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Latent Growth Curve and Latent Change Score Modeling of Developmental Relationships Between Executive Functioning and Math Achievement in Early Elementary School

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Abstract

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Keywords: math, executive function, latent change score, latent growth curve

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For my late father, Donald Keith Lewis, who called me STAR

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Chapter 1: Introduction and Literature Review

There has been an increasing number of studies examining the effect of executive functioning (EF) skills on mathematics achievement. "The importance of executive functioning skills in mathematical achievement is well established" (Bull & Lee, 2014, p. 36) is a declaration that is seemingly visible in the investments made in the early education of children. There is increased societal interest in children's mathematics achievement as society becomes more technologically advanced and intelligence based. Children's early math competency has been identified as a good predictor of their school achievement in later grades (as cited in Nguyen, Watts, Duncan, Clements, Sarama, Wolfe, & Spitler, 2016, p. 550). Math achievement shapes career ambitions in high school and beyond, where students who performed well in math aspired to more prestigious careers than their poorer performing counterparts (Shapka, Domene, & Keating, 2006). Mathematics development and proficiency in young children is an "important predictor of later labor market success" (as cited in Nguyen et al., 2016, p. 550). Therefore, math achievement could be a strong predictor of future success in school as well as in professional life, as increasingly, jobs are requiring greater math proficiency (National Mathematics Advisory Panel (NMAP), 2008). The Program for International Student Assessment (PISA), among other variables, measures math ability in 15-year-old students in 71 countries. In 2015, the United States ranked only 38th in math achievement (PEW Research Center, 2017). Similarly, the Trends in International Mathematics and Science Study (TIMS) measured math achievement in fourth and eighth grade students in 48 countries. In 2015, among grade four students the United States ranked 11 and among grade eight students ranked 8 out of 37 countries in mathematics scores (PEW Research Center, 2017). It is important to comprehend and explore variables that can impact and improve math achievement.

Literature states that executive functioning skills in young children can predict their later math achievement (Bindman, Pomerantz & Roisman, 2015; Clark, Pritchard & Woodward, 2010). Developing executive functioning skills help children to focus on and continue in the attainment of goals, which are critical components to attain academic success (as cited in Little, 2017). Executive function is described as a set of core cognitive skills that allow children to manage their attention and behavior (as cited in Bindman et al, 2015). These core cognitive skills are a critical component in children's academic achievement and executive functioning at the preschool and kindergarten levels predict children's later math and literacy achievements (as cited in Bindman et al., 2015). The EF of preschool children contributes to growth in math competencies (Clark et al., 2010).

EF is widely described as a set of interrelated cognitive processes, namely, inhibitory control, working memory, and attention shifting which work together to contribute to the development of problem-solving skills and self-regulatory behavior in children (Best & Miller, 2010; Clark et al., 2010; Garon, Bryson, & Smith, 2008). By this definition, it may be proposed that math skills could have a reciprocal effect on the development of children's executive functioning skills. Children's use of math skills may facilitate their performance on EF tasks, thus allowing for the co-development of the two domains. EF does not develop at a single point in a child's life, rather it develops over a period of time. Importantly, EF does not develop in isolation of other domains. It, in fact, develops within an ecosystem where there is a bidirectional relationship. This interactive and interconnecting relationship can be examined to determine the effects of one on the other and how they change over time; such that the prior level of the variable allows for dynamic time-dependent prediction (McArdle & Nesselroade, 2014, p.268). Thus, examining previous change in the variables can determine future changes. However, research has focused on a unidirectional relationship from EF to math achievement. A more comprehensive understanding of any dynamic relationship entails prediction of the present status based on prior status and not just static analysis which measures the current state or event. A goal of this study is to identify and understand the dynamics between EF and math achievement; that is, the ways in which these variables are recursively associated over time (Ferrer & McArdle, 2010, p 150).

Studying these relationships using longitudinal methods will allow for the measurement of change. "The study of phenomena in their time-related constancy and change is the aim of longitudinal methodology" (Nesselroade & Baltes, 1979, p. 2). The objectives of longitudinal research include the direct identification of intra-individual (within-person) change and analysis of the determinants (causes) of such change, direct identification of inter-individual differences in intra-individual change and the analysis of the determinants of this change, and examination of interrelationships in behavioral change (Grimm, Ram, & Estabrook, 2017).

Latent growth curve (LGC) modeling can capture group-level development of executive functioning on math achievement, individual developmental trajectory, and differences across time-points as well as allow for the study of predictors of these differences (Duncan, Duncan, & Strycker, 2006). At least three time-points are required to assess the validity of the linear growth trajectory, as well as the accuracy of the parameter estimates (Duncan et al., 2006). LGC modeling can demonstrate changes in mathematics achievement in relation to executive functioning. A LGC models the trajectory of changes in mathematics achievement over time. However, it is hypothesized that there is a reciprocal relationship between mathematics achievement and executive functioning and not just a unidirectional relationship from the latter to the former but rather a co-developmental relationship. A dynamic relationship may exist between the two domains with identifiable leading and lagging indicators. While it is important to understand the unidirectional relationship as efforts are made to improve learning outcomes, care must be taken to model and comprehend the full extent of this relationship. LGC modeling is not sufficiently flexible to model dynamic relationships.

Latent change score (LCS) modeling has the unique ability to model the complex, dynamic relationship between the two constructs. LCS (or latent difference score) modeling makes time-dependent change the outcome and not the observed score (time-dependent states) (Grimm et al., 2017) and measures within-person change and between-person differences in the rate of within person change (Grimm, Mazza, & Mazzocco, 2016; Grimm, Zhang, Hamagami, & Mazzocco, 2013). LCS models, unlike LGC models, are flexible enough to examine dynamic relations between one or more variables over time (McArdle, 2001; McArdle & Hamagami, 2001) and to identify leading indicators, that is, variables that are the predictors of subsequent change in the lagging indicator (Ferrer & McArdle, 2010). LCS models provide the capacity to explore bidirectional and co-development relationships between the variables currently absent from the literature.

Both statistical techniques (LGC and LCS) can model unidirectional change over time, and both model within-person change. However, only LCS can model the dynamic relationship between mathematics achievement and executive functioning and can examine bidirectional relationships where each construct shares a co-developmental relationship. This co-developmental ecosystem is often modeled using coupled difference equations (Van der et al., 2006). Data from the nationally representative Early Childhood Longitudinal Study, Kindergarten Class of 2010-11 (ECLS-K) was used in this study to explore these growth models.

Problem Statement

The literature has provided evidence that EF in small children is a predictor of their later math achievement (Bindman et al., 2015; Clark et al., 2010). However, there is a lack of investigation into whether a reciprocal relationship exists between math achievement and executive functioning skills. And, studies of achievement often focus on limited time points (Best & Miller, 2010; Greenman, Bodovski & Reed, 2011; Reilly, Neumann, & Andrews, 2015). Education reform efforts in the United States have been largely influenced by the need to improve math outcomes (Schiller, Schmidt, Muller, & Houang, 2010). Therefore, exploration of the relationship between EF and math achievement is important in helping to inform researchers and practitioners in math education. Examining the relationship between EF and math achievement from a longitudinal perspective can help to assess the change in math achievement and the effects of EF over time among children, as well as investigate the existence of any dynamic relationship between the two constructs. Additionally, studies seem to focus on the examination of the effect of EF on math achievement as a whole; however, the effect of the different components of EF on math achievement are not well studied. Where longitudinal studies do exist, they are primarily conducted using LGC models. LGC modeling is a statistical analysis technique that can demonstrate such change (Duncan, Duncan, & Strycker, 2006). However, in LCS, observed scores are modeled as a function of true scores and measurement error (Ferrer & McArdle, 2003). That is, the difference between the true score at the present time and the previous time point is modeled. Hence, at each time point, the LCS can model the scores directly by quantitatively separating the latent true scores y and x from e_y and e_x (the measurement error) (Ferrer & McArdle, 2003). Importantly, LCS has the unique ability to model dynamic relationships and examine bidirectional relationships between the constructs, and to address a gap in the literature which can lead to more targeted strategies to improving the learning outcomes of young children.

Purpose Statement

The purpose of the current study was to explore the use of LCS models as a more advanced tool for developmental research and to examine the relationship of the constructs EF and math achievement, determine how their development affect each other and if a dynamic relationship exists between the two. EF was individually examined as two separate but related components, namely, cognitive flexibility (using the *Dimensional Change Card Sort* subscale) and working memory (using the *Woodcock-Johnson III Tests of Cognitive Abilities* subscale). This was examined in a cohort of kindergarten students over four time points. Two different models of change, LGC modeling and LCS modeling, were applied to a large-scale nationally representative dataset on early childhood development. An incremental model-fitting approach was used to determine whether EF or math achievement serves as a leading indicator of change and if subsequent changes in one construct is influenced by the previous state of the other construct. The model fit and performance of LGC models and LCS models were assessed.

Literature Review

Executive Function - Theories and Measurement

While children are not born with executive skills its development is innate and slow, ranging from shortly after birth until approximately the mid-twenties (De Luca & Leventer, 2008). Age 4 years is said to be the beginning and most vital period in the development of executive functions (Garon, Bryson, & Smith, 2008). Here, the critical components of executive functions take a more prominent role in cognitive function (Garon, Bryson, & Smith, 2008). However, the first signs of EF can be detected when babies are as young as 8 to 9 months old and they try reaching out for something (Carpenter, Nagell, & Tomasello, 1998; Diamond, 1990a, 1990b). Around the age of 2 years their inhibition mechanism begins to function and by 3 to 5 years of age they begin to develop problem solving skills and move between the execution of different activities (Diamond, 2006). Development in the ability to switch between tasks, and to store information and use it occurs between 5 and 11 years (Diamond, 2006).

Jean Piaget's cognitive theory postulates that when children are born they have an inherent inquisitiveness to explore and master their environment, and through this exploration and drive to mastery they develop self-confidence (Nixon & Aldwinckle, 2003; Nixon & Gould, 1999). Piaget posited four stages of development. The first is the sensory motor stage which ranges from birth to 2 years of age where cognitive understanding is being developed through the use of motor skills and senses. The second is the pre-operational stage which ranges from 2 to 7 years age when children are considered illogical thinkers, relying less on motor skills and senses. The third is concrete operations and ranges from 7 to 12 years of age where children begin to think logically. The fourth stage is the formal operations stage which ranges from 12 to 28 years of age and sees more capable logical thinking and problem solving (Nixon & Aldwinckle, 2003; Nixon & Gould, 1999).

Unlike Piaget, Lev Vygotsky saw children's discovery of knowledge not as an isolated entity but existing and developing within their existing social environment (Armstrong, Ogg, Sundman-Wheat, & Walsh, 2014). He, however, agreed with Piaget that children were active rather than passive learners and this activeness increased along with their ability to interact with their environment. He too posited four stages of the logical and conceptual thinking of a child (Nixon & Aldwinckle, 2003). He suggested age ranges but focused on the developmental characteristics that the child would experience at different developmental milestones, such as the preschool stage. The first stage is thinking in unordered heaps where conceptual thought begins to develop, with children using problem solving techniques at the preschool stage. The second stage is complex thinking where children start to make connections between objects, though not consistently. The third stage is the thinking in concepts stage where children start to make associations and think about and understand single abstract concepts one at a time. The fourth stage is the thinking in true concepts stage reflecting more mature thinking and manipulating more than one abstract concept at a time (Nixon & Aldwinckle, 2003). Perhaps Vygotsky's sharpest disagreement with Piaget is the adult's role in extending children's skills beyond their current ability or capacity through the use of external materials or stimuli. These "developmental theories are useful towards understanding how children learn and grow, and by what means their trajectories can be supported" (Armstrong et al., 2014, p. 21).

Executive functioning is a set of higher order cognitive processes that inform goal-oriented behavior (Anderson, 2002; Carlson, 2005; Garon, Bryson, & Smith, 2008; Olson & Luciana, 2008). Inhibition is the restraining of a motor response and suppressing distracting information (Bull & Scerif, 2001; Garon, Bryson, & Smith, 2008). Working memory is the ability to retain and manipulate information over a short period of time without the need for cues or aids (Alloway, Gathercole, & Pickering, 2006; Huizinga, Dolan, & Van der Molen, 2006). Cognitive flexibility is the switching between tasks, set rules, and mental state and requires a great deal of inhibition (Miyake et al., 2000). These skills are critical for academic thriving (Morrison, Cameron Ponitz, & McClelland, 2010).

Diamond (2006) argues that executive functions include three separate components: (1) inhibition; (2) working memory; and (3) switching and cognitive flexibility (as cited in Bindman et al., 2015). These distinct components are dissociable processes and indicate differential developmental trajectories (Diamond 2002; Garon, et al, 2008; Rosso, Young, Femia, & Yurgelun-Todd, 2004). Studying preschoolers, it was discovered that executive functioning skills components were differentiated even by this age group (Hughes, 1998). On the other hand, Zelazo and Frye (1998) and Munakata (2001) theorized that EF is a unified construct. There has been an absence of agreement about whether EF is a single construct or comprises independent domains (Baddeley, 1992; Barkley et al, 2001; Brocki & Bohlin, 2004; Dempster, 1992; Isquith et al., 2004; Miyake et al., 2000). Confirmatory factor analysis was used in a study with young adults to determine the underlying nature of EF and found both a unitary construct and dissociable components and that the three components though correlated, made distinct contributions, and were used differentially depending on the task to be performed (Miyake et al., 2000). Similar findings were observed in studies with younger populations (Huizinga, Dolan, and Van der Molen, 2006; Lehto, Juujarvi, Kooistra & Pilkkinen, 2003). This model of executive functioning embraces both the unity and diversity of executive functioning where the components are simultaneously separated but correlated, where the best fitting model has partially dissociable components and has a common underlying mechanism which is likely inhibition (Miyake et al., 2000). It has been argued that inhibition may not be a distinct component (Miyake, 2009; van der Sluis et al., 2007) and inhibition tasks may not be true measures of inhibition as they rely on the use of other executive functioning skills to accomplish the tasks (Simpson & Riggs, 2005). To that point, the Dimensional Change Card Sort (DCCS) measures cognitive flexibility in the dataset used for the current study (ECLS-K); however, during the post-switch phase when the children are required to sort the cards by shape and no longer by color as (they did in the pre-switch phase) inhibition is actually being measured (Best & Miller, 2010).

In a school psychology context, however, EF as measured by the Behavior Rating Inventory of Executive Function (BRIEF) is expanded. There are three indexes and nine clinical scales (Gioia, Isquith, Guy and Kenworthy, 2015). The behavior regulation index includes two scales, inhibit which measures the children’s ability to stop their behavior by not acting on their impulse at the appropriate time, and self-monitor which is the monitoring of their own behavior and measuring against a standard (Gioia, et al., 2015). The emotion regulation index has two scales, shift or the ability to move freely from one task or situation to another as required, and emotional control which is the ability to moderate emotional response. The third and final index is the cognitive regulation index which includes five scales, namely, initiate which has do with the ability to start task and to generate ideas independently; working memory which allows the children to hold information in mind to be used to complete an activity; plan/organize which is the ability to plan and manage current and future tasks; task monitor allows for the checking of one's performance of a task during and upon completion to determine if goals were met; and finally, organization of materials. Which has to do with maintaining an orderly work space. This demonstrates the lack of consensus around a definition of EF. At one point as much as 33 separate executive skills have been identified (Eslinger, 1996).

The current study adopted Miyake's approach and examined EF as partially dissociable components with inhibition as an underlying mechanism not treated as a distinct component. The dataset used in the analysis used only the DCCS and Woodcock-Johnson Tests as measures limiting the type of measures used in this study. Therefore, DCCS will measure cognitive flexibility with inhibition as an underlying component and the Woodcock-Johnson Tests used to measure working memory with inhibition not measured as a construct. This allowed for the investigation of varying relationships with math achievement.

Mathematics and Association with Executive Functioning

Studies have found an association between EF and math achievement and EF as a predictor of math achievement growth (Bindman, Pomerantz & Roisman, 2015; Blair, Ursache, Greenberg, Veron-Feagans, & The Family Life Project Investigators, 2015; Bull & Lee, 2014; Clark, Pritchard & Woodward, 2010; Friso-van den Bos, van der Ven, Kroesbergen, & van Luit, 2013; Shaul & Scwartz, 2014). There has been limited examination of the effect of the different components of EF on math achievement and where they exist there is a lack of convergence of findings. One study found that inhibition showed a higher association with math achievement than either working memory or switching and cognitive flexibility (Espy, McDiarmid, Cwik, Stalets, Hamby & Senn, 2004). Later, another study found that working memory has a stronger correlation with math achievement than inhibition or switching and cognitive flexibility (St. Clair-Thompson & Gathercole, 2006). Yet another study found a strong association between working memory and math achievement, but no relationship with the other components (Van der Ven, Kroesbergen, Boom, & Leseman, 2011). These three studies used different methods. The first used a cross-sectional research design with a sample of 96 children (Espy et al., 2004), the second was an experiment with a sample of 51 children (St. Clair-Thompson & Gathercole, 2006), and the third was a longitudinal study with a sample of 227 children ((Van der Ven, 2011). However, the title of this longitudinal study, *The development of executive functions and early mathematics: A dynamic relationship* was a bit misleading as the authors used latent growth curve modeling which is not flexible enough to achieve an understanding of the dynamic relationship. Latent change score modeling can be used to examine dynamic relationships. Studies of the role of EF and math achievement have primarily been correlational or experimental, learning studies which included limited longitudinal explorations, and meta-analyses (Cragg & Gilmore, 2014). Therefore, there is a need for the examination of the effect of the executive functioning components on math achievement longitudinally with a large enough sample size using the appropriate growth model which can investigate dynamic relationships.

The causal effect of executive functioning on later math achievement has been questioned in a meta-analysis (Jacob & Parkinson, 2015,). This study found “no compelling evidence that a causal association between the two exists” (Jacob & Parkinson, 2015, p. 512). In addition to these questions about the nature of the relationship between EF and math achievement, it has been noted that both constructs develop strongly during childhood and a mutually developmental relationship is likely to exist where one influences the other (Bull & Lee, 2014; Jones, Gobet, & Pine, 2007; Messer, Leseman, Boom, & Mayo, 2010; Ottem, Lian, & Karlsen, 2007; Van der Maaset al., 2006), supporting the need for the examination of this relationship in a longitudinal way using a dynamic framework.

If children’s skills develop in stages then it should follow that any efforts towards examining and determining how “their trajectories can be supported” (Armstrong, Ogg, Sundman-Wheat, & Walsh, 2014, p. 21) should be done within a longitudinal framework.

> Longitudinal methodology involves repeated, time-ordered observation of an individual or individuals with the goal of identifying processes and causes of intraindividual change and of interindividual patterns of intraindividual change (Baltes & Nesselroade, 1979, p.7).

The objectives of longitudinal research include the direct identification of intra-individual (within-person) change, direct identification of inter-individual (between-person) differences in intra-individual change, analysis of interrelationships in behavioral change, analysis of causes (determinants) of intra-individual change, and analysis of causes of inter-individual differences in intra-individual change (Grimm, Ram, & Estabrook, 2017).

To identify intra-individual change requires repeated measurement or observation of the same subject. To identify inter-individual differences in intra-individual change there needs to be a comparison of the different processes of change for each subject under repeated observation. To analyze the interrelationships of behavioral change requires a multivariate framework where variability can be measured. To analyze the causes of intra-individual change requires the identification of antecedent factors. Finally, to analyze the causes of interindividual differences in intraindividual change the researcher must understand both that causes may vary among subjects, and the nature of the antecedent-consequent relationships of the subjects may vary (McArdle & Nesselroade, 2014; & Grimm, Ram, & Estabrook, 2017).

Structural Equation Modeling

Structural equation modeling (SEM) is a general multilevel multivariate analysis framework (Raudenbush & Bryk, 2002). It is a confirmatory framework rather than an explanatory one which includes path analysis, discriminant analysis, and factor analysis (Hox, 2002, Bollen, 1989). Variables used in SEM are observed and unobserved or latent in nature. The model has two distinct parameters, a measurement and a structural parameter. Models are theory driven and can be specified and re-specified to test different hypotheses. This theory driven approach is suitable to examine the various hypotheses of this study.

SEM allows the modeling of longitudinal data within its framework (Kline, 2016). Both latent growth curve models and latent change score models are typically specified within the SEM framework and are used to examine change as within-person and between-person models (Grimm, Mazza, & Mazzocco, 2016; Kline, 2016; Meredith & Tisak, 1990).

Latent Growth Curve Modeling

Longitudinal data, because of its repeated measures nature, allows for the analysis of change over time. Waves of data collected at only two time-points do not provide adequate information, as LGC models uses multi-wave data which allow for the effective testing of systematic inter-individual variability in change (Byrne, 2010). Latent growth curves are adequately modeled within the SEM framework with a continuous scale dependent variable, data collected in three or more waves, with either even or uneven time lags between time of data collection, and a sample size of at least 200 individuals at each time-point (Byrne, 2012). Within-person (intra-individual) growth trajectories over time, that is, the direction and extent of change for each person from one time-point to another, can be modeled. If a straight line is fitted to the data, there are two individual growth parameters, the intercept (initial status (η_i) on the outcome variable at time-point 1) and slope (rate of change over time) (η_s). Both the intercept and slope latent factors have a mean (*Mi* and *Ms*, respectively) and a variance (*Di* and *Ds*, respectively). The mean intercept is where the average individual starts while the mean slope concerns the average rate of change. The intercept variance is concerned with how much the individuals vary in their initial status while slope variance models their rates of change variation (Duncan, 2006). The time-points are modeled as observed variables with each having a random measurement error term. The values of the regression paths between the intercept and the observed variables are 1 to indicate it is constant across time, while the values of the regression paths between the slope and the observed variables denote the different time-points. For example, year 1 = 0, year 2 = 1, and year 3 = 2 for equal time intervals (Byrne, 2010). The measurement model, that is, the regression paths, the factor variances and covariances, and the observed variables' random measurement errors (ε_1, ε_2, and ε_3) are used to model intra-individual change (Byrne, 2010). The LGC model equation

$$y_i = \Lambda \eta_i + \varepsilon_{i,} \qquad (1)$$

where *Y* is the outcome variable, Λ is the association between the latent factors and observed variables or factor loadings, η embodies the initial status and the rate of change over time which includes *Mi*, *Ms*, *Di* and *Ds*. Alternately, the equation for three time-points can be written as

$$Y_1 = b_0 + b_2 * t_1 + \mathcal{E}_1 \qquad (2)$$

$$Y_2 = b_0 + b_2 * t_2 + \mathcal{E}_2$$

and

$$Y_3 = b_0 + b_2 * t_3 + \mathcal{E}_3,$$

where b_0 is the initial status, b_2 is the rate of change, and t_1, t_2, and t_3 are the values of time (slope regression loadings) (Duncan, 2006; Duncan & Duncan, 2009; Grimm, Zhang, Hamagami & Mazzocco, 2013). B_0 is the M_i score, b_2 is the Ms score, t_1 is set at 0, t_2 at 1 and at t_3 2, and $\mathcal{E}$ is fixed at 0.

The structural model, that is, the relationship between the factors and their residuals measures the variability across individuals (inter-individual (between-person) differences in change) due to the differences in their intercept and slope (Byrne, 2010). The mean models the average intercept and slope values and the variances model individual differences in the intercept and slope thus allowing for the "estimation of inter-individual differences in change" (Byrne, 2010, p. 309). In other words, to model inter-individual differences the intercept means and variance, slope means and variance, and the covariance are used. The means estimate the population starting point and mean increment over time, thus measuring the average population values for the factors. The variances estimate if there are between-person differences in the initial status and growth trajectories (rate of change), thus measuring the variation of individual intercept and slope from the population mean. The covariance between the two factors (or the population covariance) estimate if individuals whose initial status is higher tends to grow at a higher rate, thus measuring the variation between starting point and rate of change (Byrne, 2010).

Additionally, LGC models allow for the inclusion of predictors of change. With a predictor introduced, it becomes a conditional model "because the fixed and random effects are now conditioned on the predictors" (Curran, Obeidat, & Losardo, 2010, p. 125). The intercept and slope become endogenous factors and have disturbances (Kline, 2016). For example, race-ethnicity or gender can be included to determine if differences exist across the various races-ethnicities or between males and females Race-ethnicity (like gender) would be an exogenous time-invariant covariate.

Difficulty may arise when modeling the changes in one construct, such as math achievement, as a function of a second construct, such as EF, while both are changing (Grimm, Mazza, & Mazzocco, 2016). Here, any association between math achievement and EF remain time-invariant or static. These time-invariant associations can only be examined at the between-person level or growth factor level. Therefore, any examination of a developmental relationship between math achievement and EF that yields positive rates of growth reflects a between-person association whose effect is static, indicating "that the effect lacks subsequent movement, action, or change" (Grimm, Mazza, & Mazzocco, 2016, p. 343). It must be noted that a positive correlation between math achievement and EF slopes is not an indication that changes in EF "precede or lead to subsequent changes in" math achievement (p. 343).

Autoregressive Models

An autoregressive (or residual change) model is another approach to studying change (Duncan, Duncan, & Strycker, 2006). In an autoregressive model the past values of a variable are used to predict future values (Kline, 2016), where T1 score is used to predict T2 score (Felt, Depaoli, & Tiemensma, 2017). This is considered controlling for autoregression, as the previous state of a construct (math achievement) is included to predict the future state of the construct (Quinn, Wagner, Petcher, & Lopez, 2015). Spurious associations between variables can result from not controlling for autoregression (Gollob & Reichardt, 1987). However, autoregressive models have a major flaw when dealing with longitudinal data as they lack LGC modeling's ability to model trajectories of change over different time-points.

Latent Change Score Modeling

LCS, like LGC, models within-person change and analyzes means and covariance structures. While the literature is predominantly focused on LCS as the model through which we can find answers for developmental research, LCS models seem to offer a more comprehensive option which includes the possibilities of LGC and more. Latent change score modeling is a combination of autoregressive models' ability to model the degree to which a prior status is related to or influences a future status, and LGC models' ability to model trajectories of change over time (McArdle, 2009). These two abilities are important to modeling developmental changes. LCSmodeling allows for the variables to be endogenous and the examination of any dynamic association between them across different time-points (McArdle, 2009; Grimm, Mazza, & Mazzocco, 2016; Ferrer & McArdle, 2010).

Classical test theory has some influence in latent change score modeling

$$y_{ti} = ly_{ti} + e_{ti}, \qquad (3)$$

where y_{ti} is the observed score of individual *i* at time *t* and is composed of the ly_{ti} which is the latent true score of individual *i* at time *t*, and e_{ti} is the residual score of individual *i* at time *t*. This reflects a linear combination or growth that is considered as the additive (α) parameter. ly_{ti} is further decomposed to reflect

$$ly_{ti} = ly_{t-i} + dy_{ti}. \qquad (4)$$

Where y_{ti} is the observed score of individual *i* at time *t*, ly_{t-i} is the true score of individual *i* at time *t-1*, and dy_{ti} is the true score of individual *i* from t-1 to time *t*. This reflects an autoregressive model. Here, ly_{t-i} is the predictor and reflects a nonlinear growth component that is considered as the proportional change parameter (β or π). It is the change in y from time *t*-1 to time *t* that is the outcome rather than the status on *y* at time *t* thus allowing the examination of "within-person change and between-person differences in within-person change" (Grimm, Mazza, & Mazzocco, 2016, p. 343). Within the latent change framework, the model for latent change score modeling is

$$y_{ti} = ly_{t-i} + \sum_{r=2}^{r=t}(dy_{ri}) + e_{ti}, \qquad (5)$$

where y_{ti} is the observed score of individual *i* at time *t*=1 (initial status), and $\sum_{r=2}^{r=t}(dy_{ri})$ is the sum of the latent change scores of individual *i* from *t*=2 to time *t*=nth, reflecting the individual *i* score at time *t* is composed of individual *i* true score at *t*=1 (initial time point), the accumulation of changes up to time *t*, and the unique score (or residual score) of individual *i* at time *t* (Grimm, Mazza, & Mazzocco, 2016; Grimm, Ram, Estabrook, 2017).

Where change is being examined between two constructs (in a latent change score bivariate framework) over different time-points, both the α and the π parameters are estimated as well as this framework allows for the estimation of the coupling (ϒ) parameters. The dynamic association between the two constructs math achievement and EF in a latent change score modeling bivariate framework has two types of indicators. Namely, a leading and a lagging indicator with change in the former leading to change in the latter; thus, their development is coupled. Therefore, the effect of the mean level of EF at T1 influences the changes (level of growth) in math achievement at T2 (Ferrer & McArdle, 2010; McArdle, 2009; Quinn, Wager, Petscher, & Lopez, 2015).

Univariate Models

A framework for specifying and estimating latent change score models has been postulated (Grimm, Mazza, & Mazzocco, 2016; Grimm, Ram, & Estabrook, 2017). When specifying univariate models, one model often estimated is the no change model where both expected means and variances are 0

$$dy_{ti} = 0. \qquad (6)$$

Next is the constant change model in which the amount of change is allowed to vary between-persons (across individuals) but fixed within-person (b_{1i} is the constant change component)

$$dy_{ti} = b_{1i}. \qquad (7)$$

Integrating this equation with the model for latent change score ($y_{ti} = ly_{t-i} + \sum_{r=2}^{r=t}(dy_{ri}) + e_{ti}$) a series of latent true scores equations can be developed

$$ly_{1i} = ly_{1i} \tag{8}$$

$$ly_{2i} = ly_{1i} + b_{1i}$$

$$ly_{3i} = ly_{1i} + b_{1i} + b_{1i}.$$

Here, ly_{1i} represents the intercept of the latent variable while b_{1i} is the slope. Then there is the proportional change model

$$dy_{ti} = \pi * ly_{t-1i}. \tag{9}$$

Here, the change detected between *t* – 1 and *t* is directly proportional to its start or status at *t* – 1. Merging this equation with the model for latent change score ($y_{ti} = ly_{t-i} + \sum_{r=2}^{r=t}(dy_{ri}) + e_{ti}$) a series of latent true scores equations can be developed

$$ly_{1i} = ly_{1i} \tag{10}$$

$$ly_{2i} = ly_{1i} + \pi * ly_{1i}$$

$$ly_{3i} = ly_{1i} + \pi * ly_{1i} + \pi *(ly_{1i} + \pi * ly_{1i})$$

$$ly_{4i} = ly_{1i} + \pi * ly_{1i} + \pi *(ly_{1i} + \pi * ly_{1i}) + \pi *(ly_{1i} + \pi * ly_{1i} + \pi * (ly_{1i} + \pi * ly_{1i}))$$

both ly_{1i} (initial latent true score) and π (proportional change parameter) determine the latent true score, this model is much like the exponential change model (Grimm, Mazza, & Mazzocco, 2016). Additionally, there is the dual change model which combines the constant change and the proportional change models

$$dy_{ti} = b_{1i} + \pi * ly_{t-1i}. \tag{11}$$

Combining this equation with the model for latent change score ($y_{ti} = ly_{t-i} + \sum_{r=2}^{r=t}(dy_{ri}) + e_{ti}$) a series of latent true scores equations can be developed

$$ly_{1i} = ly_{1i} \quad (12)$$

$$ly_{2i} = [ly_{1i}] + b_{1i} + \pi * ly_{1i}$$

$$ly_{3i} = [ly_{1i} + b_{1i} + \pi * ly_{1i}] + b_{1i} + \pi *(ly_{1i} + b_{1i} + \pi * ly_{1i})$$

$$ly_{4i} = [ly_{1i} + b_{1i} + \pi * ly_{1i} + b_{1i} + \pi *(ly_{1i} + b_{1i} + \pi * ly_{1i})] + b_{1i} + \pi *(ly_{1i} + b_{1i} + \pi * ly_{1i} + b_{1i} + \pi * (ly_{1i} + b_{1i} + \pi * ly_{1i})),$$

(Grimm, Mazza, & Mazzocco, 2016).

Bivariate Models

Where math achievement and EF as a bivariate model is concerned the equation would reflect

$$dy_{ti} = b_{1i} + \pi_y * ly_{t-1i} \quad (13)$$

$$dn_{ti} = g_{1i} + \pi_n * ln_{t-1i}.$$

The first line of equation 13 would represent math and the second line represent number reverse, for example. Here, the constant change component and proportional change parameter are incorporated for both variables. From here the dynamic associations can be modeled as in equation 14 where equation 13 is further developed to examine coupling

$$dy_{ti} = b_{1i} + \pi_y * ly_{t-1i} + \Upsilon_y * ln_{t-1i} \quad (14)$$

$$dn_{ti} = g_{1i} + \pi_n * ln_{t-1i} + \Upsilon_n * ly_{t-1i}$$

In equation 14, the coupling parameters are Υ_y and Υ_n and serve to identify how the prior true score is related to the subsequent true changes, and leading and lagging indicators can be identified (Grimm, Mazza, & Mazzocco, 2016).

Of LCS modeling, Petcher, Quinn, and Wagner, (2016) highlighted that in education and psychology it can potentially yield “more theoretically interesting findings about how individuals change” (p. 1691) than other types of growth models. Developmental changes concerning reading, memory and depression outcomes at two time-points were shown to be more comprehensively studied using LCS (Hawley, Zuroff, Ho, & Blatt, 2006; Quinn, Wagner, Petscher, & Lopez, 2015). Additionally, LCS was used to address the discrepancy in the literature on the nature of the relationship between reading and writing for children between the first and fourth grade by establishing a unidirectional rather than a bidirectional relationship (Ahmed, Wagner, and Lopez, 2014). Studies have shown that LCS is not without its limitations, including failure to converge due to its complexity and if the sample size is small (Jackson, 2007). There have been difficulties with the requirement to constraining variance which has also caused convergence issues (Stoel et al., 2006) and proved unrealistic with real world data. In some cases where convergence occurs they may include negative variances and correlations that exceed one (Heywood case) with no clear solution (Joreskog, 1999). Clark, Nuttall, and Bowles (2018) in their Monte Carlo simulation study found that just the specification of a single constraint to achieve estimation can result in biased estimates but that these estimates still proved effective at capturing change and growth trajectories.

This study investigated the nature of the relationship between EF and math achievement using two models of change, namely, LGC modeling and LCS modeling. While latent growth curve modeling is the primary model used for studying longitudinal data, it does not model dynamic relationships and identify leading and lagging indicators to better get at the heart of co-developmental relationships. The current study proposed latent change score modeling as the model of choice to examine: to what extent prior status affects future status (autoregression); the trajectories of change over time (LGC); and the dynamic relationship between the variables that may be observed in developmental change research. LCS models can examine co-developmental relationships between the variables and will add to the literature on EF and mathematics achievement and better help practitioners identify informed strategies to advance learning.

Research Questions and Hypotheses

The research questions that this study answered are:

(1) What are the patterns of growth and interrelationships in the development of executive function and math achievement?

(2) Is one construct a leading indicator of the other and are executive function and math achievement dynamically dependent?

It was hypothesized that the patterns of association and the growth trajectory for EF and mathematics achievement are positively related, in that, the development of EF impacts change in mathematics achievement; and thus, EF skills developed in young children is a predictor of growth in mathematics achievement. LGC models can demonstrate changes in mathematics achievement in relation to EF. It was hypothesized that the developmental trajectories of EF and math achievement are co-developmental. Ensuing changes in one construct are influenced by the current and previous levels of the other construct. LCS modeling will more accurately capture the co-developmental nature of the variables, and model growth and dynamics between EF and math achievement (Ferrer & McArdle, 2010).

Chapter 2: Method

Data/Participants

This study used data from the national Early Childhood Longitudinal Study, Kindergarten Class of 2010-11 (ECLS-K:2011) collected by the National Center for Education Statistics (NCES). It followed a cohort of students from kindergarten throughout their elementary school years. The ECLS-K:2011 focuses on children's early education experiences and their development (Tourangeau, et al., 2017). A total of 18,174 young children participated from 1,352 schools (1,052 public and 300 private), resulting in a nationally representative sample of those who started kindergarten 2010-11. The sample demographics (Table 1) reflected 49.4% and 50.6% females and males, respectively. The children's race/ethnicity included White non-Hispanic (53.3%), Hispanic (24), Black non-Hispanic (13.1), Asian non-Hispanic (3.8%), American Indian or Alaska Native non-Hispanic (0.9%), Native Hawaiian/other Pacific Islander/non-Hispanic (0.7%), two or more races (4.1%). Eighty-seven percent were five years of age when they entered kindergarten for the first time, while 10% and 3% were age 6 years or older and younger than 5 years, respectively (Mulligan, McCarroll, Flanagan, & Potter, 2014).

Table 1

Sample Demographics

Characteristics	Percentage
Sex	
Females	49.4%
Males	50.6%
Race/Ethnicity	
White non-Hispanic	53.3%
Hispanic	24%
Black non-Hispanic	13.1%
Asian non-Hispanic	3.8%
American Indian or Alaska Native non-Hispanic	0.9%
Native Hawaiian/other Pacific Islander/non-Hispanic	0.7%
Two or more races	4.1%
Age entered kindergarten	
5 years	87%
6 years or older and	10%
Younger than 5 years	3

Procedure

The ECLS-K:2011 used a multi-stage clustered sampling technique and assessed the cohort at nine different time points, namely, kindergarten during fall 2010 and spring 2011, first grade during fall 2011 and spring 2012, second grade during fall 2012 and spring 2013, third grade spring 2014, fourth grade spring 2015, and finally fifth grade spring 2016. However, for the purposes of this study four-time points will be included, that is, kindergarten with a national representative sample of 18,170 (time points 1 and 2), and first grade (time points 3 and 4) with a fall 2011 subsample of 6,110 and a full spring 2012 of 18,174 (Najaran, Tourangeau, Nord, Wallner-Allen & Mulligan, 2018) (Table 2); with general patterns of missing data across time points. The sample at time-point three was only a third of the sample size of the other timepoints. Prior to the second-grade assessment, children were assessed using a paper-based test, then with the second grade the assessment was computerized, scored differently as well as differed in how the construct was assessed. While the type of test was age appropriate based on the level of difficulty, the overall computed scores of the paper-based test cannot be directly compared with the overall computed score of the computerized version (Najaran et al., 2018). Thus, the second grade has been excluded and deemed not appropriate to be included as it would not give accurate information. ECLS-K:2011 data are collected through multiple methods and from multiple informants including child assessments, and interviews and questionnaires for parents, teachers, and school administrators. Both direct and indirect children assessment data were collected. For the current study, only the measures of interest will be discussed.

Table 2

Study Time Points and Sample Size

Time Points	Grade	School Term	Sample Size
1	Kindergarten	Fall 2010	18,170
2		Spring 2011	18,170
3	First Grade	Fall 2011	6,110
4		Spring 2012	18,174

Measures

Math Achievement Measures

A two-stage direct cognitive assessment for mathematics was used in the ECLS-K: 2011 dataset. In stage one, 20 questions ranging in difficulty and based on the children's scores served to route children to their next level of difficulty (low, medium, or high) for stage two (Tourangeau et al., 2017). There were 113 items covering "number sense, properties, and operations; measurement; geometry and spatial sense; data analysis, statistics, and probability; and patterns, algebra, and functions" and measured "skills in conceptual knowledge, procedural knowledge, and problem solving" (Tourangeau et al., 2017, p.2-4). The IRT-based math scores had high reliability ranging from .92 to .94 (Tourangeau et al., 2017).

Executive Function Measures

Executive function, in the ECLS-K:2011, was directly measured through two separate constructs, cognitive flexibility and working memory (Tourangeau et al., 2017). Zelazo's (2006) *Dimensional Change Card Sort* (DCCS) was used to measure cognitive flexibility. There were two distinct phases, a pre-switch phase and a post-switch phase. In the pre-switch trials children were required to sort 22 picture cards into one of two trays first by color in what was called the color game. Thereafter, in the post-switch trials children were required to sort the picture cards by shape in a shape game. To move to the third and final trial, the border game, children had to correctly sort four of the six picture cards in the shape game. The border game required children to sort the cards into either the color or the shape tray depending on the presence or absence of a border (Tourangeau, et al., 2017). The outcome is the number of cards correctly sorted (Garon, Bryson, & Smith, 2008). These three games were each scored as correct or incorrect and a scale score provided for each; additionally, the three scores were sum into a combined scale score. The combined score, which can be a maximum of 18 correct answers, provides the total number of picture cards sorted correctly in all three games and is the recommended score to use to measure performance (Tourangeau et al., 2017).

After the completion of the card sort games, the Numbers Reversed task was administered to measure working memory using the *Woodcock-Johnson III Tests of Cognitive Abilities*. The children were required to repeat a series of numbers (up to 8) in reverse order with each level becoming increasingly difficult (Tourangeau et al., 2017). The numbers reversed items were marked as correct or incorrect. Three different scores were produced for analysis of the numbers reversed data, grade and age percentile scores, grade and age standard scores, and *W*-ability scores. Both the percentile and standard scores are suited for analysis for a single point in time whereas test publishers recommend the *W* score (a growth scale) as the most suitable for longitudinal analysis and measuring growth. *W* is a standardized score that is a special transformation ability scale with a mean of 500 and a standard deviation of 100 (Tourangeau et al., 2017). The *W* score is an equal interval scale that captures the child's ability as well as the difficulty of the task and any increase would indicate growth., A *W* score of 403 corresponds to a raw score of 0 (Tourangeau et al., 2017). The mean score of 500 represents the mean performance of a child 10 years of age. For the time points of interest for the current study, the sample is younger than 10 years and their performances are being compared to that of their older peers. According to the test developers, this comparison may show that the younger children are underperforming; however, this is not necessarily the case. *W* scores are a function of the number of correct answers and not age. They are available for all children where the standard and percentile scores are not (Tourangeau et al., 2017).

Data Analysis

Large datasets are prone to missing data due to nonresponse. Using SPSS, the data were checked for missingness and the nature of the missing data. That is, were the data missing at random (MAR), missing not at random (MNAR), or missing completely at random (MCAR) (Rubin, 1976). Full information maximum likelihood (FIML) was used where appropriate (Quinn, Wagner, Petscher, & Lopez, 2015) as by default Mplus (Muthén & Muthén, 2018) uses FIML where participants have scores on at least some of the variables, hence, cases with missing data on all cases are automatically excluded from the analysis (Geiser, 2013). Where missing data patterns are concerned, there were 13 missing data patterns (Tables 3 and 4). Table 5 presents the proportion of data present that contribute to the calculation of the variance or covariance (Geiser, 2013). All the values are above the Mplus minimum covariance coverage value of 0.100 (10%).

Table 3

Missing Data Patterns (x = not missing)

	1	2	3	4	5	6	7	8	9	10	11	12	13
CHILDID	x	x	x	x	x	x	x	x	x	x	x	x	x
X1MSCAK2	x	x	x	x	x	x	x	x	x	x	x	x	x
X2MSCAK2	x	x	x	x	x	x							
X3MSCAK2	x	x	x	x			x	x	x	x			
X4MSCAK2	x	x			x		x	x			x		
X1NRWABL	x		x		x		x		x		x	x	
X2NRWABL	x	x	x	x	x	x							
X3NRWABL	x	x	x	x			x	x	x	x			
X4NRWABL	x	x			x		x	x			x		
X1DCCSTO	x		x		x		x		x		x	x	
X2DCCSTO	x	x	x	x	x	x							
X3DCCSTO	x	x	x	x			x	x	x	x			
X4DCCSTO	x	x			x		x	x			x		

Table 4

Missing Data Pattern Frequencies

Pattern	Frequency	Pattern	Frequency	Pattern	Frequency
1	70	6	6	11	1
2	3	7	12	12	2
3	117	8	1	13	8
4	27	9	11		
5	1	10	1		

Table 5

Proportion of Data Present

Covariance Coverage

	CHILDID	X1MSCAK2	X2MSCAK2	X3MSCAK2	X4MSCAK2
CHILDID	1.000				
X1MSCAK2	1.000	1.000			
X2MSCAK2	0.862	0.862	0.862		
X3MSCAK2	0.931	0.931	0.835	0.931	
X4MSCAK2	0.338	0.338	0.285	0.331	0.338
X1NRWABL	0.823	0.823	0.723	0.808	0.323
X2NRWABL	0.862	0.862	0.862	0.835	0.285
X3NRWABL	0.931	0.931	0.835	0.931	0.331
X4NRWABL	0.338	0.338	0.285	0.331	0.338
X1DCCSTO	0.823	0.823	0.723	0.808	0.323
X2DCCSTO	0.862	0.862	0.862	0.835	0.285
X3DCCSTO	0.931	0.931	0.835	0.931	0.331
X4DCCSTO	0.338	0.338	0.285	0.331	0.338

Covariance Coverage

	X1NRWABL	X2NRWABL	X3NRWABL	X4NRWABL	X1DCCSTO
X1NRWABL	0.823				
X2NRWABL	0.723	0.862			
X3NRWABL	0.808	0.835	0.931		
X4NRWABL	0.323	0.285	0.331	0.338	
X1DCCSTO	0.823	0.723	0.808	0.323	0.823
X2DCCSTO	0.723	0.862	0.835	0.285	0.723
X3DCCSTO	0.808	0.835	0.931	0.331	0.808
X4DCCSTO	0.323	0.285	0.331	0.338	0.323

Covariance Coverage

	X2DCCSTO	X3DCCSTO	X4DCCSTO
X2DCCSTO	0.862		
X3DCCSTO	0.835	0.931	
X4DCCSTO	0.285	0.331	0.338

To answer the two research questions, an incremental model-fitting approach with varying degrees of freedom was specified in Mplus. LGCmodeling was used to examine the unconditional growth trajectory of the variables as it allows for the modeling of linear change over time and corrects for random error, which in turn allows for the "estimation of interindividual differences in true intraindividual change in trust over time" (Coovert, Miller, & Bennett, 2017, p. 9). The univariate unconditional LGC for each variable, DCCS, number reverse (NR) and mathematics (math) achievement were modeled and assessed. The measures of interest included four repeated measures each. The final LGC model is a variation of the associative model where both univariate models were combined, and the regression paths modeled instead of covariances. In all the models the random errors were initially fixed to be equal to each other and the factor loadings between the slope and time points as well as the factor loadings for the intercept were allowed to be freely estimated (thus unconstrained). In the associative model, for identification purposes the means and standard deviations of the intercept and slopes of the constructs obtained from the univariate models were used to inform the model. A series of LCS models were used to examine the growth trajectory and went further to examine the dynamic relationship between math achievement and executive functioning. Univariate LCS modeling for the three variables included (a) constant change models, (b) proportional change models, and (c) dual change models; these were followed by bivariate LCS modeling (d) bivariate dual uncoupled models (Math & DCCS, and Math & NR), (e) bivariate dual change unidirectional coupling model (DCCS to change in Math, NR to change in Math, Math to change in DCCS, and Math to change in NR) , and (f) bivariate dual change bidirectional couplings models (Math & DCCS, and Math & NR).

The models were evaluated for identification, model fit, means, and comparisons made across models. Model fit were assessed using multiple indices. The exact-fit hypothesis was assessed using the chi-square test of fit ($p > 0.05$) which is a measure of the deviation from the perfect model fit (Kline, 2016) with a significant chi-square value resulting in the rejection of the null hypothesis which indicates the exact fit of the model to the population (Geiser, 2013). However, the chi-square test often indicates that the null hypothesis (model fits perfectly) is to be rejected in large samples due to statistical power (Bollen & Curran, 2006). The comparative fit index (CFI) measures the relative improvement in the fit of the current model over the previous model with values between .95 and 1.00 indicating excellent fit and values between .90 and .95 indicating adequate fit (Garver & Mentzer, 1999). The Tucker-Lewis Index (TLI) of .95, indicates the model of interest improves the fit by 95% relative to the null model and TLI of 1 indicates ideal fit; but TLI is preferable for smaller samples (Bollen & Curran, 2006). The Root mean square error of approximation (RMSEA) is scaled as a badness-of-fit index where zero indicates best fit, with p-value > .05, a 90% C. I. (particularly a lower and upper threshold of .05 and .1), and values between 0 and .6 indicating excellent fit and values between .6 and .8 reflecting adequate fit, and values between .8 and 1 indicating moderate fit (Bollen & Curran, 2006; Browne and

Cudeck (1993) Hu & Bentler, 1995; Kline, 2016; MacCallum, Browne, & Sugawara, 1996). However, care must be exercised as RMSEA can be too conservative with a large sample size (Kline, 2016). The standardized root mean square residual (SRMR) was used to measure the mean absolute correlation residual and determines the overall difference between the observed and predicted correlations and uses Hu and Bentler's threshold of 0.08 (Kline, 2016), while values less than .05 indicates excellent fit (Steiger, 1990). The Akaike information criterion (AIC) and the Bayesian information criteria (BIC) considers goodness-of-fit and parsimony and helps to determine the best model, with a good model having the smallest value among all the models; however, AIC penalizes for more complex models (Dziak, Coffman, Lanza, & Li, 2012). Sample-size adjusted BIC is useful for large samples and was used in this study.

Research Question One

To answer research question one, three LGC unconditional models were specified. For math achievement the direct measure math score was specified at T1 – T4. Where EF is concerned, two models were specified; the DCCS scores which measures cognitive flexibility, and the number reversed scores which measures working memory. Univariate LCS models were also specified. First, the constant change models, which are equivalent to the LGC with a linear growth model, were specified for math scores, DCCS scores, and number reversed scores. Second, the proportional change models were specified to determine if growth is a function of the performance at the previous level. Third, dual change models, which incorporates the two previous models, were specified for the three variables. This allowed for nested model testing (Quinn, Wagner, Petscher, & Lopez, 2015).

To examine patterns of association between the variables and determine their growth trajectory as well as assess if the development of executive functioning impacts the development of math achievement, a variation of the bivariate LGC associative model in which regression paths are modeled instead of covariances were specified. Associative models investigate interrelationships (correlations) among the growth factors (development parameters) between pairs of measures but do not examine causation (Duncan et al., 2006). They first require the modeling of univariate growth curves to comprehend the change over time.

LCS bivariate dual uncoupled models were specified for the three variables, that is, math achievement and DCCS, and math achievement and number reverse. Both the additive and proportional change parameters were estimated. The coupling parameters were fixed to not allow for any examination of dynamic relations while the slopes for both variables were allowed to correlate as well as the intercept for both variables. The latent change score bivariate dual uncoupled model is equivalent to the fitting slope and intercept parameters in an associative LGC model.

Research Question Two

Research question two was specified using LCS models. It was hypothesized that ensuing changes in math achievement are influenced by the current level of executive functioning. In the first set of models specified, the slope and intercepts of both variables were allowed to correlate. The coupling parameters from executive functioning to change in math achievement was not fixed to allow for estimation of coupling, however, the coupling parameters from math achievement to change in executive functioning was fixed at zero to not allow for the estimation of coupling. It is also hypothesized that ensuing changes in executive functioning are influenced by the current level of math achievement. Therefore, the coupling parameters from math achievement to change in executive functioning was not fixed to allow for coupling estimation, however, the coupling parameters from executive functioning to change in math achievement was fixed at zero to not allow for the estimation of coupling. Finally, bidirectional coupling models were specified where coupling parameters from math achievement to change in executive functioning and from executive functioning to change in math achievement were simultaneously estimated to model dynamic relationships. Again, the slope and intercepts of both variables were allowed to correlate.

Model Rationale

Based on the analytic technique required to understand the dynamics among the measures, several models were specified to allow for incremental testing (Tables 6-8). Each construct was first individually modeled to ascertain the growth trajectories. Hence, the first models to be specified were the unconditional LGC and the constant change model which are comparable. These were followed by the proportional change model to determine if scores are predicted by scores from the previous time-point. This was followed by the dual change model which includes both the two previous LCS models. The first set of bivariate models, associative model and the dual change uncoupled model which are comparable, were then specified. The associative model helped to identify if DCCS and or number reverse influences math scores. This was the last LGC model to be specified as LGC is not robust enough to examine dynamic relationships. A series of LCS models were then specified to allow for the testing of co-development and leading and lagging indicators. Leading and lagging indicators were identified using the coupling parameters where best fit helped to determine if these indicators exist. The LCS dual change score uncoupled models are specified and were used as a baseline for comparison with the coupled models. This model is nested within the unidirectional coupled model and its bivariate parameters are housed within the correlations of the growth factors and the time-specific factors. This model, for example math achievement to changes in DCCS, determines if subsequent changes in DCCS are partially accounted for by current levels of math achievement. Finally, the full coupling models were specified. where statistically significant coupling parameter from math achievement to DCCS, for example, indicates growth in DCCS was partially accounted for by the level of math achievement.

Table 6

Latent Growth Univariate Models – Research Question 1

Analysis Type	Model Category	Model	Purpose
Univariate	Research Question 1 What are the patterns of growth and interrelationships in the development of executive function and math achievement?		
	1	Models 1-3 – LGC unconditional models (Math, DCCS, Number Reverse)	Models univariate growth curves to comprehend the change over time. Required before modeling associative models
	2	Models 4-6 – LCS Constant change models (Math, DCCS, Number Reverse)	Equivalent to the LGC with a linear growth model. This allows for comparison of the results
	3	Models 7-9 – LCS Proportional change models (Math, DCCS, Number Reverse)	Determine if growth is a function of the performance at the previous level
	4	Models 10-12 – LCS Dual Change (Math, DCCS, Number Reverse)	Incorporates the two previous LCS models. This will allow for nested model testing

Table 7

Latent Growth Bivariate Models – Research Question 1

Analysis Type	Model Category	Model	Purpose
Bivariate	Research Question 1 What are the patterns of growth and interrelationships in the development of executive function and math achievement?		
	5	Models 13-14 – LGC associative models (DCCS & Math, Number Reverse & Math)	Examine patterns of association between the variables and determine their growth trajectory as well as assess if the development of executive functioning impacts the development of math achievement. They investigate interrelationships among the growth factors (development parameters) between pairs of measures but do not examine causation.
	6	Models 15-16 – LCS bivariate dual change score uncoupled (DCCS & Math, Number Reverse & Math)	Equivalent to the fitting slope and intercept parameters in an associative latent growth curve model. This allows for comparison of the results

Table 8

Latent Growth Bivariate Models – Research Question 2

Analysis Type	Model Category	Model	Purpose
Bivariate		Research Question 2 Is one construct a leading indicator of the other and are executive function and math achievement dynamically dependent?	
	7	Models 17-20 – LCS Bivariate Dual Change Model Coupling (Coupling from: DCCS to Math; Number Reverse to Math; Math to DCCS; Math to Number reverse)	Allows for the examination of leading and lagging indicators. Test how prior true scores affect subsequent scores
	8	Models 21-22 – LCS Bidirectional Coupling (DCCS & Math; Number Reverse & Math)	A complete marrying of autoregression and LGC

Chapter 3: Results

Descriptive Statistics and Correlations

Sample statistics for the measures math achievement, DCCS and number reverse are presented in Table 9 for the four time-points. Across time-points math achievement scores ranged from 7.2 to 111.58 with the average scores increasing over time after an initial decline at time-point 2, DCCS average scores ranged from 0 to 18 while number reverse average scores ranged from 393 to 596 with some fluctuation. Correlation between DCCS and number reverse raw scores ranged from no relationship to strongly correlated (Table 10).

Table 9

Descriptive Statistics

Measure	Min	Max	Mean	SD	n
Math T1	7.2	111.58	31.67	11.37	15595
Math T2	7.2	88.76	45.28	12.19	17143
Math T3	17.14	108.7	52.9	14.87	5222
Math T4	16.5	109.53	66.8	15.35	15103
DCCS T1	0	18	14.2	3.33	15604
DCCS T2	0	18	15.14	2.79	17149
DCCS T3	0	18	15.7	2.44	5222
DCCS T4	0	18	16.1	2.31	1509
NR T1	393	581	433.01	30.21	15598
NR T2	393	572	449.7	30.52	17147
NR T3	393	596	456.96	28.74	5222
NR T4	393	596	469.33	25.82	15107

Table 10

Raw Scores Correlation by Measures Across Time-Points

	M1	M2	M3	M4	NR1	NR2	NR3	NR4	D1	D2	D3	D4
M1												
M2	.79**											
M3	.86**	.91**										
M4	.77**	.86**	.86**									
NR1	.64**	.37**	.46**	.41**								
NR2	.49**	.58**	.68**	.62**	.29**							
NR3	.66**	.63**	.65**	.68**	.48**	.65**						
NR4	.55**	.63**	.67**	.70**	.31**	.55**	.69**					
D1	.55**	.37**	.34**	.41**	.78**	.25**	.38**	.35**				
D2	.34**	.47**	.43**	.37**	.12	.56**	.28**	.28**	.19**			
D3	.34**	.36**	.38**	.33**	.26*	.36**	.37**	.25*	.23*	.14		
D4	.31**	.35**	.33**	.45**	.19**	.24**	.2	.35**	.19*	.23**	.28**	

** $p < 0.01$ (2-tailed).

* $p < 0.05$ (2-tailed).

Univariate Models

LGC Unconditional and LCS Constant Change Model

The first set of models fit to the data was the LGC unconditional and the LCS constant change models (figures 1-6). Initially, the errors were constrained to be equal across time but due to a lack of fit the models had to be re-specified to not be constrained. Additionally, the low correlations between DCCS and number reverse across times also justified relaxing this constraint. The overall fit of the models to the data then indicated good fit. The LGC unconditional math achievement model had good fit, X^2 *(3) = 5.81, p = .121* and RMSEA = .061 (.000, .135) with 95% confidence (CI), SRMR =.05, with CFI and TLI close to 1, all supporting good fit with AIC and BIC at 5631.2 and 5670, respectively (Table 11). The LCS constant change math achievement model had better fit than the unconditional math achievement model, X^2 *(4) = 4.59, p = .332* and RMSEA = .024 (.000, .101), with CFI and TLI close to 1 supporting good fit with AIC and BIC at 5627.9 and 5663.3, respectively. The LGC unconditional and LCS constant change NR models had identical estimates, X^2 *(5) = 10.48, p = .063* and RMSEA = .066 (.000, .122) SRMR =.096, with CFI and TLI close to 1, all supporting adequate fit and the AIC and BIC at 7794.4 and 7831.2, respectively. The LGC unconditional DCCS model had better fit than the LGC math achievement and number reverse models and was identical to the fit statistics of the LCS constant change DCCS model, X^2 *(5) = 4.78, p = .444* and RMSEA = .000 (.000, .086) SRMR =.054, with CFI and TLI = 1, all supporting excellent fit and the AIC and BIC at 4070.2 and 4102, respectively. The math achievement LGC unconditional and LCS constant change models have different indices while the other models were identical due to the need to modify the models for fit. A summary of the fit indices of these models is presented in Table 11.

The LGC unconditional math achievement mean intercept or the initial score (*Mi* = 31.33) and the mean slope or growth over time (*Ms* =13.44) were statistically significant ($p < .001$) (see Appendix, Table 17) indicating there was a systematic change in the children's math achievement from Kindergarten to 1st grade. Their level of math achievement increased on average by 13 units per time point. While the variance of the intercept was statistically significant (*Di* = 120.19, $p < .001$) the variance of the slope was not (2.41, $p = .38$) indicating significant variability among children's math achievement when they started kindergarten but not in their growth rates over the observed time points. The correlation between the intercept and slope was not statistically significant indicating no significant association between the initial and growth factors. In addition, the error variances which were freely estimated across the time-points were statistically significant.

The LCS constant change math achievement model mean intercept (*Mi* = 31.61) and mean slope (*Ms* = 13.09) were both statistically significant ($p < .001$) with a statistically significant variance of the intercept (*Di* = 124.73, *p* = < .001) and a variance of the slope that was not statistically significant, supporting the findings of the LGC unconditional math model (see Appendix, Table 20). Additionally, the correlation between the intercept and slope was not statistically significant while the error variances were statistically significant.

The LGC unconditional DCCS and the LCS constant change DCCS parameter estimates were identical with mean intercept (*Mi* = 14.03) and slope (*Ms* =.62) both statistically significant ($p < .001$) and the variance of the intercept (*Di* = 3.54, $p = .025$) but not that of the slope (*Ds* = .178, $p = .610$) (Tables 18 and 21). The freely estimated error variances were statistically significant while the correlation was not indicating there was no significant association between initial level of performance and rate of change for DCCS.

The LGC unconditional number reverse and the LCS constant change number reverse models also had identical parameters (see Appendix, Tables 19 and 22). Both the mean intercept and the mean slope were statistically significant (*Mi* = 432.69, *Ms* = 11.62) as well as the variance of the intercept (*Di* = 1360.22, $p < .001$) however the variance of the slope was not statistically significant indicating the model-predicted rate of change was the same among the participants. The correlation was negative and not statistically significant suggesting there was not a significant association between initial level of performance and rate of change for number reverse.

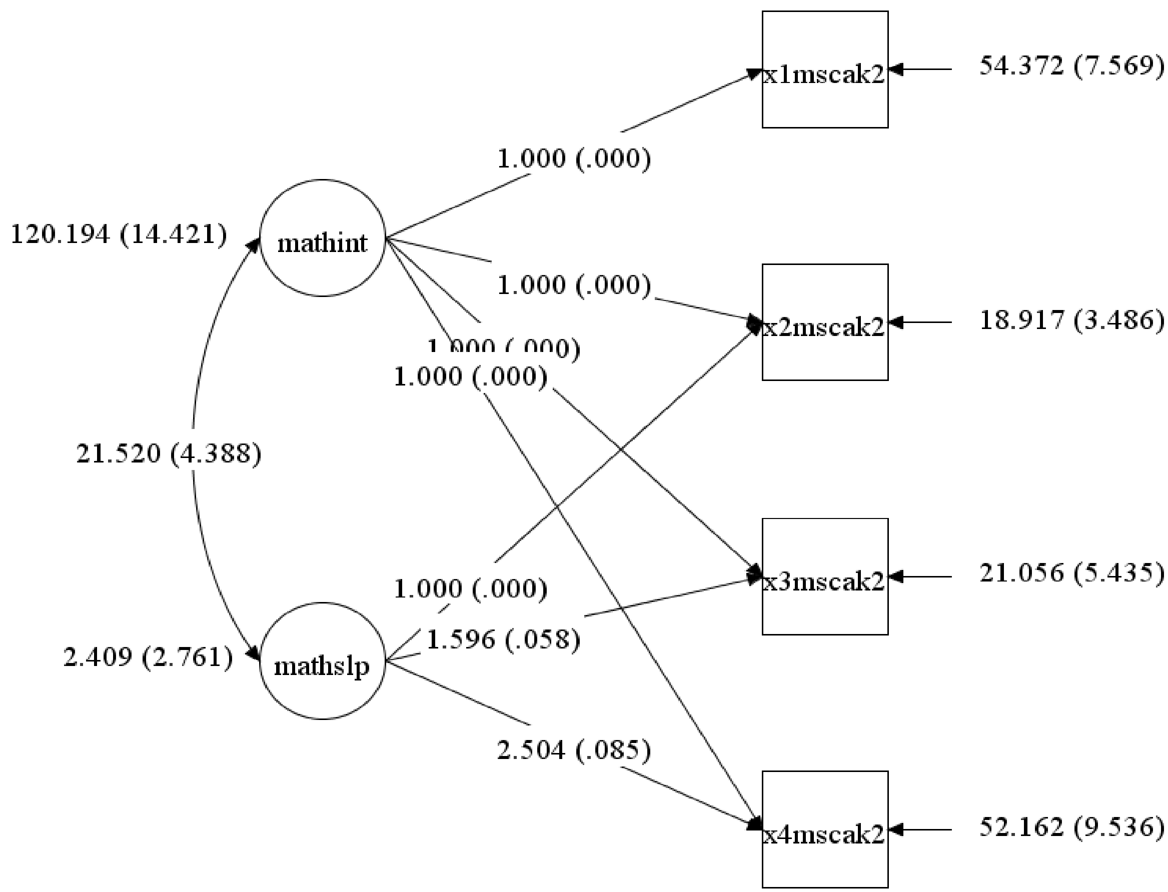

Figure 1. LGC unconditional growth curve – Math achievement with standardized estimates.

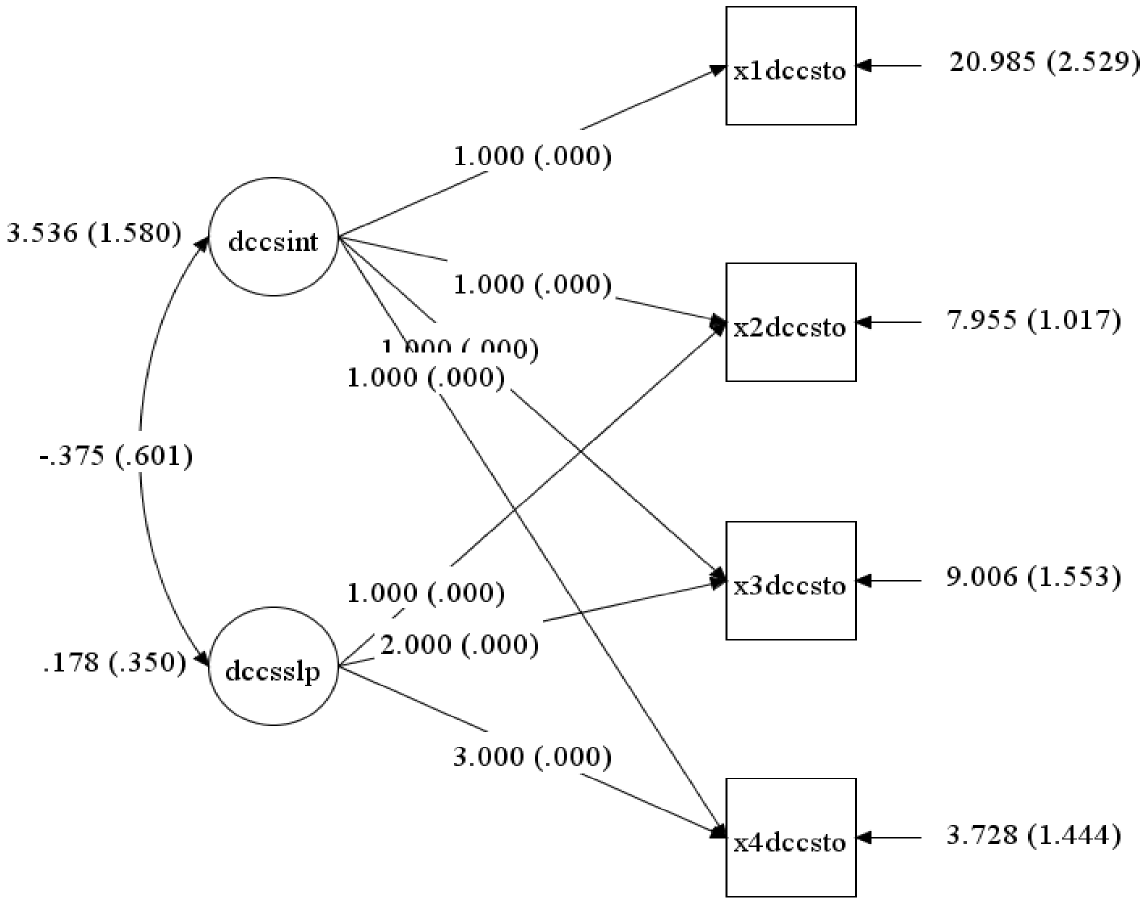

Figure 2. LGC unconditional growth curve model – DCCS with standardized estimates.

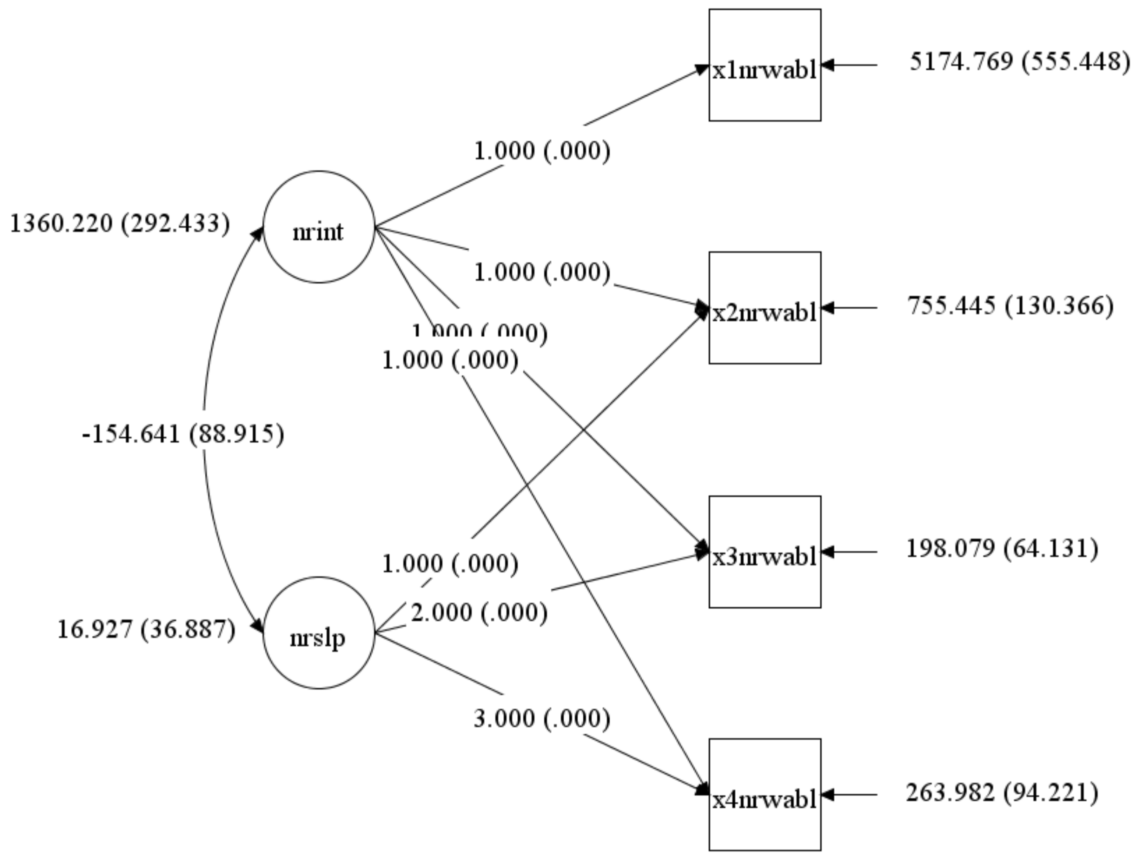

Figure 3. LGC unconditional growth curve – Number reverse with standardized estimates.

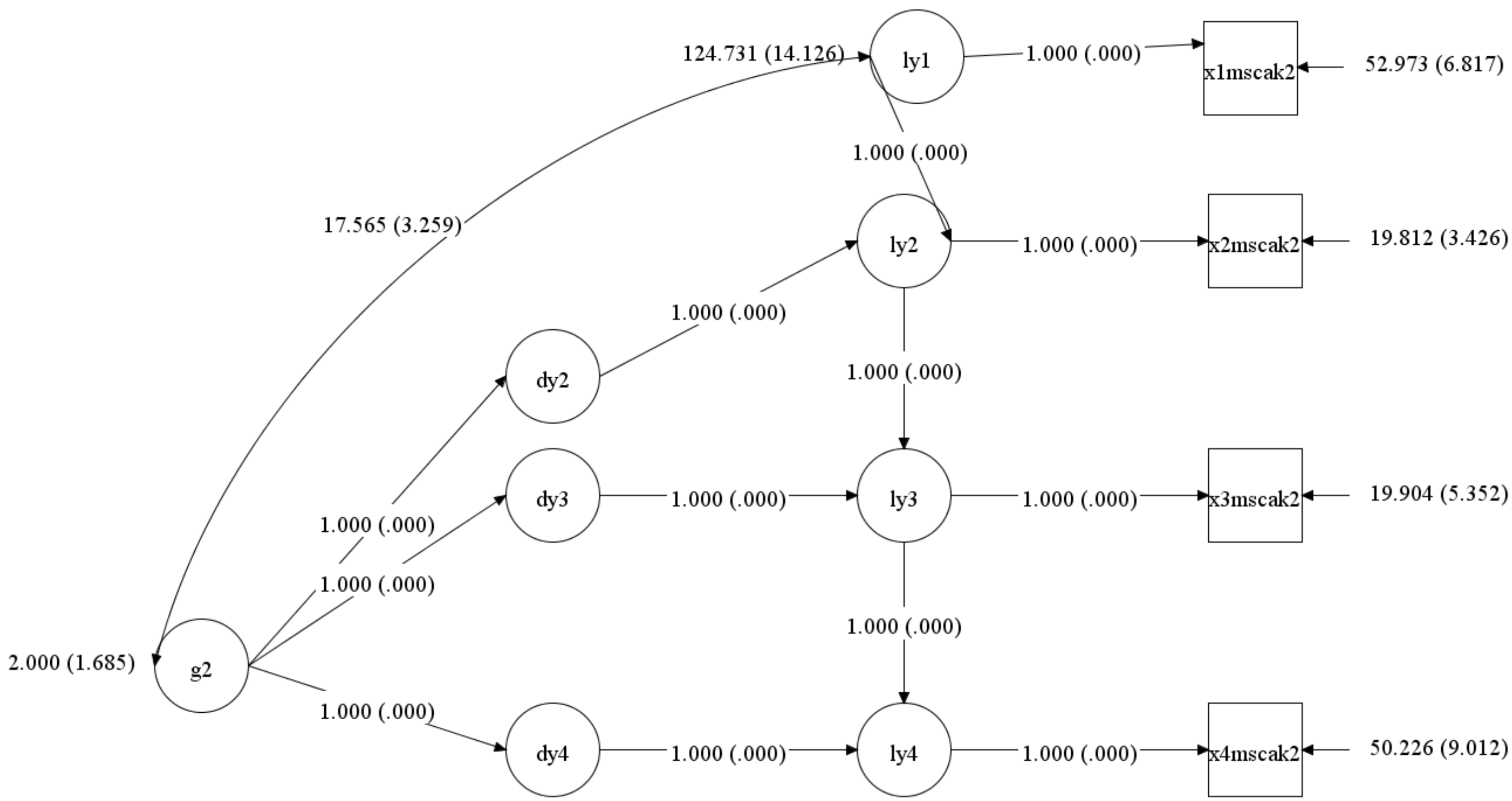

Figure 4. LCS constant change model – Math achievement with standardized estimates.

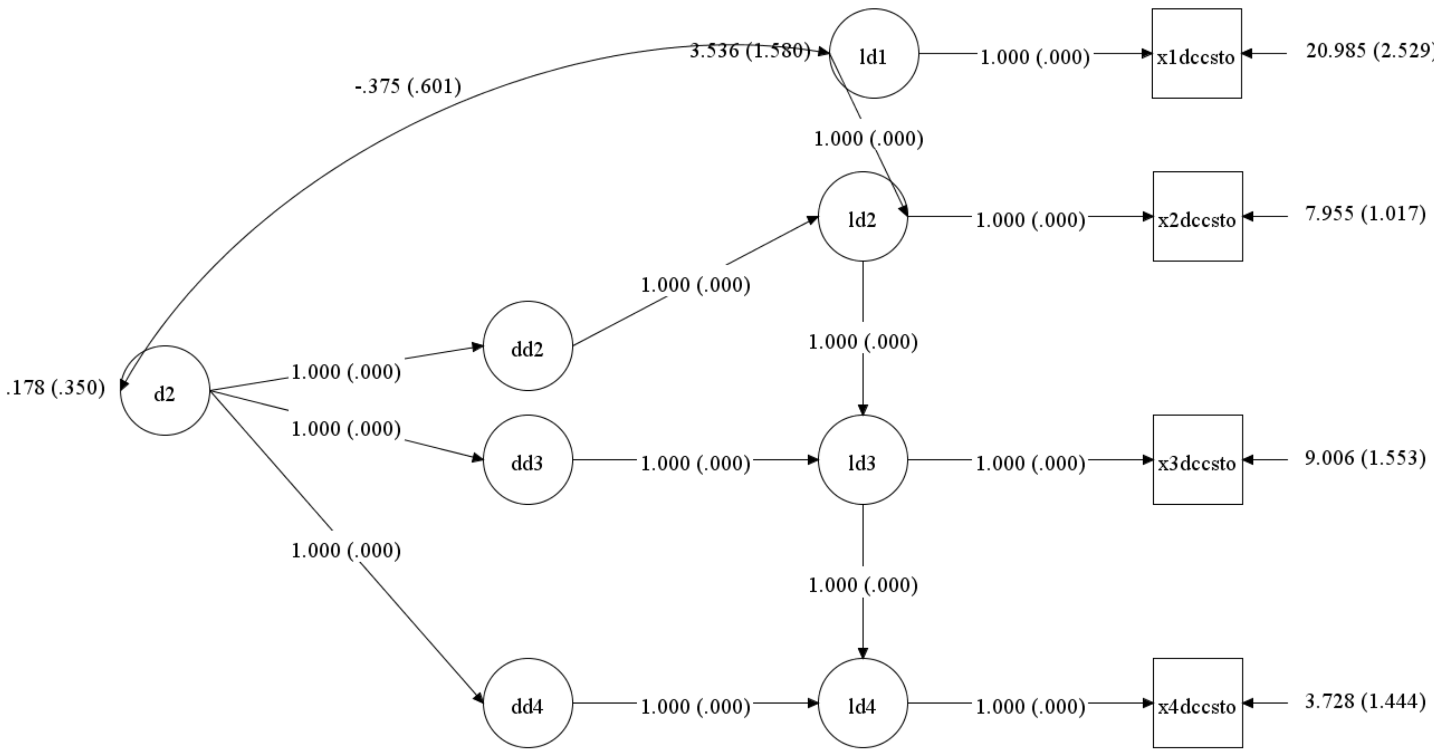

Figure 5. LCS constant change model – DCCS with standardized estimates.

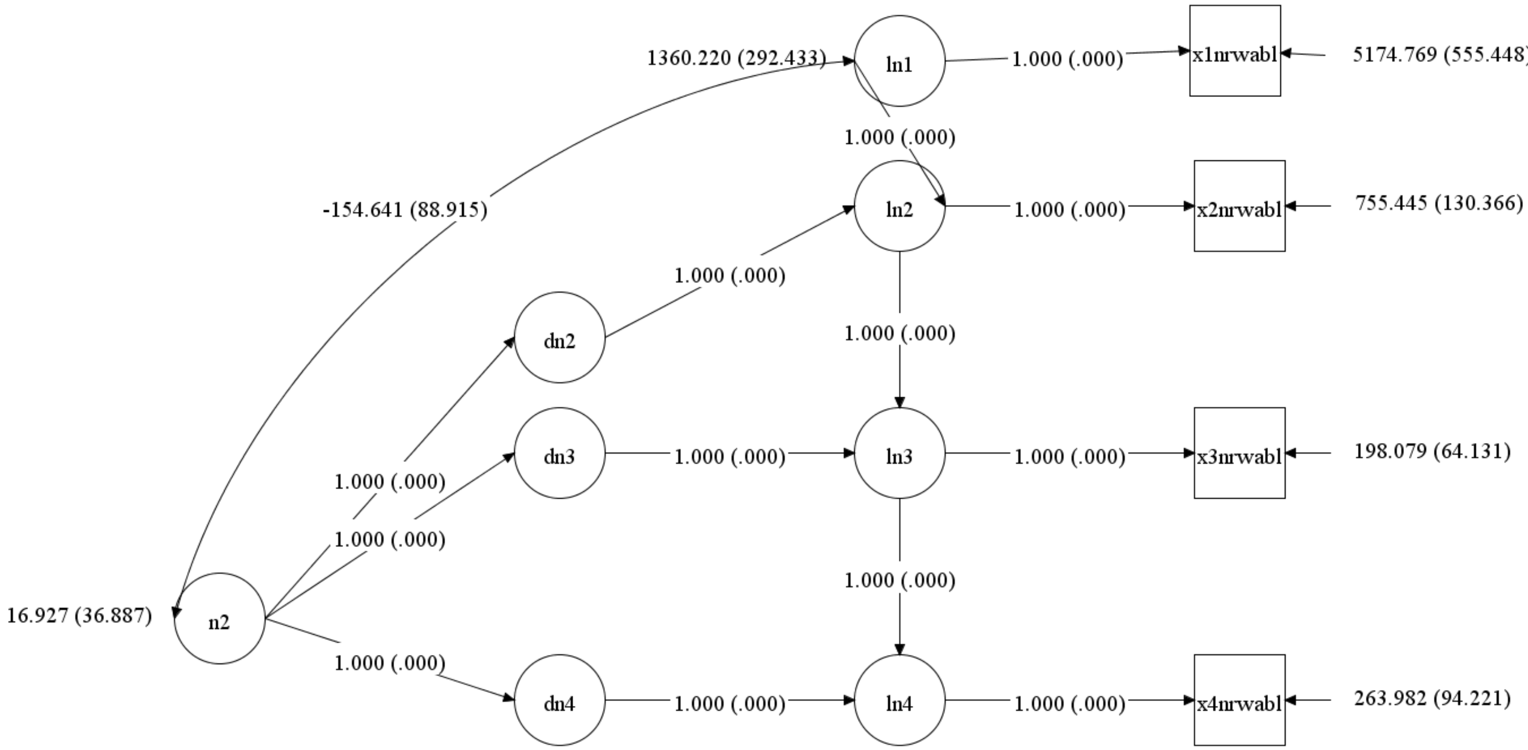

Figure 6. LCS constant change model – Number reverse with standardized estimates.

Table 11

Fit Statistics - LGC Unconditional and LCS Constant Change Models

Construct	Model	X^2	df	*p*	CFI	TLI	RMSEA with 95% CI	*p*	SRMR	AIC	BIC	ABIC
MATH	LGC	5.81	3	.121	.996	.992	.061 (.000, .135)	.320	.076	5631.20	5670.00	5635.10
	LCS	4.59	4	.332	.999	.999	.024 (.000, .101)	.613	050	5627.99	5663.28	5631.58
DCCS	LGC	4.78	5	.444	1.00	1.00	.000 (.000, .086)	.736	.054	4070.20	4102.00	4073.50
	LCS	4.78	5	.444	1.00	1.00	.000 (.000, .086)	.736	.054	4070.24	4102.00	4073.47
NR	LGC	10.48	5	.063	.967	.960	.066 (.000, .122)	.265	.096	7794.40	7831.20	7802.70
	LCS	10.48	5	.063	.967	.960	.066 (.000, .122)	.265	.096	7799.42	7831.19	7802.66

Proportional Change Model

Next the proportional change models were fit to test if growth is a function of performance at the previous level. Thereafter, the LCS dual change models were specified which incorporated both the constant change and the proportional change models and allowed for nested model testing (figures 7-9). The fit statistics and the parameter estimates for the proportional change models are presented in Table 12. The exact fit hypothesis was not rejected for the math achievement LCS proportional change model X^2 *(3) = .509, p = .92*. Good fit was also supported by RMSEA = .000 (.000, .039) with 95% CI, SRMR =. 022, CFI and TLI = 1, and AIC and BIC of 5625.9 and 5664.7, respectively. The model had an ABIC of 5629.86 compared to the LGC unconditional math achievement model ABIC = 5631.6 and LCS math achievement constant change model ABIC = 5635.1, suggesting the better model is the LCS proportional change math achievement model. The LCS proportional change DCCS model had good fit X^2 *(5) = 1.99, p = .851*, RMSEA = .000 (.000, .048) with 95% CI, SRMR =. 048, CFI and TLI = 1, and AIC and BIC of 4067.4 and 4099.2, respectively. The model ABIC = 4070.7 compared to the LGC unconditional DCCS and LCS constant change DCCS models (ABIC = 4073.5), suggesting the better model is the LCS proportional change DCCS model. The LCS proportional change number reverse model's exact fit hypothesis was rejected X^2 *(5) = 14.72, p = .012*. The RMSEA = .088 (.038, .142) with 95% CI and the upper bound criteria exceeded, SRMR =. 223, while CFI and TLI were .942 and .930 respectively, with AIC and BIC of 7803.7 and 7835.4, respectively. The model ABIC = 7806.9 compared to the LGC unconditional number reverse and LCS constant change number reverse models (ABIC = 7802.7), suggesting the LGC unconditional number reverse and LCS constant change number reverse models were better than the LCS proportional change number reverse model.

Both the mean intercept and the variance of the intercept for math achievement were statistically significant (Mi = 31.30, $p < .001$; Di = 129.27, $p < .001$) (see Appendix, Table 23). Indicating significant initial math achievement scores and children differed in their initial scores. The largest and only statistically significant proportional effect was between spring and fall of 2011, indicating that significant changes occurred between these time-points (β = .176, $p < .001$). The error variances for math achievement were statistically significant like that of the DCCS and number reverse models (see Appendix, Table 24 and 25). The mean intercept and the variance of the intercept for DCCS were statistically significant (Mi = 13.6, $p < .001$; Di = 2.14, $p < .001$). The change score was predicted by the previous time-point between fall 2010 and spring 2011 (β = .94, $p < .05$).

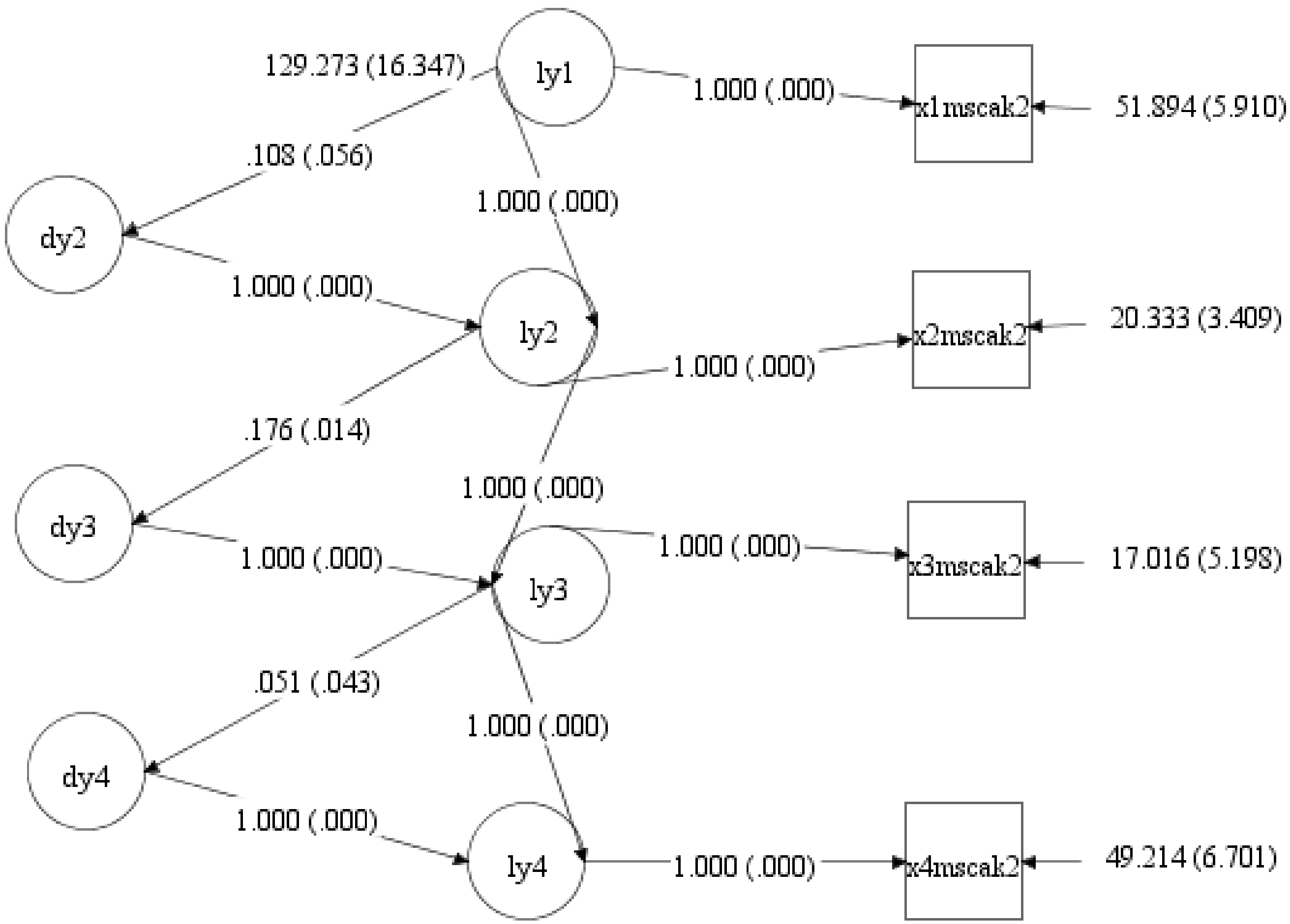

Figure 7. LCS proportional change model – Math achievement with standardized estimates.

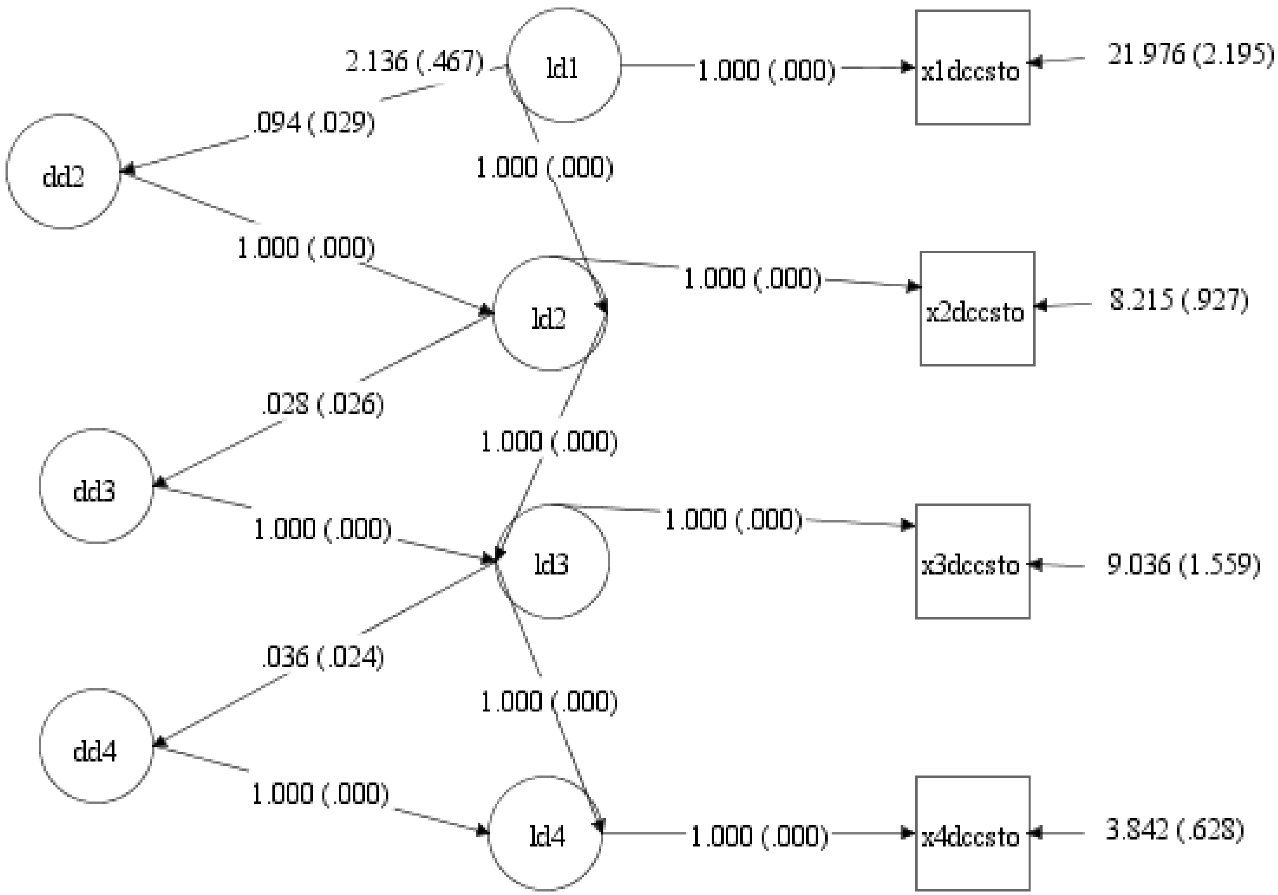

Figure 8. LCS proportional change model – DCCS with standardized estimates.

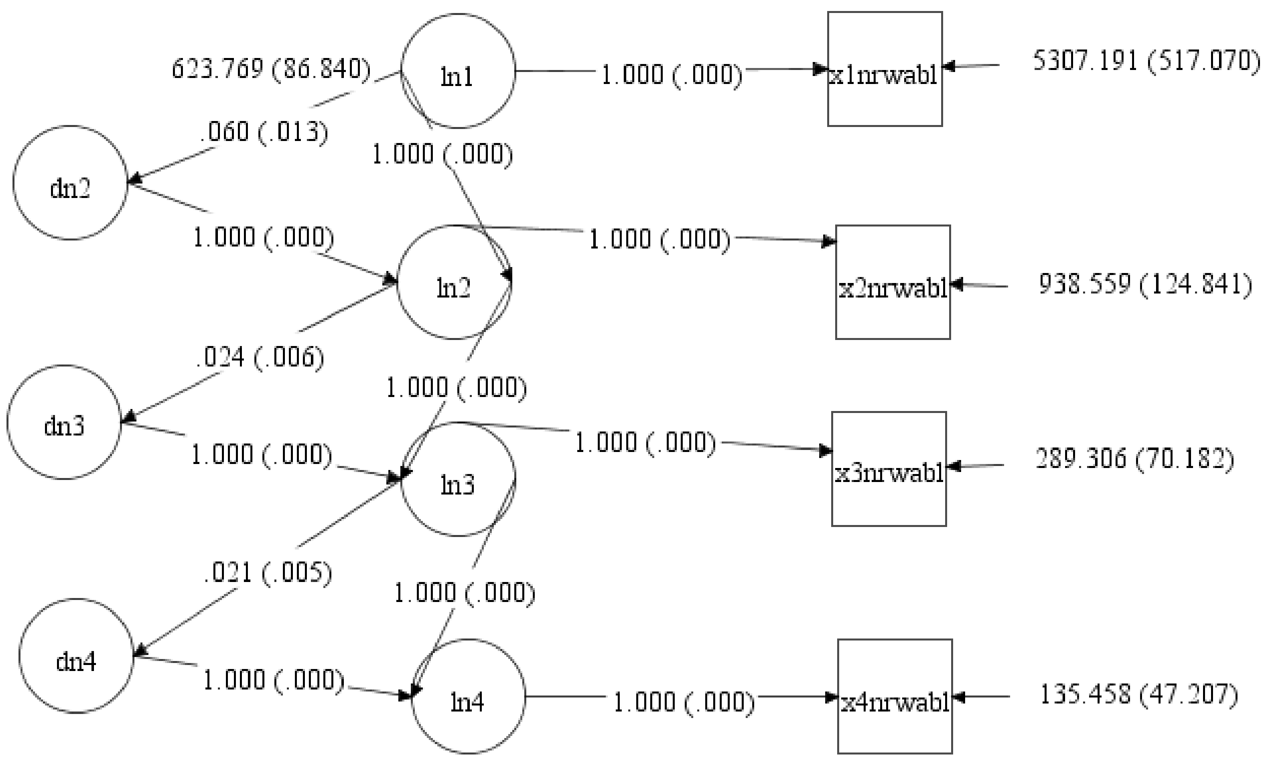

Figure 9. LCS proportional change model – Number reverse with standardized estimates.

Table 12

Fit Statistics - LCS Proportional Change Models

Construct	Model	X^2	df	*p*	CFI	TLI	RMSEA with 95% CI	*p*	SRMR	AIC	BIC	ABIC
MATH	LCS	.509	3	.917	1.00	1.01	.000 (.000, .039)	.965	.022	5625.90	5664.73	5629.86
DCCS	LCS	1.99	5	.851	1.00	1.13	.000 (.000, .048)	.954	.048	4067.44	4099.21	4070.68
NR	LCS	14.72	5	.012	.942	.930	.088 (.038, .142)	.096	.223	7803.66	7835.43	7806.89

Dual Change Model

Three LCS dual change models were specified (Figures 10-12). The fit statistics are presented in Table 13. The exact fit hypothesis was rejected for the math achievement LCS dual change model X^2 *(3) = 3.11, p = .038*. However, adequate fit was supported by RMSEA = .012 (.000, .108) with 95% CI and the upper bound criteria exceeded, SRMR =. 055, CFI and TLI = 1. The model had an ABIC of 5632.5 compared to LCS proportional change math achievement model ABIC = 5629.9, suggesting the better model is the LCS proportional change math achievement model. The LCS dual change DCCS model had good fit X^2 *(4) = 1.58, p = .812,* RMSEA ≈.000 (.000, .059) with 95% CI, SRMR =. 053, and CFI and TLI = 1. The ABIC = 4072.6 compared to the LCS proportional change DCCS model (ABIC = 4070.7) suggesting the better model was the LCS proportional change DCCS model. The LCS dual change number reverse model's exact fit hypothesis was not rejected X^2 *(4) = 7.05, p = .133*. The RMSEA = .055 (.000, .120) with the upper bound criteria exceeded (95% CI), SRMR = .136 suggesting inadequate fit, while and CFI and TLI were .982 and .972 respectively. The ABIC = 7801.6 compared to the LGC unconditional number reverse and LCS constant change models (ABIC = 7802.7) indicating the former model was better.

Both the mean intercept and mean slope for all three models were statistically significant (math achievement: *Mi* = 31.29, *Ms* = 15.17, *p < .001*; DCCS: *Mi* = 13.62, *p < .001* and *Ms* = 7.6, *p < .01*; number reverse *Mi* = 425.09, *Ms* = 159.26, *p < .001*) (see Appendix, Table 26 -28). Therefore, the average start math score was 31 with an average increase of 15 units across time-points, while the average initial DCCS score was 14 with an average increase of 8 units, and average initial number reverse scores of 425 with average increase of 159 units. The variance of the intercept for two of the models were statistically significant (math achievement: *Di* = 122.77, number reverse: *Di* = 14458.29, $p < .001$), but the variances of the slopes were not. This result indicates that while there was significant variability among children's performance at the initial stage there was not significant variability in their growth over the observed time-points which is similar to the results of the unconditional and constant change models but different from the proportional change model (which had the better fit) where both parameters were statistically significant. There was no difference in children's initial DCCS scores or their growth over time unlike the unconditional and constant change DCCS models where there was variance in the initial score, and this is also reflected in the proportional change model (which was the better fitting model). However, the dual change models revealed that math achievement change was negative suggesting deceleration in scores (β = -0.053), but this was not statistically significant. There was statistically significant change in DCCS and number reverse scores, however, like math achievement there was deceleration in growth overtime (DCCS: $\beta = -0.468$, $p < .05$; number reverse: $\beta = -0.328$, $p < .05$). The errors for math achievement were statistically significant like that of the DCCS and number reverse models. The dual change model also indicated that math achievement correlation was statistically significant between individual child differences (intercept) and slope suggesting that children with higher initial math scores will likely have higher growth rates.

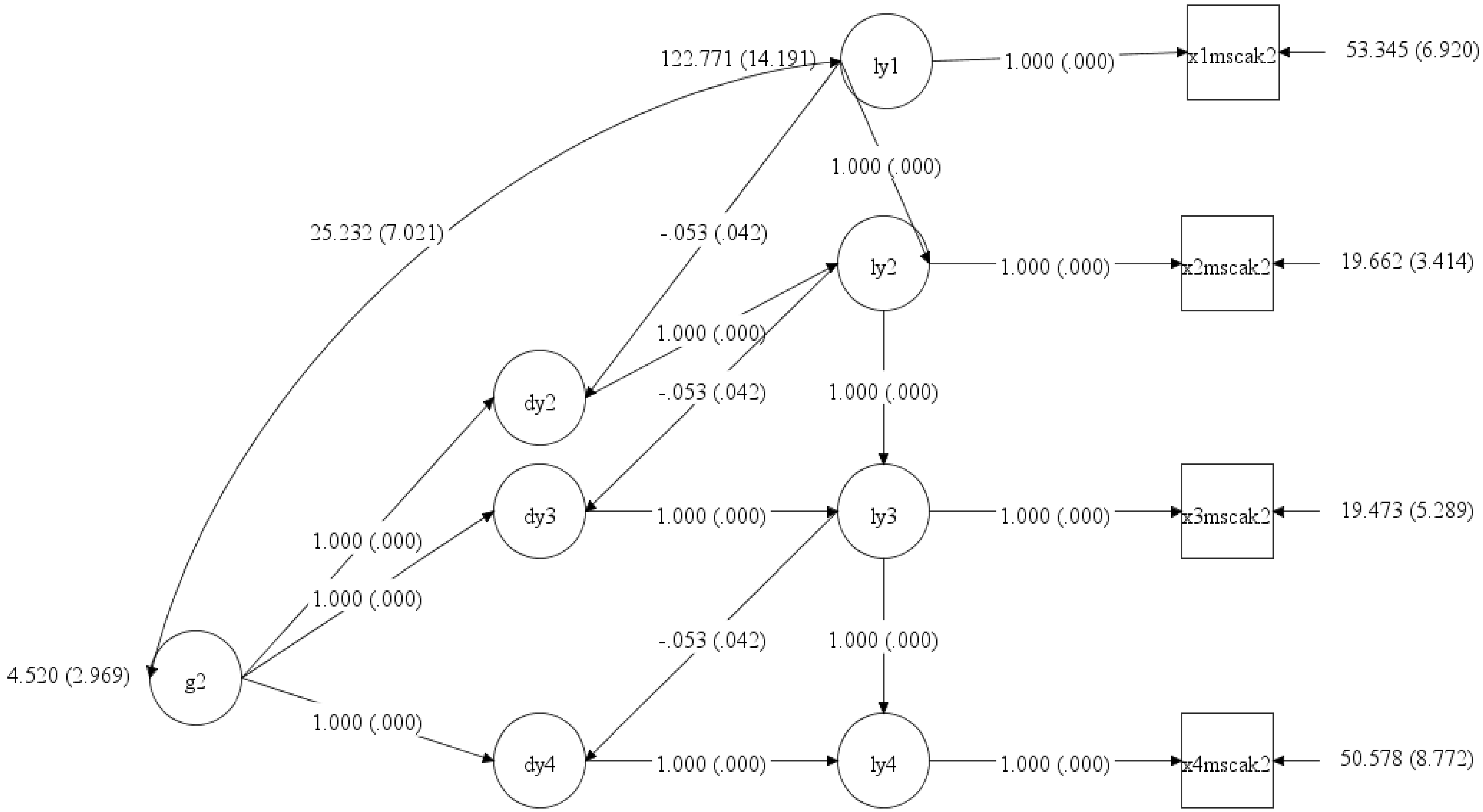

Figure 10. LCS Dual change model – Math achievement with standardized estimates.

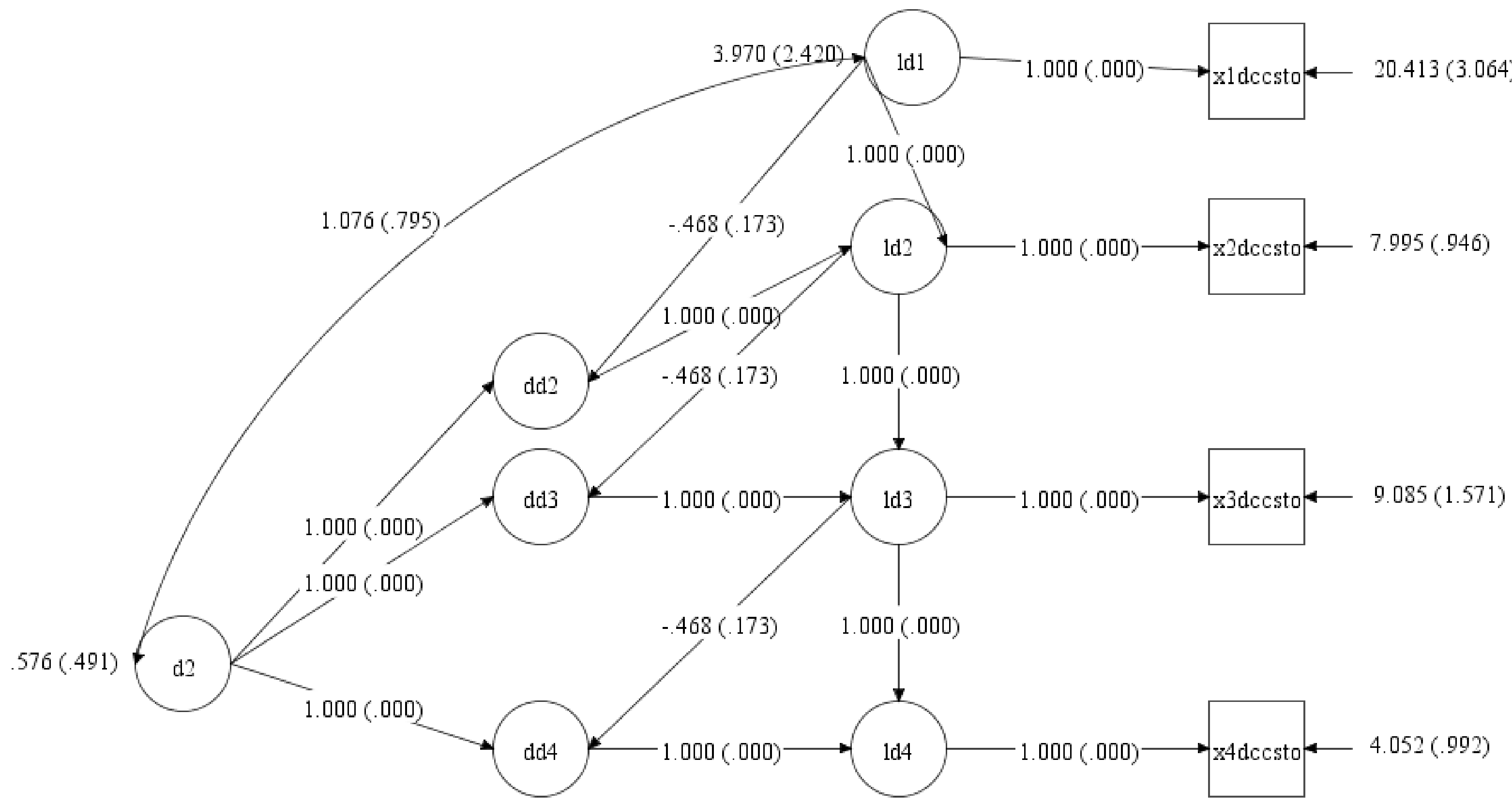

Figure 11. LCS dual change model – DCCS with standardized estimates.

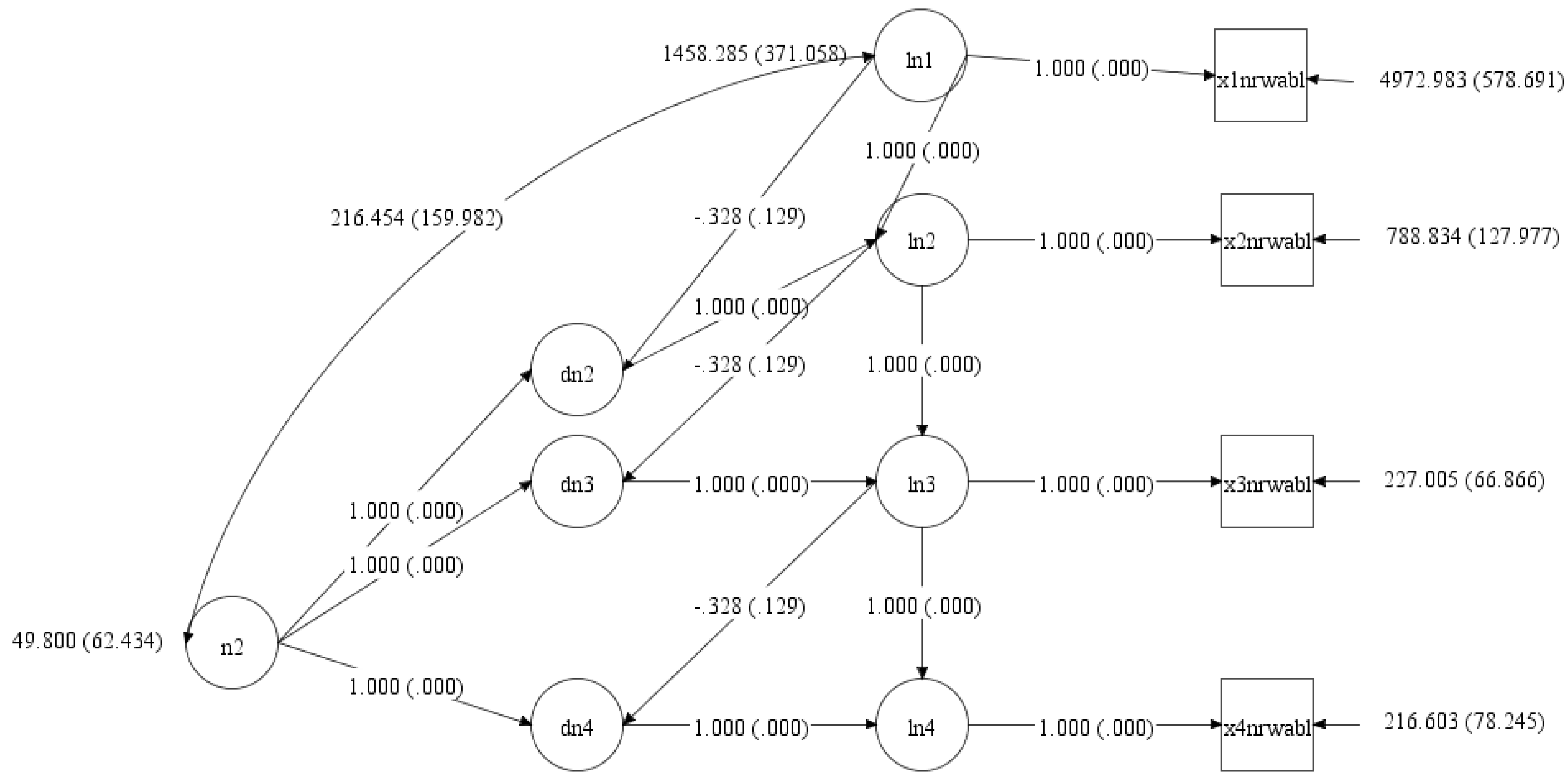

Figure 12. LCS dual change model – Number reverse with standardized estimates.

Table 13

Fit Statistics - LCS Dual Change Models

Construct	Model	X^2	df	*p*	CFI	TLI	RMSEA with 95% CI	*p*	SRMR	AIC	BIC	ABIC
MATH	LCS	3.11	3	.038	1.00	1.00	.012 (.000, .108)	.617	.055	5628.50	5667.33	5632.46
DCCS	LCS	1.58	4	.812	1.00	1.23	.000 (.000, .059)	.928	.053	4069.04	4104.33	4072.63
NR	LCS	7.05	4	.133	.982	.972	.055 (.000, .120)	.373	.136	7797.99	7833.29	7801.58

Bivariate Models

LGC Associative and LCS Bivariate Dual Change Uncoupled Models

The LGC associative and the LCS bivariate dual change uncoupled models were the first set of bivariate models to be specified (Figures 13-16). They tested for any association between the variables and assessed the growth trajectory and determined if the development of executive functioning influences the development of math achievement. The fit statistics are presented in Table 14. The exact fit hypothesis was rejected for the math achievement and DCCS LGC associative model X^2 *(26) = 149.36, p < .001*. Poor model fit was demonstrated by RMSEA = .137 (.116, .159) with 95% CI and SRMR = .187 which exceeded the thresholds for adequate fit. Additionally, CFI = .861, TLI = .851, AIC and BIC were 9636.6 and 9700.1, respectively. The math achievement and DCCS LCS bivariate dual change uncoupled model had better fit than its LGC counterpart but still less than satisfactory. The exact fit hypothesis was also rejected X^2 *(17) = 56.19, p < .001* and SRMR = .187, however the RMSEA suggested moderate fit (.096 (.069, .124) with 85% CI, CFI and TLI were .956 and .927, respectively and AIC and BIC of 9561.4 and 9656.7, respectively. The ABIC = 9571.14 which was lower than the associative model (ABIC = 9643). The math achievement and number reverse LGC associative model did not fit the data and performed even more poorly than the LGC math achievement and DCCS associative model, X^2 *(26) = 218.58, p < .001*, RMSEA .171 (.151, .193) with 955 CI, SRMR = .441, CFI = .834, TLI = .821, and AIC and BIC of 13297.1 and 13360.6, respectively. The math achievement and number reverse LCS bivariate dual change uncoupled model had better fit than the LGC associative model. The exact fit hypothesis remained rejected X^2 *(15) = 36.25, p = .002* and SRMR = .164, however the RMSEA was adequate (.075 (.044, .106) with 95% CI, CFI = .982 and TLI .966, and AIC and BIC of 13136.7 and 13239, respectively. The ABIC = 13147.14 which was higher than the math achievement and DCCS LCS bivariate dual change uncoupled mode

Where the LCS Math achievement and DCCS bivariate dual change uncoupled model was concerned the mean intercept, the mean slope, the variance of the intercept and the variance of the slope were all statistically significant. Math achievement *Mi* = 32.15, *Ms* = 14.71, p < .001, DCCS *Mi* = 13.79, p < .001, *Ms* = 6.31, p = .029 (see Appendix, Table 29). Indicating the average start math score was 32 with an average increase of 15 units, while the average initial DCCS score was 14 with an average increase of 6 units. The variance of the intercept and the variance of the slope were statistically significant for math achievement *Di* = 118.67, p < .001, *Ds* = 7.53, p = .047, and for DCCS *Di* = 5.48, p = .012, *Ds* = .695, p = .025. This demonstrates that children differed in their initial scores and in their growth over time. The math achievement change score was negative indicating deceleration over time, but this was not statistically significant. However, the DCCS change score was negative and statistically significant (β = -0. 383). The correlation between math achievement mean intercept and slope was a strong positive statistically significant (r = .81, p < .001) while the DCCS mean intercept and slope was negative and not statistically significant. The errors were statistically significant. For the math achievement and number reverse LCS bivariate dual change uncoupled model, the mean intercept, and mean slope for math achievement and number reverse were statistically significant (math achievement Mi = 31.99, Ms = 10.86, p < .001, number reverse Mi = 426.77, Ms = 201.60, p < .001) (see Appendix, Table 30), reflecting that the average starting math score was 32 with an average unit increase of 11 while the average initial number reverse score was 427 with 202 unit increase over time. The variance of the intercept was also statistically significant, but the variances of the slopes were not (math achievement Di = 134.88, $p < .001$, number reverse $Di = 1558.69$, $p < .001$). These indicated there was no variability in the children's growth over the time-points. The math achievement change score was negative indicating deceleration over time, but this was not statistically significant. However, the number reverse change score was negative and statistically significant (β = -0. 427), indicating change decelerated over time or as the children got older. The errors were statistically significant.

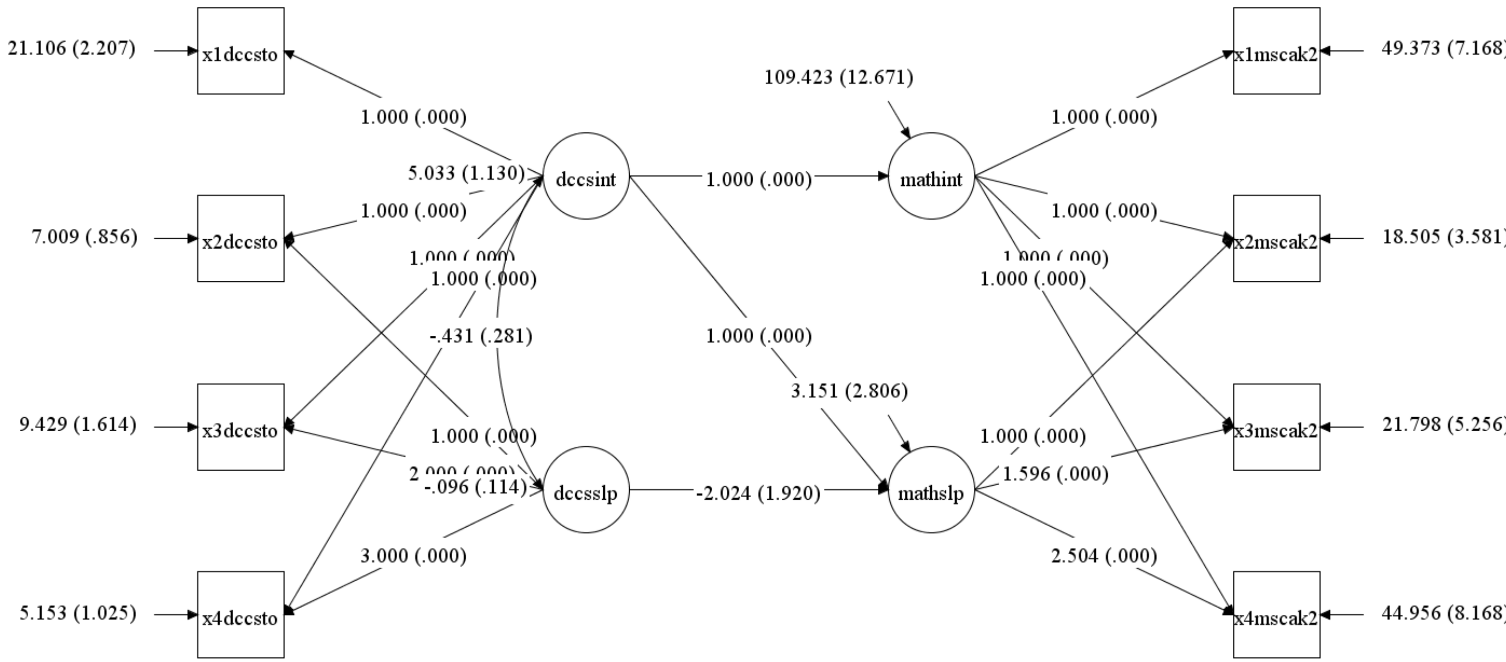

Figure 13. LGC associative model – Math and DCCS with standardized estimates.

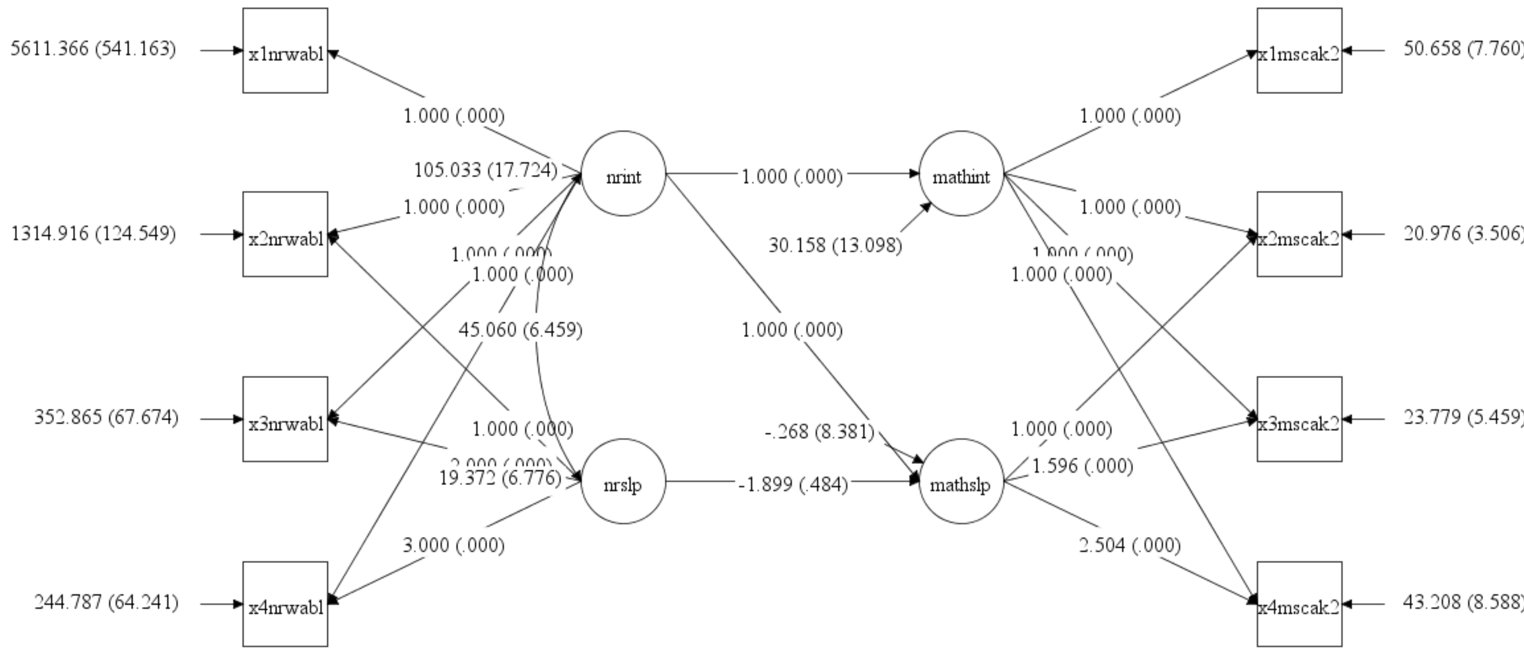

Figure 14. LGC associative model – Math and number reverse with standardized estimates

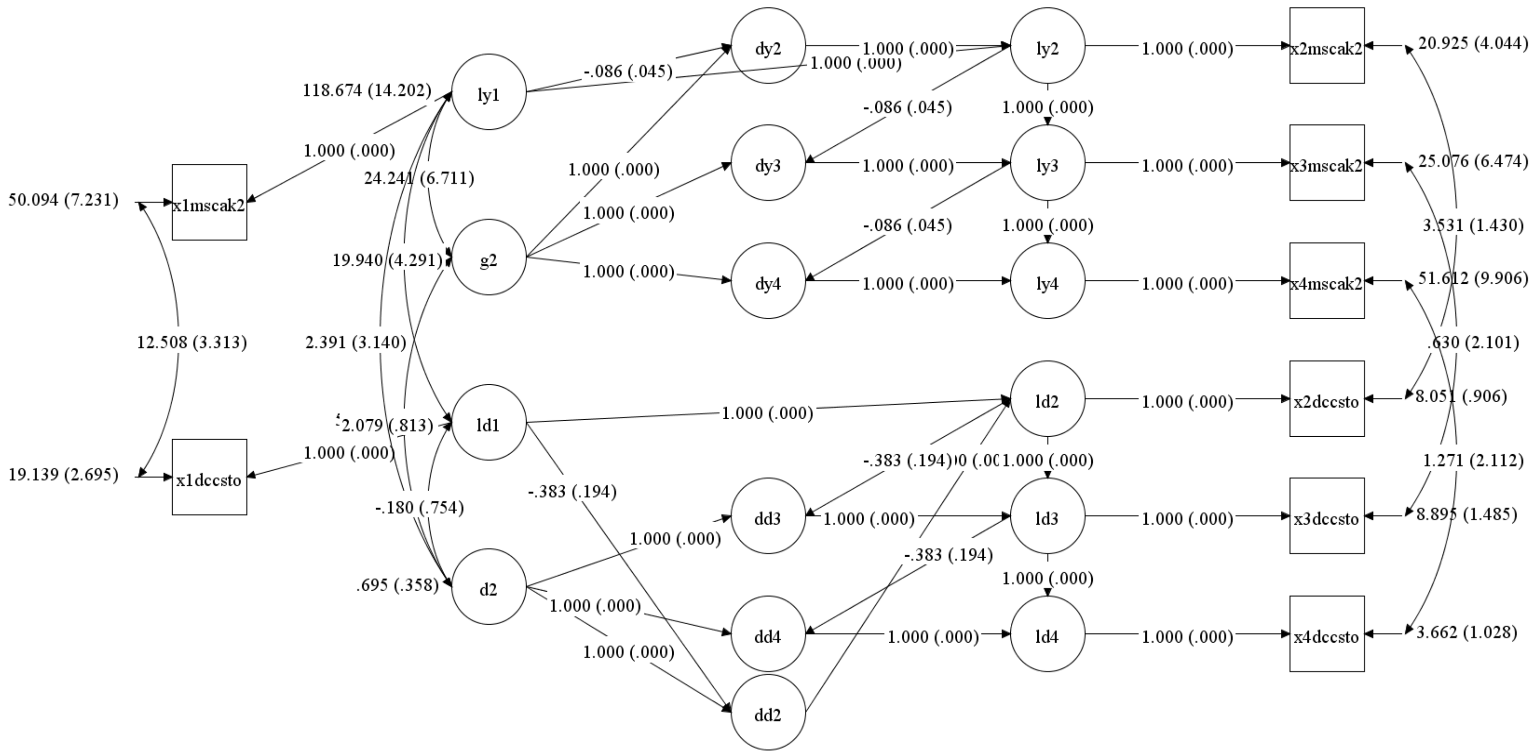

Figure 15. LCS bivariate dual change no coupling model – Math and DCCS with standardized estimates

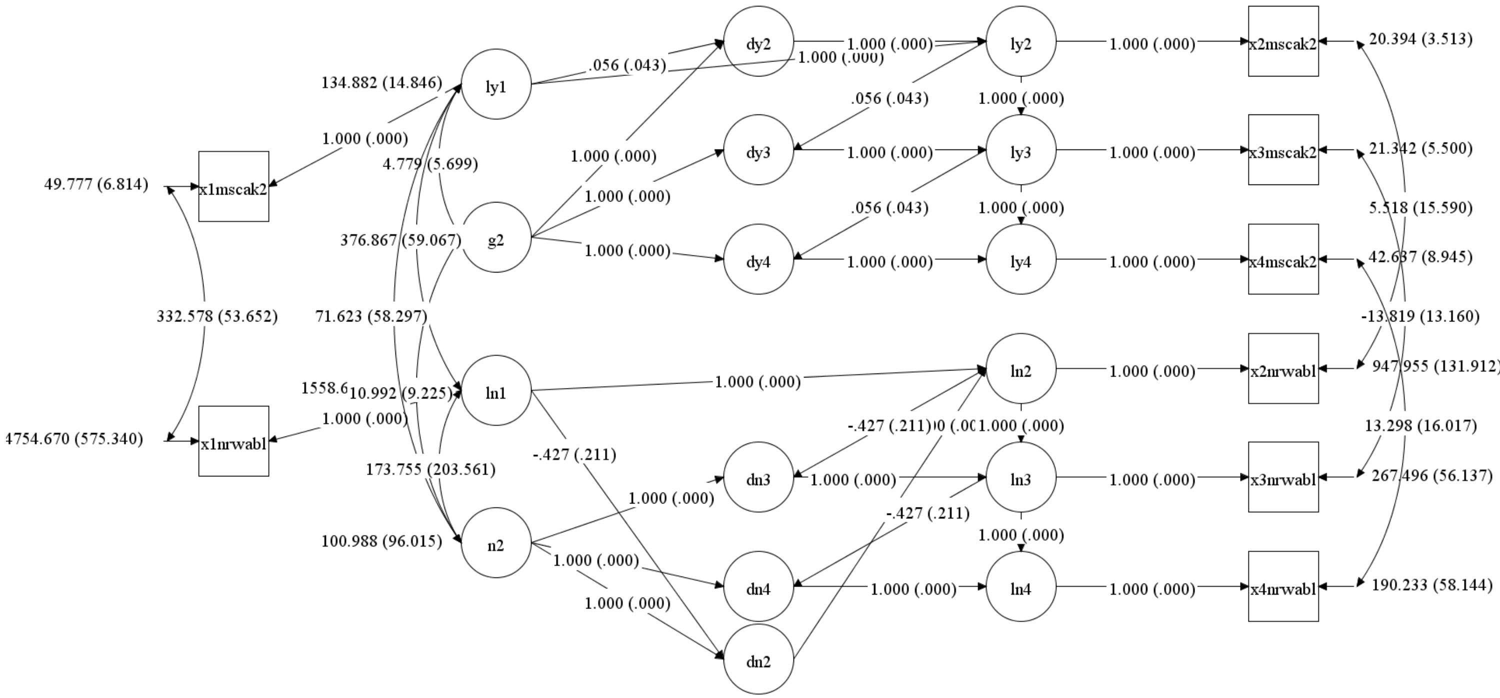

Figure 16. LCS bivariate dual change no coupling model – Math and number reverse with standardized estimates.

Table 14

Fit Statistics - LGC Associative and LCS Bivariate Dual Change Models

Construct	Model	X^2	df	*p*	CFI	TLI	RMSEA with 95% CI	*p*	SRMR	AIC	BIC	ABIC
MATH & DCCS	LGC	149.32	26	.000	.861	.851	.137 (.116, .159)	<.001	.187	9636.57	9700.09	9643.04
	LCS	56.19	17	.000	.956	.927	.096 (.069, .124)	.004	.112	9561.44	9656.74	9571.14
MATH & NR	LGC	218.58	26	.000	.834	.821	.171 (151, .193)	<.001	.441	13297.05	13360.58	13303.52
	LCS	36.25	15	.002	.982	.966	.075 (.044, .106)	.087	.164	13136.72	13239.02	13147.14

Bivariate Dual Change Unidirectional Coupled Model

Latent change score bivariate dual change unidirectional coupled models were specified to examine leading and lagging indicators (Figures 17-19). The fit statistics are presented in table 15. The exact fit hypothesis was rejected for the model of DCCS to changes in math achievement X^2 *(16) = 44.23, p < .001*. Adequate model fit was supported by RMSEA = .084 (.055, .114) although the upper boundary was exceeded (with 95% CI), and SRMR = .080; CFI = .968, TLI = .944, AIC and BIC of 9551.5 and 9650.3, respectively and ABIC was 9561.54 which showed an improvement over the DCCS and math achievement no coupling model. The exact fit was rejected for the model of number reverse to changes in math achievement X^2 *(19) = 101.14, p < .001*. Poor model fit was supported by RMSEA = .131 (.107, .157) with 95% CI, SRMR = .201; CFI = .929, TLI = .896, AIC and BIC of 13193.6 and 13281.9, respectively. The exact fit hypothesis was rejected for the model of math achievement to changes in DCCS X^2 *(16) = 55.67, p < .001*. with RMSEA = .099 (.072, .128) as well as the upper boundary was exceeded (with 95% CI), and SRMR = .111; CFI = .955, TLI = .922, AIC and BIC of 9562.9 and 9661.8, respectively. Where math achievement to changes in number reverse was concerned, the exact fit hypothesis was again rejected X^2 *(16) = 38.82, p = .002*. Adequate model fit was supported by RMSEA = .071 (.042, .101) with 95% CI, however, SRMR = .111; CFI = .981, TLI = .959, AIC and BIC of 13135.3 and 13230.6, respectively and ABIC was 13144.99 and was an improvement over the number reverse no coupling model. It was observed that the number reversed models performed poorer than their DCCS counterparts with the model of DCCS changes in math achievement having the lowest ABIC.

Both mean intercepts for the DCCS to changes in math achievement model were statistically significant, math achievement $Mi = 32.13$, $p < .001$ and DCCS Mi = 14.24, p < .001 indicating children's initial math and DCCS scores were on average 32 and 14, respectively (see Appendix, Table 31). The mean slopes were negative and not statistically significant. Math achievement variance of the intercept was statistically significant indicating differences among the children's starting scores. The errors were statistically significant. The coupling parameter from DCCS to changes in math achievement was estimated and was not statistically significant indicating that subsequent changes in math achievement were not partially accounted for by current levels of DCCS. The proportional change parameter for math achievement was negative and statistically significant (-0.215, $p < .05$), indicating that while math scores are influenced by its previous state, change in scores decelerate as scores increased.

Where math achievement to changes in DCCS is concerned, the mean intercepts and mean slopes were statistically significant (math achievement $Mi = 32.13$, $p < .001$ and $Ms = 14.82$, $p < .001$ and DCCS $Mi = 13.85$, $p < .001$ and $Ms = 8.42$, $p = .016$) (see Appendix, Table 32). Children had an initial math score of 32 which would likely increase by 15 units while DCCS initial score was 14 and likely increase by 8 units over time. The variance of the intercepts were statistically significant (math achievement: $Di = 118.7$, $p < .001$; DCCS: $Di = 6.17$, $p = .042$), but only the math achievement slope was statistically significant (*Ds* = 7.48, $p = .047$). There was variability in children's initial scores for both measures but not in their growth rate. All the errors were statistically significant. In the math achievement to changes in DCCS, the proportional change parameters for math achievement (-0.089, $p < .05$) and DCCS (-0.583, $p < .05$) indicated that they were influenced by their previous state but that these changes slowed over time as scores increased. The coupling parameter was not statistically significant indicating that subsequent changes in DCCS were not partially accounted for by current levels of math achievement. The intercept and slope of math achievement were strongly correlated ($r = .84$, $p < .001$) indicating that children with a higher initial math score will likely show more change over time. The math achievement and DCCS intercepts were strongly correlated ($r = .78$, $p < .001$), indicating children with a higher initial DCCS score will likely show more change over time. Additionally, the slopes of the two constructs were strongly correlated ($r = .82$, $p < .05$)

In the math changes in number reverse model the mean intercepts were statistically significant (math achievement *Mi* = 32.04, $p < .001$ and number reverse *Mi* = 427.34, $p < .001$ (see Appendix, Table 33), indicating an initial math score of 32 and an initial number reverse score of 427. Only the math achievement mean slope was statistically significant (*Ms* = 10.79, $p < .001$) indicating that over time math scores were likely to increase by 11 units. The variance of the intercepts were statistically significant while the variance of the slopes were not (math achievement *Di* = 133.39, $p < .001$ and number reverse *Di* = 1417.22, $p < .001$), indicating children differed in initial scores but not in their growth rate. All the errors were statistically significant. Where the math changes in number reverse is concerned neither the proportional parameter nor the coupling parameters were statistically significant. Indicating that scores were not influenced by previous state and current math scores does not predict subsequent number reverse scores.

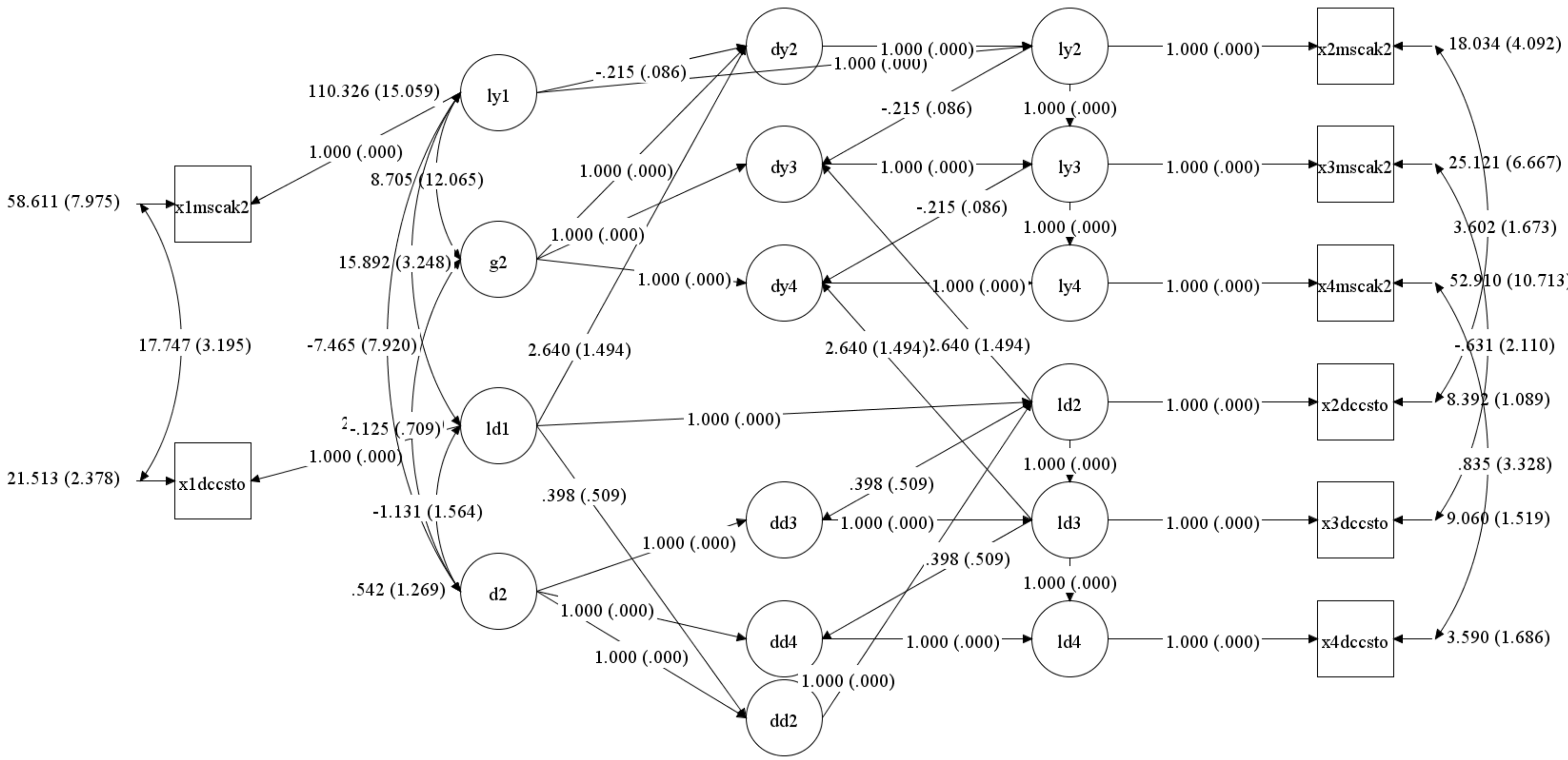

Figure 17. LCS bivariate dual change unidirectional coupling model - DCCS changes in math with standardized estimates.

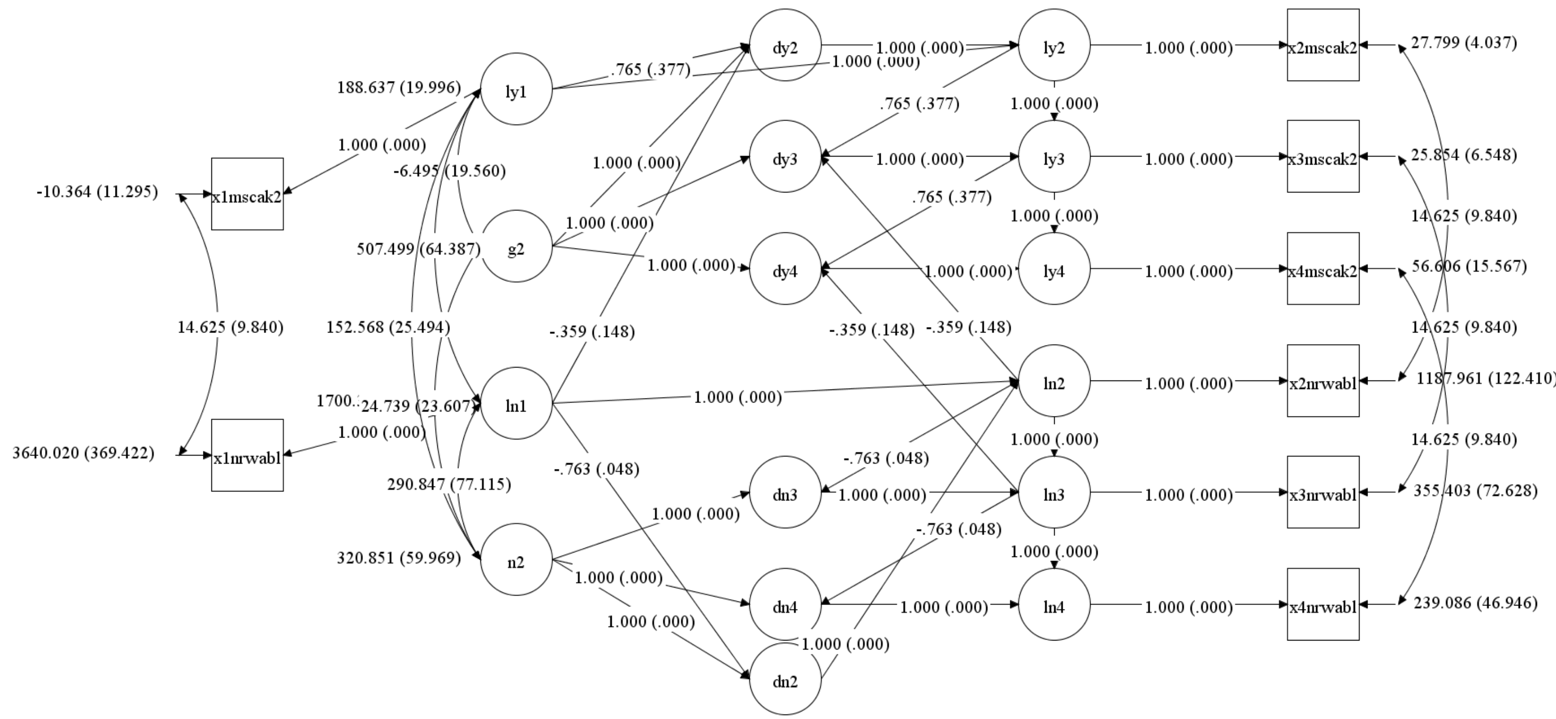

Figure 18. LCS bivariate dual change unidirectional coupling model – Number reverse changes in math with standardized estimates.

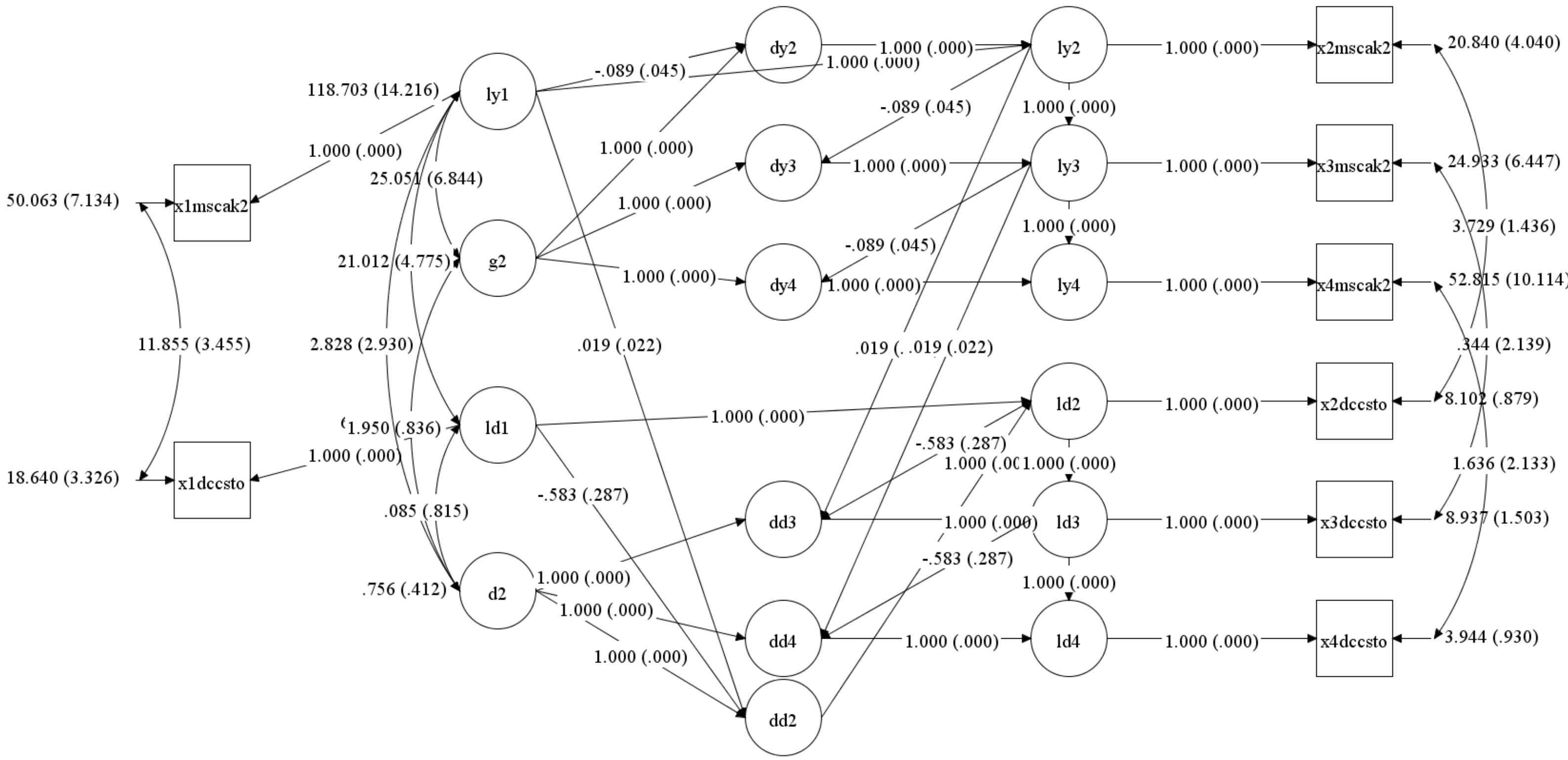

Figure 19 LCS bivariate dual change unidirectional coupling model - Math changes in DCCS with standardized estimates.

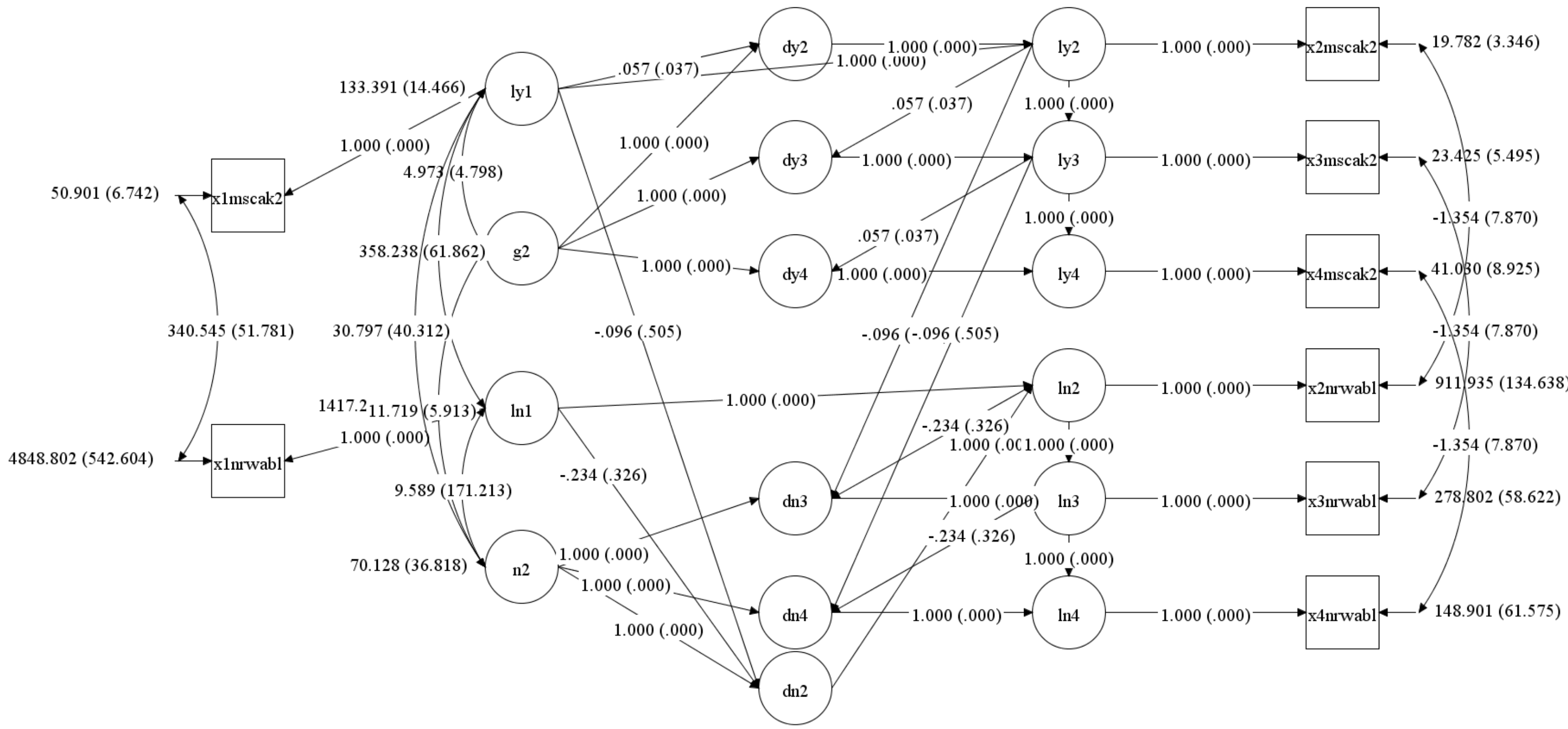

Figure 20. LCS bivariate dual change unidirectional coupling model - math changes in number reverse with standardized estimates.

Table 15

Fit Statistics - LCS Bivariate Dual Change Unidirectional Coupling Models

Construct	Model	X^2	df	*p*	CFI	TLI	RMSEA with 95% CI	*p*	SRMR	AIC	BIC	ABIC
DCCS & MATH	DCCS to Math	44.23	16	<.001	.968	.944	.084 (.055, .114)	.029	.080	9551.48	9650.30	9561.54
NR & MATH	Number Reverse to Math	101.14	19	<.001	.929	.896	.131 (107, .157)	<.001	.201	13193.62	13281.85	13202.59
MATH & DCCS	Math to DCCS	55.67	16	<.001	.955	.922	.099 (.072, .128)	.003	.111	9562.93	9661.75	9572.98
MATH & NR	Math to Number Reverse	38.82	16	.002	.981	.959	.071 (.042, .101)	.109	.119	13135.29	13230.59	13144.99

Bidirectional Coupling Model

The final set of models to be fit to the data were the LCS bidirectional coupling models (Figures 21-22) which were a full combination of LGCM and autoregression. The exact fit hypothesis was rejected for the DCCS and math achievement model X^2 *(23) = 57.92, p < .001* and SRMR = .223, however the RMSEA was acceptable (.078 (.053, .103), CFI and TLI were .961 and .952, respectively and AIC and BIC were 9551.2 and 9625.3, respectively (Table 16). This is an improvement over the unidirectional models. The exact fit hypothesis was also rejected for the math achievement and number reverse model X^2 *(18) = 59.82, p < .001* and SRMR = .319, however the RMSEA showed moderate fit (.096 (.079, .123)), CFI and TLI were .964 and .944, respectively and AIC and BIC were 13154.3 and 13246.1, respectively. The DCCS and math achievement bidirectional model's ABIC = 9558.7 and indicated better model fit than the unidirectional models, DCCS changes in math achievement (ABIC = 9561.5) and math achievement changes in DCCS (ABIC = 9572.9). The math achievement and number reverse bidirectional model's ABIC = 13163.6 had slightly poorer fit than the unidirectional math achievement to changes in number reverse model (ABIC = 13144.9).

The mean intercepts and the mean slopes were all statistically significant (math achievement *Mi* = 36.77, *p < .001*, *Ms* = 22.19, *p < .001* and DCCS *Mi* = 13.60, *p < .001*, *Ms* =14.09, *p < .001*) in the DCCS and math achievement bidirectional coupling model (see Appendix, Table 34). These indicated children had a mean starting math score of 37 and with everyone unit change in DCCS would likely see a 22 unit increase in math scores over time. Mean DCCS starting scores were 14 with a 14 unit increase in

DCCS scores for every unit change in math scores over time. The variance of the intercepts were statistically significant (math achievement *Di* = 201.37, $p < .001$, and DCCS *Di* = 17.56, $p < .001$), but only the DCCS slope was statistically significant (*Ds* = 16.30, $p < .025$). This shows that there were differences in children's initial math and DCCS scores but only DCCS scores showed any differences in growth rate. All the errors were statistically significant. Both the change parameters for math achievement and DCCS were statistically significant (math achievement: .48, $p < .001$; *DCCS* = -1.09, $p < .001$). Significant change parameters are an indication of change being predicted by the previous state, hence changes in math achievement and DCCS are predicted by performance at the previous level. The positive parameter for math achievement indicated acceleration in math scores and the negative parameter for DCCS indicates that change decelerates as DCCS scores increased. Both the coupling parameters were statistically significant (math achievement to changes in DCCS: 0.061, $p < .01$; DCCS to changes in math achievement: -2.36, $p < .001$). Therefore, subsequent changes in DCCS were partially accounted for by current levels of math achievement and subsequent changes in math achievement were partially accounted for by current levels of DCCS with DCCS being the leading predictor. The math achievement intercept and slope had a moderate inverse association ($r = -0.54$, $p < .001$) indicating that children with a higher initial math score showed less change in DCCS over time. The math achievement intercept and the DCCS intercept had a strong correlation ($r = .723$, $p < .001$) indicating that children with a higher initial DCCS score showed greater change over time. The

DCCS intercept and slope association (r = 357, $p < .01$) indicated that children with a higher initial DCCS scores tended to show greater change over time.

The number reverse and math achievement model had mean intercepts and slopes that were statistically significant (math achievement Mi = 30.98, $p < .001$, Ms = 298.26, $p = .002$ and number reverse Mi = 416.58, $p < .001$, Ms = 730.33, $p < .001$) (see Appendix, Table 35). The variance intercepts were statistically significant (math achievement Di = 256.35, $p < .001$ and number reverse Di = 1423.13, $p < .001$) while only the number reverse variance slope was statistically significant (Ds = 296.76, $p < .001$), indicating there were differences in children's initial math and number reverse scores but only number reverse scores showed any differences in growth rate. All the errors were statistically significant.

Both the change parameters for math achievement and number reverse were statistically significant indicating the changes were predicted by the previous state (math achievement: 1.36, $p < .001$; number reverse = -1.85, $p < .001$). The positive parameter for math achievement indicated acceleration in math scores and the negative parameter for number reverse indicates that change decelerated as scores increased. Both the coupling parameters were statistically significant (math achievement to changes in number reverse: 2.40, $p < .001$; number reverse to changes in math achievement: -0.784, $p < .001$). Therefore, subsequent changes in number reverse were partially accounted for by current levels of math achievement and subsequent changes in math achievement were partially accounted for by current levels of number reverse, respectively, with math achievement being the leading indicator. The math achievement intercept and slope had a weak inverse association (r = -0.28, $p < .001$) indicating children with a higher initial math score while there would be growth the change slowed over time. The math achievement intercept and the number reverse intercept had a strong correlation (r = .908, $p < .001$) indicating that children with a higher initial math score showed greater change over time. The number reverse intercept and slope association (r = 231, $p < .01$) indicated that children with a higher initial number reverse scores tended to show greater change over time. The slopes of both constructs were strongly associated (r = .873, $p < .001$).

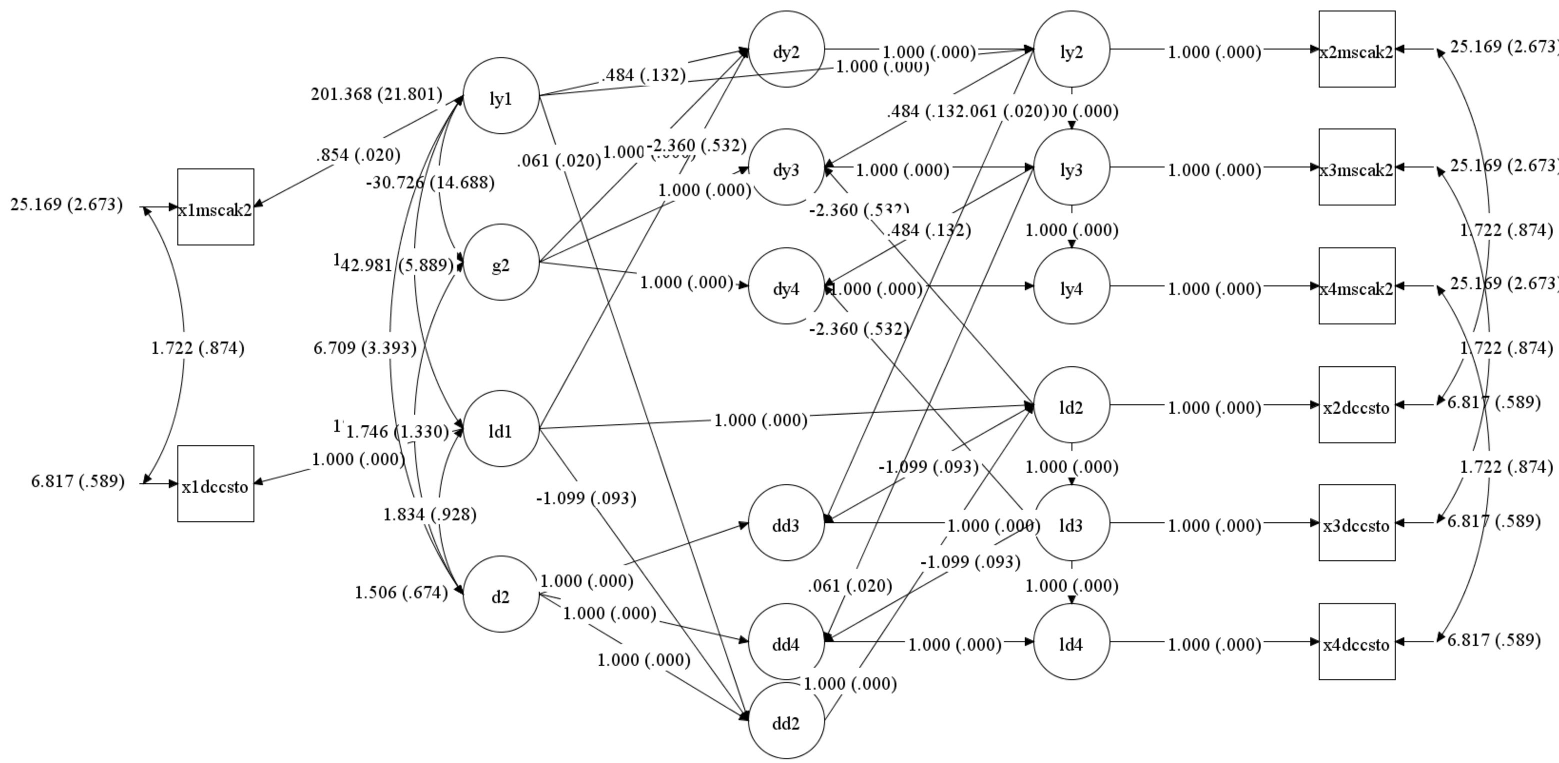

Figure 21. LCS bivariate dual change bidirectional coupling models – DCCS & math with standardized estimates.

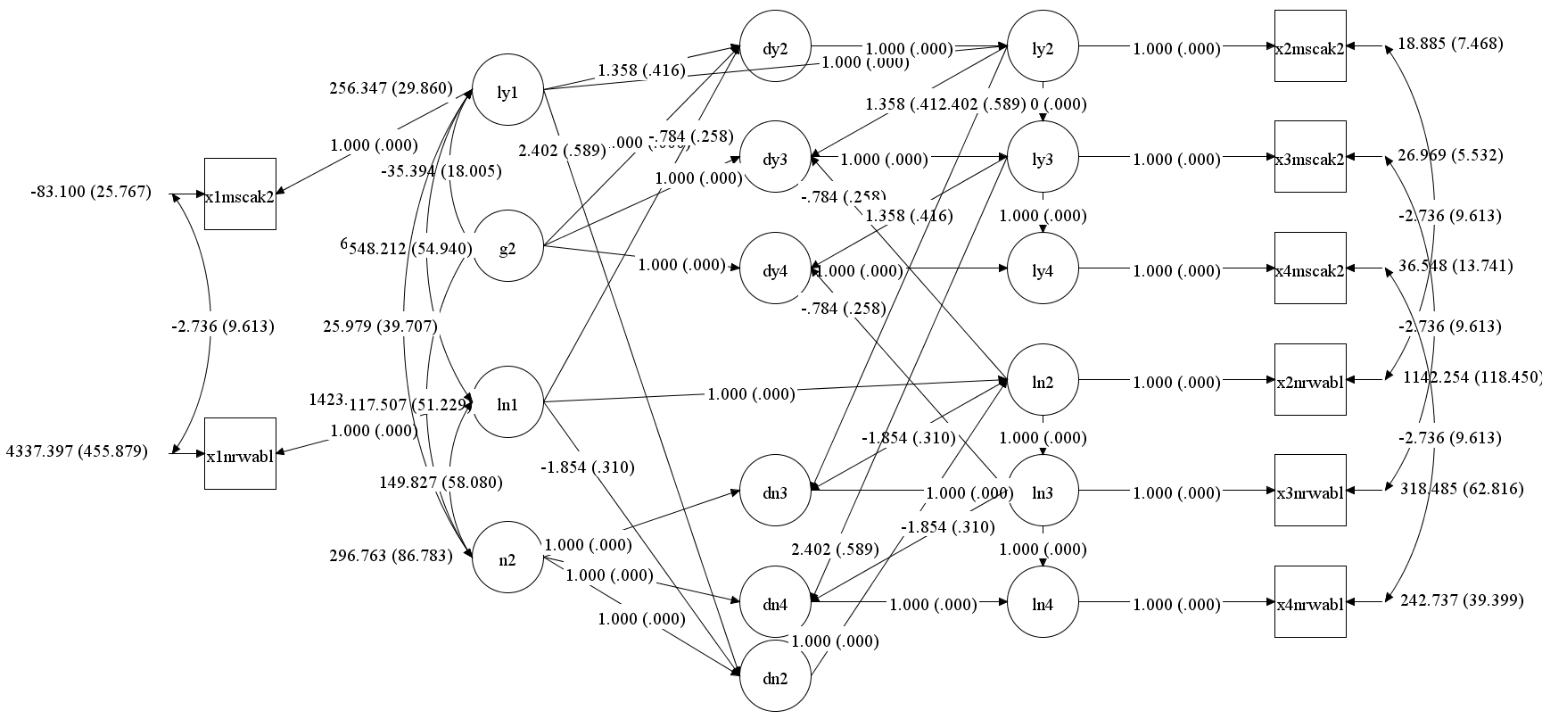

Figure 22. LCS bivariate dual change bidirectional coupling models – Number reverse & math with standardized estimates.

Table 16

Fit Statistics - LCS Bivariate Dual Change Bidirectional Coupling Models

Construct	Model	X^2	df	*p*	CFI	TLI	RMSEA	*p*	SRMR	AIC	BIC	ABIC
DCCS & MATH	DCCS & MATH	57.92	23	<.001	.961	.952	.078 (.053, .103)	.034	.223	9551.17	9625.29	9558.72
NR & MATH	NR & MATH	59.82	18	<.001	.964	.944	.096 (.070, .123)	.003	.319	13154.29	13246.06	13163.64

Chapter 4: Discussion

The purpose of this study was to explore the use of LCS as a more advanced tool for developmental research than LGC and to examine the growth trajectories and in particular, the unidirectional and bidirectional longitudinal relationship between math achievement and EF (cognitive flexibility and working memory) during kindergarten and 1st grade years. LGC and LCS modeling techniques were used to model the data to (1) investigate the relationship by component, (2) compare LGC and LCS models within a development framework, (3), explore LCS' complex co-development capabilities and identify leading and lagging indicators. Prior research has predominantly focused on the effect of EF on mathematics (Bindman, Pomerantz & Roisman, 2015; Blair, Ursache, Greenberg, Veron-Feagans, & The Family Life Project Investigators, 2015; Bull & Lee, 2014; Clark, Pritchard & Woodward, 2010; Friso-van den Bos, van der Ven, Kroesbergen, & van Luit, 2013; Shaul & Scwartz, 2014). This study explored a bidirectional relationship with the hypothesis that both the constructs develop within an ecosystem and not in isolation.

Findings by Research Questions

What are the patterns of growth and interrelationships in the development of executive function and math achievement?

The growth trajectories as explored by the three models (math achievement, DCCS, and number reverse) was linear. LGC and LCS models, which modeled univariate growth curves to estimate growth over time, revealed that for the constructs math achievement and number reverse there were significant inter-individual differences at the start but not in the growth rates, whereas there were differences at the start and in the slope of growth for DCCS. While the previous LGC and LCS models were able to model linear growth, the proportional change models' ability went further and investigated if growth was a function of performance at a prior level. The proportional model fits were compared with those of the LGC unconditional and LCS constant change models for the same construct and the better fitted model was used to interpret the data. The results highlighted that previous time-points successfully predicted the change score at certain levels for math achievement and DCCS, between spring and fall of 2011 and between fall 2010 and spring 2011, respectively. The change scores for number reverse was predicted by the previous time-points at each level. Therefore, growth was a function of the performance at the previous level, particularly for number reverse. Whereas students continuously built on number reverse foundations laid in prior stages, DCCS and math achievement saw students build on the foundations in the very early stages and this pattern did not continue in later grades. The LCS proportional change math achievement and DCCS models were better fitted to the data than their LGC unconditional and LCS constant change model counterparts suggesting their change is proportional while the LGC unconditional and LCS constant change models for number reverse had slightly better fit than the LCS proportional change, suggesting change is linear.

The dual change model for math achievement when compared to the constant change model had similar fit and when compared with the proportional change model, the proportional change had a slightly better fit suggesting growth in math is more constant (or linear) than proportional. This was the same for DCCS. However, the dual change number reverse model fit the data better than the constant and proportional change models, reflecting that the development of number reverse is both linear and proportional change with a greater acquisition of number reverse skills in the earlier time. The negative change parameters for the constructs indicated the rate of growth decreased yearly. Where math achievement was concerned, the highest performing students at the start were never caught by the lower performing students while the opposite obtained for DCCS and number reverse. The LCS dual change models are very useful in providing this type of information.

The univariate models discussed above indicated that each set of models played a different role. The LGC unconditional and LCS constant change models are useful to model linear growth only, LCS proportional change models are useful to model autoregression but not linearity, while dual change univariate models paired both the constant change and autoregression parameter in one model to determine if change is constant, proportional, or both. To determine if this was the case it necessitated all three LCS univariate models be specified to allow for comparison by construct and the model with best fit indicated the type of growth that the children were experiencing. The LGC univariate models were important in determining the growth trajectory before modeling patterns of association.

The LGC associative models with math achievement and DCCS and math achievement and number reverse performed poorly despite multiple re-specifications to obtain better fit. This seemed to suggest that associative models are not appropriate to use with this data. It is at this point also that we begin to clearly notice that number reverse performed differently compared to the other constructs, as the math and number reverse performed worse than the LGC associative models. Where the LCS bivariate dual change uncoupled models were concerned, the strong positive correlation between the math achievement intercept and slope in the LCS math achievement and DCCS bivariate dual change uncoupled model supported that the highest performing students in math and DCCS remained ahead of their lower performing counterparts. Again, the negative change score for both the variables indicated deceleration in growth for the subsequent levels. The LCS bivariate dual change uncoupled number reverse model's performance was very poor, even worse than the LGC associative math achievement and DCCS models, highlighting the need to look critically at the EF construct.

Is one construct a leading indicator of the other and are executive function and math achievement dynamically dependent?

The unidirectional models were not able to highlight leading indicators among the constructs but was able to reinforce that math achievement was influenced by the previous level and change decelerated with subsequent levels, which indicated that children with a higher initial math score would likely show more growth on math over time, and children with a higher initial DCCS score will likely show more growth on DCCS over time. However, in the bidirectional math achievement and DCCS coupled model changes were predicted by their previous state with math achievement changes accelerating compared to the deceleration in changes for DCCS. The coupling parameters revealed that there is clearly a bidirectional relationship between math achievement and DCCS where DCCS is a leading indicator of the changes in math achievement, but the changes decelerate with subsequent levels. The bidirectional math achievement and number reverse coupled model were also predicted by their previous state, that is math achievement was predicted by its previous state and number reverse was predicted by its previous state, with math achievement changes accelerating compared to the deceleration in changes for number reverse. There is also a bidirectional relationship between math achievement and number reverse with math achievement being the leading indicator of the changes in number reverse with changes accelerating with subsequent levels as indicated by the coupling parameter. This finding is consistent with the literature indicating that EF functions develop earlier than math skills. However, what it indicates is that once they both start to develop math achievement becomes a leading indicator over number reverse specifically.

While the DCCS and math achievement model results support the view that executive functioning skills in young children can predict their later math achievement (Bindman, Pomerantz & Roisman, 2015; Clark, Pritchard & Woodward, 2010), it highlighted that the EF components perform differently. Not only does DCCS and number reverse perform differently throughout model specifications, but the bidirectional relations are also different by construct. DCCS was a stronger construct throughout and culminated as the leading indicator in its relationship with math achievement. This supports previous studies that found that the EF components made distinct contributions (Huizinga, Dolan, and Van der Molen, 2006; Lehto, Juujarvi, Kooistra & Pilkkinen, 2003, Miyake et al., 2000).

Theoretical Implications

Investigations into the relationship between EF and learning outcomes primarily talk about the effect of "executive function." This use of the term may give the impression that that there exists a tool that measures EF as a construct in and of itself. There are different views on what specifically comprises EF with school psychology exposing a wider description of EF (Gioia, Isquith, Guy and Kenworthy, 2015) than education (Diamond 2002; Garon, et al, 2008; Rosso, Young, Femia, & Yurgelun-Todd, 2004). Likewise, there are different schools of thought as to whether EF is a single construct or comprises independent domains (Baddeley, 1992; Barkley et al, 2001;

Brocki & Bohlin, 2004; Dempster, 1992; Diamond 2002; Garon, et al, 2008; Isquith et al., 2004; Miyake et al., 2000; Rosso, Young, Femia, & Yurgelun-Todd, 2004). Despite these differences there does not exist an instrument that measures EF as a construct but rather instruments exist that measures the different components. This study has highlighted that these components are far from identical in their development and growth.

The dataset used in this study relied on two specific measures of EF (DCCS and NR). Both have a bidirectional relationship with math but only DCCS is a leading indicator of math achievement. Math achievement turned out to be a leading indicator of number reverse. Number reverse is purported to measure working memory and DCCS measures cognitive flexibility and both are components of the latent construct EF. However, there seems to be justification to stop uniformly referring to EF as if it is a single construct as demonstrated by the different performance in this study. Importantly, it seems to suggest that the constructs contribute differently to learning outcomes.

With this discovery, one must wonder what is happening with number reverse in kindergarten. It seems to depend on how familiar or fluent children are with their numbers or their exposure to numbers for them to be successful. Importantly, development theory suggested that between 2-7 years children are considered illogical thinkers and it is between 7 and 12 years that children begin to think logically (Nixon & Aldwinckle, 2003; Nixon & Gould, 1999). Most children who entered kindergarten for the first time, in this study were five years of age (87%) with a smaller set being 6 years or older (10). Based on the theory, a majority of these children are not yet at the place where they can be logical thinkers as noted by Piaget and Vygotsky in their theories of child development. The measures used in this study do require some logic for success and theory suggests that at this age the children are not equipped to use logic.

Practical implications

The results of this study indicated that the achievement gap persisted during the first two years of elementary school as poor performing students at the entry point did not close these gaps. Early educational interventions must be provided for such at risk students who may likely show early struggles as this may follow them throughout later grades. Math achievement as a leading indicator of number reverse suggest that instructional practice should include early math skills in a codevelopment framework as EF and math achievement codevelops in an ecosystem and the development of math skills will promote the development of EF. The literature states that the importance of EF to math achievement has been well established (Bull & Less, 2014). However, this study documents that the importance of math achievement to EF should not be understated or ignored. There needs to be a targeted approach toward students so that their development of math and EF skills can co-support each other. This new instructional practice which models learning within an ecosystem framework must not only include math achievement and the components of EF but must include other key and relevant developmental processes codeveloping much like math achievement and EF, such as language acquisition and development and motor skills development.

School curriculum needs to be assessed for age or development alignment. The discussion on child development theory and the widely accepted premise of child development should be used to guide curriculum development. In conjunction with child development theory, the findings of the current study suggest that curriculum should not only reflect the previous notion of a unidirectional relationship between EF and math achievement but also a bidirectional one. Additionally, curriculum should be influenced by the knowledge that the EF components perform differently and if they perform differently where leading and lagging indicators are identified then those leading indicators should be focused on in the curriculum as the drivers for the other indicators.

Methodological Implications

LGC and LCS models were applied to the same data in order to provide a comparison of capacities. They were used to determine the trajectory of growth and the longitudinal relationships among math and the two EF constructs. The literature indicated that LGC unconditional model is equivalent to the LCS constant change model. This study was able to demonstrate that the models were indeed equivalent and had identical estimates in some cases.

Three LCS proportional change models were estimated and fit the data better than the LGC unconditional and LCS constant change models for math achievement and DCCS with LCS proportional change number reverse model performing the worst as the LGC unconditional and LCS constant change models had better fit. However, only the LCS dual change number reverse model had better fit than the LCS proportional change model. The LGC associative models performed poorly compared to the LCS bivariate dual change uncoupled models and did not fit the data. Of the two LCS bivariate dual change uncoupled models, math achievement and DCCS fit the data better than math achievement and number reverse once more highlighting that the number reverse construct performed poorly compared to its DCCS counterpart, perhaps implying underlying issues with the construct.

Four bivariate dual change unidirectional coupled models were specified with DCCS affecting changes in math achievement fitting the data better followed by math achievement affecting changes in DCCS and lastly math achievement to changes in number reverse. In these models the relationship sequence is determined by the presence of a significant path from the one variable to the regressor. Again, the number reverse models performed poorer than the DCCS models with number reverse to changes in math achievement not fitting the data. DCCS to changes in math achievement, math achievement to changes in DCCS, and math achievement to changes in number reverse all performed better than the LCS bivariate dual change uncoupled models. These models were used to determine if there were associations between the variables and if one affects the other, but this model does not have the ability to determine if there is a dynamic relationship between the two constructs as only one parameter was allowed to be estimated at a time. The specification of the unidirectional model was important to later provide comparison with the full coupled model to determine the nature of the relationship, unidirectional or bidirectional. Finally, the DCCS and math achievement bidirectional coupled model fit the data better than the math achievement and number reverse model. More importantly, the DCCS and math achievement bidirectional coupled model fit the data better than the unidirectional models as well as they were not able to make any determination. The DCCS and math achievement bidirectional coupled model was an improvement over the unidirectional models (DCCS to changes in math achievement and math achievement to changes in DCCS models). However, the math achievement and number reverse bidirectional coupled model had a slightly higher ABIC than the math achievement to changes in number reverse unidirectional model, but the former model was able to model the relationships where the latter did not.

LGC models more readily fit the data and did not require much modifications with the exception of the associative models which never fit the data even when estimates from the unconditional models were used to specify the models. The LGC unconditional model was initially specified with fixed error variances but were later allowed to be different across time for better fit as well as the low correlations between DCCS and number reverse across times also justify relaxing this constraint. The LCS constant change, proportional change, dual change, and bivariate dual change models also did not require significant modifications except estimating another path where necessary to achieve good fit.

Additional methodological observations included that as the LCS models grew more complex they required more modifications and iterations. While LGC models required limited iterations the complex LCS models had to be increased to 5000 iterations to allow for convergence. This was particularly true for models that included the number reverse construct, again highlighting that the number reverse variable would in some instances perform differently than DCCS. The number reverse to changes in math achievement did not fit the data despite multiple modifications suggesting there is a ceiling effect. Several modification indices were provided with the outputs, but many did not meet theoretical justifications. Where theoretically sound modifications were specified, and convergence achieved there were instances where correlations exceeded 1 and respecification did not fix this issue.

Limitations and Directions for Future Research

This study has made significant methodological and substantive contributions to the literature, but it is not without its limitations. LCS requires time invariance in the change parameters which caused convergence issues. Model re-specifications and multiple iterations helped to achieve convergence but, in the end, not all models fit the data. In some cases, as a result of the time invariance requirement, the model converged with correlation exceeded one (Joreskog, 1999). Additionally, there are concerns that the modifications made to accomplish estimation and fit may result in bias estimation. Although it has been shown that despite any bias that may occur, LCS models can still capture change and estimate growth trajectories (Clark, Nuttall, & Bowles, 2018). However, there still needs to be a thorough investigation of these bias effects on real world data. Literature acknowledges that convergence issues due to LCS models' complexity can be further exacerbated by small sample size. This study had some missingness however the use of FIML was suitable and even with missing data the sample remained large. Therefore, there needs to be more investigation with real world data to fully understand how the convergence issues experienced in this study will behave in small datasets like those used in school psychology.

Another limitation of this study is that it did not address the need to investigate EF across a wider age group span than is often found in the literature. The ECLS-K 2011 dataset used, follows children between kindergarten and elementary school. Additionally, the measures used in this study are those used with the ECLS-K 2011 dataset and therefore limited the examination of a wider and more comprehensive definition of EF. This is particularly important as the results of the current study indicates that EF does not affect math as a single construct but that its components behaved differently. Number reverse needs further examination as a measure of EF as well as how it will relate and perform with a different measure. Therefore, EF measurement quality needs to be examined particularly with using DCCS and number reverse only. Given the two constructs' distinctly different functioning in the models, there's a need for a more holistic tool to measure EF for this age group.

One of this study's key limitation is the time-points used. Due to the change in the testing environment grades 2 and later years could not be included in the analysis. The literature could benefit from the replication of the current study across a wider time span. Therefore, future research must find a psychometric way to make the data before and after the second-grade equivalent to foster the modeling of the data beyond early elementary years.

Finally, future research must examine these variables in greater depth. With the findings of this research concerning the math achievement and EF relationships further work should examine how it might apply to language and motor skills development. Using the BRIEF scale from school of Psychology as a possible measure should prove useful as these relationships are explored further. Future work must explore learning and curriculum development within a codevelopmental ecosystem framework.

The current study provided a comparison between LGC and LCS models to assess their abilities. It went beyond the correlational and experimental studies and used longitudinal data with a large nationally representative sample to investigate growth. In light of the existing literature, this study went further and used appropriate advanced quantitative analysis to investigate dynamic relationships; examined bidirectional relationships between the constructs and identified DCCS to be a leading indicator and math to be lagging in the DCCS and math achievement relationship and math achievement as a leading indicator and number reverse as lagging in the number reverse and math achievement relationship. Importantly, it showed LCS to be more robust at modeling the data than LGC. Additionally, it examined the EF components and was able to add to the literature on how differently the EF measures performed.

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Appendix

Parameter Estimates

Table 17

Parameter Estimates – Math Achievement LGC Unconditional

	LGC Math	S.E.
Means		
-Intercept	31.33***	.851
-Slope	13.44***	.582
Variance		
-Intercept	120.19***	14.42
-Slope	2.41	2.76
E1	54.37***	7.57
E2	18.92***	3.49
E3	21.06***	5.44
E4	52.16***	9.54
Correlation	1.27	.860
Covariance	21.52***	4.39
R^2		
- T1	.689***	.044
- T2	.898***	.019
- T3	.903***	.903
- T4	.823***	.823

Table 18

Parameter Estimates – DCCS LGC Unconditional

	LGC DCCS	S.E.
Means		
-Intercept	14.03***	.239
-Slope	.623***	.094
Variance		
-Intercept	3.54*	1.58
-Slope	.178	.350
E1	20.99***	2.53
E2	7.96***	1.02
E3	9.01***	1.55
E4	3.73*	1.44
Correlation	-0.472	.357
Covariance	-0.375	.601
R^2		
- T1	.144*	.063
- T2	.271***	.068
- T3	.234***	.055
- T4	.437*	.213

Table 19

Parameter Estimates – Number Reverse LGC Unconditional

	LGC NR	S.E.
Means		
-Intercept	432.69***	3.72
-Slope	11.62***	.985
Variance		
-Intercept	1360.22***	292.43
-Slope	16.93	36.89
E1	5174.77***	555.45
E2	755.45***	130.37
E3	198.08*	64.13
E4	263.98*	94.22
Correlation	-1.019	0.695
Covariance	-154.64	.082
R^2		
- T1	.208***	.042
- T2	.586***	.069
- T3	.803***	.055
- T4	.689***	.111

Table 20

Parameter Estimates – Math Achievement LCS Constant Change Models

	LCS Math	S.E.
Means		
-Intercept	31.61***	.819
-Slope	13.09***	.464
Variance		
-Intercept	124.73***	14.13
-Slope	2.00	1.69
E1	52.97***	6.82
E2	19.81***	3.43
E3	19.90***	5.35
E4	50.23***	9.01
Correlation	1.11*	.554
Covariance	17.57***	3.26
R^2		
- T1	.702***	.039
- T2	.891***	.020
- T3	.911***	.024
- T4	.832***	.032

Table 21

Parameter Estimates – DCCS LCS Constant Change Models

	LCS DCCS	S.E.
Means		
-Intercept	14.03***	.239
-Slope	.623***	.094
Variance		
-Intercept	3.54*	1.58
-Slope	.178	.350
E1	20.99***	2.53
E2	7.96***	1.02
E3	9.01***	1.55
E4	3.73*	1.44
Correlation	-0.472	.357
Covariance	-0.375	.601
R^2		
- T1	.144*	.063
- T2	.271***	.068
- T3	.234***	.055
- T4	.437*	.213

Table 22

Parameter Estimates – Number Reverse LCS Constant Change Models

	LCS NR	S.E.
Means		
-Intercept	432.69***	3.27
-Slope	11.62***	.985
Variance		
-Intercept	1360.22***	292.43
-Slope	16.93	36.89
E1	5174.77***	555.45
E2	755.45***	130.37
E3	198.08*	64.13
E4	263.98*	94.22
Correlation	-1.02	.695
Covariance	-154.64	88.92
R^2		
- T1	.208***	.042
- T2	.586***	.069
- T3	.803 ***	.055
- T4	.689*	.111

Table 23

Parameter Estimates – Math Achievement LCS Proportional Change Models

	LCS Math	E.S.
Means		
-Intercept	31.30*** (LY1)	.866
Variance		
-Intercept	129.27***	16.35
Proportional		
dy2 → ly1	.108	.056
dy3 → ly2	.176***	.014
dy4 → ly3	.051	.043
E1	51.89***	5.91
E2	20.33***	3.41
E3	17.01*	5.19
E4	49.21***	6.7
R^2		
- T1	.714***	.037
- T2	.886***	.020
- T3	.928***	.022
- T4	.831***	.026

Table 24

Parameter Estimates – DCCS LCS Proportional Change Models

	LCS DCCS	S.E.
Means		
-Intercept	13.6*** (LD1)	.327
Variance		
-Intercept	2.14***	.467
Proportional		
dd2 → ld1	.094*	.029
dd3 → ld2	.028	.026
dd4 → ld3	.036	.024
E1	21.98***	2.19
E2	8.22***	.927
E3	9.04***	1.56
E4	3.84***	.628
R^2		
- T1	.089***	.020
- T2	.237***	.048
- T3	.230***	.051
- T4	.430***	.078

Table 25

Parameter Estimates – Number Reverse LCS Proportional Change Models

	LCS NR	S.E.
Means		
-Intercept	421.4*** (LN1)	5.12
Variance		
-Intercept	623.77***	86.84
Proportional		
dn2 → ln1	.060***	.013
dn3 → ln2	.024***	.006
dn4 → ln3	.021***	.005
E1	5307.19***	517.07
E2	938.56***	124.84
E3	289.31***	70.18
E4	135.26*	47.21
R^2		
- T1	.105***	.016
- T2	.428***	.055
- T3	.717***	.056
- T4	.850***	.050

Table 26

Parameter Estimates – Math Achievement LCS Dual Change Models

	LCS Math	E.S.
Means		
-Intercept	31.29***	.854
-Slope	15.17***	1.71
Variance		
-Intercept	122.77***	14.19
-Slope	4.52	2.97
Proportional		
dy2 → ly1	-0.053	.042
dy3 → ly2	-0.053	.042
dy4 → ly3	-0.053	.042
E1	53.345***	6.92
E2	19.66***	3.41
E3	19.47***	5.29
E4	50.58***	8.77
Correlation LY1 with G2	1.07***	.266
Covariance LY1 with G2	25.23***	7.02
R^2		
- T1	.697***	.040
- T2	.892***	.019
- T3	.913***	.023
- T4	.830***	.031

Table 27

Parameter Estimates – DCCS LCS Dual Change Models

	LCS DCCS	E.S.
Means		
-Intercept	13.62***	.331
-Slope	7.6*	2.59
Variance		
-Intercept	3.97	2.42
-Slope	.576	.491
Proportional		
dd2 → ld1	-0.468*	.173
dd3 → ld2	-0.468*	.173
dd4 → ld3	-0.468*	.173
E1	20.41***	3.06
E2	7.99***	.964
E3	9.09***	1.57
E4	4.05***	.992
Correlation Ld1 with d2	0.712	.638
Covariance Ld1 with d2	1.08	.795
R^2		
- T1	.163	.098
- T2	.262***	.057
- T3	.223***	.059
- T4	.388*	.141

Table 28

Parameter Estimates – Number Reverse LCS Dual Change Models

	LCS NR	E.S.
Means		
-Intercept	425.09***	5.34
-Slope	159.26***	57.99
Variance		
-Intercept	1458.29***	371.06
-Slope	49.8	62.43
Proportional		
dn2 → ln1	-0.328*	.129
dn3 → ln2	-0.328*	.129
dn4 → ln3	-0.328*	.129
E1	4972.98***	578.69
E2	788.83***	127.98
E3	227.0*	66.87
E4	216.6*	78.25
Correlation Ln1 with n2	.803	.467
Covariance Ln1 with n2	216.45	159.98
R^2		
- T1	.227***	.054
- T2	.559***	.068
- T3	.771***	.057
- T4	.746***	.072

Table 29

Parameter Estimates – Math Achievement and DCCS LCS Bivariate Dual Change Models

	LCS Math & DCCS		S.E.	
Means				
-Intercept	32.15*** (LY1)	13.79*** (LD1)	.867	.325
-Slope	14.71*** (G2)	6.31* (D2)	2.05	2.89
Variance				
-Intercept	118.67***	5.48*	14.2	2.19
-Slope	7.53*	.695*	3.79	.358
Math Proportional effects				
dy2 → ly1	-0.086		.045	
dy3 → ly2	-0.086		.045	
dy4 → ly3	-0.086		.045	
DCCS Proportional effects				
dd2 → ld1	-0.383*		.194	
dd3 → ld2	-0.383*		.194	
dd4 → ld3	-0.383*		.194	
Errors				
E1 - Math	50.09***		7.23	
E2 - Math	20.9***		4.04	
E3 - Math	25.08***		6.47	
E4 - Math	51.61***		9.91	
E1 - DCCS	19.14***		2.69	
E2 - DCCS	8.05***		.906	
E3 - DCCS	8.89***		1.49	
E4 - DCCS	3.66***		1.03	
Correlation				
ly1 with g2 (math constant change)	.811***		.172	
ly1 with ld1	.782***		.148	
ly1 with d2	.263		.320	
ld1 with d2 (DCCS constant change)	-0.092		.383	
d2 with g2	.909***		.226	
Math T1 with DCCS T1	.404***		.084	
Math T2 with DCCS T2	.272*		.096	
Math T3 with DCCS T3	.042		.140	
Math T4 with DCCS T4	.092		.148	

Covariance		
ly1 with g2 (math constant change)	24.24***	6.71
ly1 with ld1	19.94***	4.29
ly1 with d2	2.39	3.14
ld1 with d2 (DCCS constant change)	-0.180	.754
d2 with g2	2.08*	.813
Math T1 with DCCS T1	12.51***	3.31
Math T2 with DCCS T2	3.53*	1.43
Math T3 with DCCS T3	.630	2.10
Math T4 with DCCS T4	1.27	2.11
R^2		
- Math T1	.703***	.043
- Math T2	.878***	.023
- Math T3	.882***	.029
- Math T4	.815***	.036
- DCCS T1	.223*	.085
- DCCS T2	.241***	.057
- DCCS T3	.212***	.055
- DCCS T4	.442*	.151

Table 30

Parameter Estimates – Math Achievement and Number Reverse LCS Bivariate Dual Change Models

	LCS Math & NR		S.E.	
Means				
-Intercept	31.99*** (LY1)	426.77*** (LN1)	.877	5.32
-Slope	10.86*** (G2)	201.60* (N2)	1.78	95.89
Variance				
-Intercept	134.88***	1558.69***	14.85	410.29
-Slope	2.26	100.99	2.01	96.02
Math Proportional Effects				
dy2 → ly1	0.056		.043	
dy3 → ly2	0.056		.043	
dy4 → ly3	0.056		.043	
NR Proportional Effects				
dn2 → ln1	-0.427*		.211	
dn3 → ln2	-0.427*		.211	
dn4 → ln3	-0.427*		.211	
Errors				
E1 - Math	49.78***		6.81	
E2 - Math	20.39***		3.51	
E3 - Math	21.34***		5.50	
E4 - Math	42.64***		8.95	
E1 - NR	4754.67***		575.34	
E2 - NR	947.96***		131.91	
E3 - NR	267.49***		56.14	
E4 - NR	190.23*		58.14	
Correlation				
Ly1 with g2 (math constant change)	.275		.298	
Ly1 with ln1	.822***		.082	
Ly1 with n2	.614*		.237	
Ln1 with n2 (NR constant change)	.438		.348	
n2 with g2	.730*		.328	
Math T1 with NR T1	.684***		.062	
Math T2 with NR T2	.040		.110	

Math T3 with NR T3	-0.183	.180
Math T4 with NR T4	.148	.168
Covariance		
Ly1 with g2 (math constant change)	4.78	5.69
Ly1 with ln1	376.87***	59.87
Ly1 with n2	71.62	58.29
Ln1 with n2 (NR constant change)	173.76	203.56
n2 with g2	10.99	9.23
Math T1 with NR T1	332.58***	53.65
Math T2 with NR T2	5.52	15.59
Math T3 with NR T3	-13.82	13.16
Math T4 with NR T4	13.29	16.02
R^2		
- Math T1	.730***	.036
- Math T2	.889***	.019
- Math T3	.903***	.024
- Math T4	.852***	.032
- DCCS T1	.247***	.061
- DCCS T2	.461***	.070
- DCCS T3	.691***	.055
- DCCS T4	.741***	.079

Table 31

Parameter Estimates - DCCS Changes in Math LCS Bivariate Dual Change Unidirectional Coupling Models

	DCCS Changes in Math		S.E.	
Means				
-Intercept	32.13*** (LY1)	14.24*** (LD1)	.823	.265
-Slope	-18.48 (G2)	-5.28 (D2)	18.94	7.51
Variance				
-Intercept	110.33***	2.64	15.06	1.52
-Slope	.837	.542	5.53	1.27
Math Proportional Effects				
dy2 → ly1	-0.215*		.086	
dy3 → ly2	-0.215*		.086	
dy4 → ly3	-0.215*		.086	
DCCS Proportional Effects				
dd2 → ld1	.398		.509	
dd3→ ld2	.398		.509	
dd4 → ld3	.398		.509	
Coupling: DCCS changes in MATH				
dy2 → ld1	2.64		1.49	
dy3 → ld2	2.64		1.49	
dy4 → ld3	2.64		1.49	
Errors				
E1 - Math	58.61***		7.98	
E2 - Math	18.03***		4.09	
E3 - Math	25.12***		6.67	
E4 - Math	52.91***		10.71	
E1 - DCCS	21.51***		2.38	
E2 - DCCS	8.39***		1.09	
E3 - DCCS	9.06***		1.52	
E4 - DCCS	3.59*		1.69	
Correlation				
ly1 with g2 (math constant change)	.906		2.09	
ly1 with ld1	.931***		.211	
ly1 with d2	-0.966		.518	
ld1 with d2 (DCCS constant change)	-0.946***		.155	
d2 with g2	-0.186		1.56	
Math T1 with DCCS T1	.500***		.074	
Math T2 with DCCS T2	.293*		.116	
Math T3 with DCCS T3	-0.042		.141	

Math T4 with DCCS T4	.061	.228
Covariance		
ly1 with g2 (math constant change)	8.71	12.07
ly1 with ld1	15.89***	3.25
ly1 with d2	-7.47	7.92
ld1 with d2 (DCCS constant change)	-1.131	1.56
d2 with g2	-0.125	.709
Math T1 with DCCS T1	-17.75***	3.19
Math T2 with DCCS T2	3.60*	1.67
Math T3 with DCCS T3	-0.631	2.11
Math T4 with DCCS T4	.835	3.33
R^2		
- Math T1	.653***	.051
- Math T2	.902***	.022
- Math T3	.892***	.028
- Math T4	.817***	.039
- DCCS T1	.109	.062
- DCCS T2	.232*	.081
- DCCS T3	.222***	.055
- DCCS T4	.460	.040

Table 32

Parameter Estimates - Math Changes in DCCS LCS Bivariate Dual Change Unidirectional Coupling Models

	Math Changes in DCCS		S.E.	
Means				
-Intercept	32.13*** (LY1)	13.85*** (LD1)	.865	.339
-Slope	14.82*** (G2)	8.42* (D2)	2.03	3.49
Variance				
-Intercept	118.70***	6.17*	14.23	3.03
-Slope	7.48*	.756	3.77	.412
Math Proportional Effects				
dy2 → ly1	-0.089*		.045	
dy3 → ly2	-0.089*		.045	
dy4 → ly3	-0.089*		.045	
DCCS Proportional Effects				
dd2 → ld1	-0.583*		.287	
dd3 → ld2	-0.583*		.287	
dd4 → ld3	-0.583*		.287	
Coupling: MATH changes in DCCS				
dd2 → ly1	0.019		.022	
dd3 → ly2	0.019		.022	
dd4 → ly3	0.019		.022	
Errors				
E1 - Math	50.06***		7.13	
E2 - Math	20.84***		4.04	
E3 - Math	24.93***		6.45	
E4 - Math	52.82***		10.11	
E1 - DCCS	18.64***		3.33	
E2 - DCCS	8.10***		.879	
E3 - DCCS	8.94***		1.50	
E4 - DCCS	3.94***		.930	
Correlation				
ly1 with g2 (math constant change)	.841***		.180	
ly1 with ld1	.776***		.179	
ly1 with d2	.299		.274	
ld1 with d2 (DCCS constant change)	.040		.377	
d2 with g2	.820**		.252	
Math T1 with DCCS T1	.388***		.091	
Math T2 with DCCS T2	.287**		.096	

Math T3 with DCCS T3	.023	.143
Math T4 with DCCS T4	.113	.140
Covariance		
ly1 with g2 (math constant change)	25.05***	6.84
ly1 with ld1	21.01***	4.78
ly1 with d2	2.83	2.93
ld1 with d2 (DCCS constant change)	.085	.815
d2 with g2	1.95*	.836
Math T1 with DCCS T1	11.86**	3.46
Math T2 with DCCS T2	3.73**	1.44
Math T3 with DCCS T3	.344	2.14
Math T4 with DCCS T4	1.64	2.13
R^2		
- Math T1	.703***	.043
- Math T2	.879***	.023
- Math T3	.883***	.029
- Math T4	.812***	.036
- DCCS T1	.249*	.118
- DCCS T2	.226***	.056
- DCCS T3	.206***	.056
- DCCS T4	.405**	.129

Table 33

Parameter Estimates - Math Changes in Number Reverse LCS Bivariate Dual Change Unidirectional Coupling Models

	Math Changes in Number Reverse		S.E.	
Means				
-Intercept	32.04*** (LY1)	427.34*** (LN1)	.861	5.08
-Slope	10.79*** (G2)	121.11 (N2)	1.54	125.94
Variance				
-Intercept	133.39***	1417.22***	14.47	379.79
-Slope	2.57	70.13	1.87	36.82
MATH Proportional Effects				
dy2 → ly1	.057		.037	
dy3 → ly2	.057		.037	
dy4 → ly3	.057		.037	
NR Proportional Effects				
dn2 → ln1	-0.234		.326	
dn3 → ln2	-0.234		.326	
dn4 → ln3	-0.234		.326	
Coupling: NR changes in MATH				
dn2 → ly1	-0.096		.505	
dn3 → ly2	-0.096		.505	
dn4 → ly3	-0.096		.505	
Errors				
E1 - Math	50.90***		6.74	
E2 - Math	19.78***		3.35	
E3 - Math	23.43***		5.49	
E4 - Math	41.03***		8.93	
E1 - NR	4848.80***		542.60	
E2 - NR	911.94***		134.64	
E3 - NR	278.80***		58.62	
E4 - NR	148.90*		61.58	
Correlation				
ly1 with g2 (math constant change)	.281		.266	
ly1 with ln1	.824***		.072	
ly1 with n2	.318		.363	
ln1 with n2 (NR constant change)	.030		.539	
n2 with g2	.912**		.312	
Math T1 with NR T1	.685***		.058	
Math T2 with NR T2	-0.010		.059	
Math T3 with NR T3	-0.017		.098	
Math T4 with NR T4	-0.017		.101	
Covariance				

ly1 with g2 (math constant change)	4.97	4.79
ly1 with ln1	358.24***	61.86
ly1 with n2	30.79	40.31
ln1 with n2 (NR constant change)	9.59	171.21
n2 with g2	11.72*	5.91
Math T1 with NR T1	340.55***	51.78
Math T2 with NR T2	-1.35	7.87
Math T3 with NR T3	-1.35	7.87
Math T4 with NR T4	-1.35	7.87
R^2		
- Math T1	.724***	.036
- Math T2	.891***	.019
- Math T3	.895***	.023
- Math T4	.858***	.032
- NR T1	.226***	.055
- NR T2	.485***	.072
- NR T3	.695***	.058
- NR T4	.795***	.085

Table 34

Parameter Estimates - DCCS & Math LCS Bivariate Dual Change Coupling Models

	DCCS & Math		S.E.	
Means				
-Intercept	36.77*** (LY1)	13.60*** (LD1)	1.19	.327
-Slope	22.19*** (G2)	14.09*** (D2)	3.57	1.02
Variance				
-Intercept	201.37***	17.56***	21.80	2.40
-Slope	16.30	1.51*	9.56	.674
Math Proportional Effects				
dy2 → ly1	0.484***		.132	
dy3 → ly2	0.484***		.132	
dy4 → ly3	0.484***		.132	
DCCS Proportional Effects				
dd2 → ld1	-1.09***		.093	
dd3 → ld2	-1.09***		.093	
dd4 → ld3	-1.09***		.093	
Coupling: MATH changes in DCCS				
dd2 → ly1	.061**		.020	
dd3 → ly2	.061**		.020	
dd4 → ly3	.061**		.020	
Coupling: DCCS changes in MATH				
dy2 → ld1	-2.36***		.532	
dy3 → ld2	-2.36***		.532	
dy4 → ld3	-2.36***		.532	
Errors				
E1 - Math	25.17***		2.67	
E2 - Math	25.17***		2.67	