radially [71]. Two different scenarios are performed on the distribution test system, each using a different load condition. The first scenario uses base-load values to represent typical load conditions, while the second scenario uses uncertain-load values to represent worst-case load conditions. Values for base and uncertain loads are derived from historical data collected over a year-long period. Uncertain load values are constrained by lower and upper bounds defined as the
lowest and highest load values over the aforementioned period. To determine the worst-case optimal hosting capacity, loads that minimize the optimal hosting capacity are selected from the uncertain-load profile. The optimal hosting capacity for uncertain loads can thus be interpreted as the optimal hosting capacity that guarantees acceptable hosting capacity regardless of load variations. The total active and reactive load on the system for the base-load condition is 4.925 MW and 2.705 MVAR, while the worst-case load profile totals to 2.708 MW and 1.487 MVAR. Exchange power flowing from the distribution network to the upstream grid is capped at 6.44 MW. The following cases are studied:

**Case 6.7.1:** Comparison with the traditional iterative method.

**Case 6.7.2:** Impact of load variations.

**Case 6.7.3:** Comparison with the linearized method.

![Figure 6.17: The IEEE 123-bus distribution system.](image)

**Case 6.7.1:** This case compares the proposed method with traditional iterative method in terms of computation time and solution accuracy. The sensitivity-based and
the traditional iterative hosting capacity methods are applied to the same distribution test system using the base-load scenario to determine the optimal DG hosting capacity. In addition, both scenarios are initialized with the same operational constraints (i.e. thermal and voltage limits) to facilitate a direct comparison. In the traditional iterative method, a DG with a 1 kW step size was selected to determine the individual hosting capacity. The optimal individual hosting capacities for buses 4, 63, and 98 (that are randomly selected to illustrate the accuracy of the proposed method) are 6.564 MW, 2.862 MW and 2.990 MW, respectively. Using the proposed sensitivity-based method, the optimal hosting capacity for the same selected buses are 6.218 MW, 2.686 MW, and 2.831 MW, respectively. The comparison of these arbitrarily-selected solutions demonstrates the acceptable accuracy of the proposed method. Checking the results for all buses, the highest deviation in results is obtained as 5.42%. The time required to determine the individual DG hosting capacity using the proposed method is less than 2 s, while the traditional method requires an average of 18 minutes. This applies a single-bus hosting capacity calculation; more time would be required for combinations of 2 and higher buses. These results demonstrate a clear improvement in the required runtime with only a slight decrease in solution accuracy.

**Case 6.7.2:** In this case, the proposed method is used to calculate the optimal hosting capacity for both base and uncertain load scenarios. The network optimization permits the installation of additional DGs in all buses simultaneously. Table 6.9 compares the obtained results from both scenarios. For the base-load optimization, the total hosting capacity is found to be 11.368 MW with 1.641 MW, 3.632 MW, 3.984 MW and 2.111
MW of DG power placed in buses 2, 3, 5, and 121. For the uncertain-load optimization, the total hosting capacity is found to be 9.151 MW with 1.156 MW, 1.215 MW, 3.966 MW, 0.721 MW, and 2.093 MW of DG power placed in buses 2, 5, 7, 117 and 121. In both scenarios the optimal hosting capacity was limited by the thermal limits, especially of the line connecting the distribution network to the upstream grid. Comparing the two scenarios, the load uncertainty reduces the optimal hosting capacity by 19.5% (from 11.368 MW to 9.151 MW). The exported DG power from the distribution network to the upstream grid in both scenarios is the same. In both scenarios, the overall runtime of the entire problem is less than 2 s.

Table 6.9: Optimal hosting capacity results for base-load and uncertain-load.

<table>
<thead>
<tr>
<th>Bus #</th>
<th>Base-load hosting capacity (MW)</th>
<th>Uncertain-load hosting capacity (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.641</td>
<td>1.156</td>
</tr>
<tr>
<td>3</td>
<td>3.632</td>
<td>0.0</td>
</tr>
<tr>
<td>5</td>
<td>3.984</td>
<td>1.215</td>
</tr>
<tr>
<td>7</td>
<td>0.0</td>
<td>3.966</td>
</tr>
<tr>
<td>117</td>
<td>0.0</td>
<td>0.721</td>
</tr>
<tr>
<td>121</td>
<td>2.111</td>
<td>2.093</td>
</tr>
<tr>
<td>Total DG (MW)</td>
<td>11.368</td>
<td>9.151</td>
</tr>
</tbody>
</table>

Case 6.7.3: In this case, the proposed method is compared with the linear optimization-based hosting capacity method that considers system losses. This comparison is needed to show if ignoring system losses in the sensitivity-based method impacts the results. In both methods, DGs are allowed to be installed in all buses simultaneously using base-load profile. The optimal grid hosting capacity in the proposed the sensitivity-based method is 11.368 MW. However, the optimal hosting capacity using the linear optimization-based hosting capacity method is calculated at 11.788 MW. The
difference between these two solutions is 3.56 %, which indicates the acceptable accuracy that can be provided using the sensitivity-based method.
Chapter Seven: Conclusion and Future Work

7.1 Conclusion

Many distribution networks are designed in a radial structure convenient for transferring electric power from a central location to consumers at the peripheries, i.e., a one-way flow of electricity. When designed, these grids were not intended to carry high levels of energy produced by DG near consumer areas. The growing integration of DGs into distribution networks has created some considerable challenges for distribution system operators such as deviations in voltage profile, network reliability, and power quality issues. However, existing infrastructure of distribution systems can accommodate some DG units. Quantifying the effect of incorporating additional generation and loads into an existing infrastructure is the hosting capacity approach.

Hosting capacity optimization is used to determine the maximum DG capacity able to be injected into a distribution network without negatively impacting its operational performance. The work presented in this dissertation overcomes issues arising from previous iterative hosting capacity optimization approaches by linearization of the nonlinear AC power flow equations. In particular, methods presented here determine a near-optimal solution in a short amount of time.

Two different methods for finding the optimal hosting capacity in a radial distribution grid are proposed. The first method uses an optimization-based mathematical method. The second uses a sensitivity-based hosting capacity calculation method. The
methods benefited from a linear power flow model which enabled a linear programming formulation of the developed methods and alleviated the need for performing iterations. The sensitivity-based hosting capacity calculation problem was developed based on the sensitivity analysis of line power flow and voltage magnitudes with respect to nodal active and reactive injections. Using sensitivity analysis further reduced the optimization’s complexity and, accordingly, the computation time. The simplicity of used equations in the proposed sensitivity-based method permits scaling-up the analysis to larger systems without requiring long runtimes.

The effects of load uncertainty were also considered to show the dependence of the hosting capacity on load variations as well as improving the robustness of DG integration in distribution networks. By using the worst-case load profile, a more conservative hosting capacity was obtained which would be valid for all variations in the load profile. Results showed that the proposed methods could outperform traditional hosting capacity methods in terms of computation time while ensuring an acceptable accuracy. This increases the speed and robustness of the hosting capacity method, allowing for real-time analysis of a radial distribution network.

Another optimization method for distribution networks with EV integration was proposed with the objective of determining the marginal hosting capacity values. Marginal values were associated with each network node and represented the marginal amount of additional generation/consumption that can be added to that specific bus without requiring additional investment on grid upgrade. Solutions were achieved quickly by employing the proposed linearized AC power flow. The proposed marginal hosting
capacity method can be used to make split-second decisions on various bus configurations and to determine which buses are more suitable for accepting additional capacities.

In this dissertation, a network reconfiguration method which can effectively increase the DG hosting capacity was also proposed. The method utilized a linear power flow model for network optimization which provides several benefits including reduced problem complexity, increased accuracy, and decreased computation time. Technical complications arising from imposing radiality constraints were addressed by utilizing results from graph theory and network optimization. The ability of the proposed method to simultaneously optimize DG placement and select the topology that would allow for maximum hosting capacity was demonstrated in numerical studies. The advantages offered by this method would be ideal for use in network management tasks which seek to maximize the accommodation of DGs within existing radial distribution grids.

7.2 Future Work

The development of several hosting capacity optimization methods have been presented here along with demonstrations of their merits and efficacy. Several possible suggestions and comments for future work can be explored, mostly dealing with modifying the objective function, redefining operational performance criteria, and exploring control strategies.

One of the consistent goals of this research was to optimize DG hosting capacity in active distribution networks. Other objectives requiring investigation are reducing line losses and minimizing operational costs. Additionally, economic constraints were ignored
in these studies. Future studies can include economic constraints by using a multi-objective, cost-efficient optimization model. Further, control strategies such as volt/var control and reactive power control could be incorporated into the method to explore their added benefits. This could allow distribution system planners and operators to also address voltage rise and line overload issues more efficiently.

Another way to extend the model would be to introduce new operational constraints. Some examples of these may be harmonic distortion limits, protection limits, and voltage stability indices in order to obtain a more comprehensive hosting capacity model. Additionally, an extension to the proposed methods could be developed which recommends which system upgrades are most beneficial. This can be accomplished by evaluating which operational performance criteria are exceeded when the optimal hosting capacity is reached.
References


[43] R. Rajaram, K. Sathish Kumar, and N. Rajasekar, “Power system reconfiguration in a radial distribution network for reducing losses and to improve voltage profile


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List of Publications:


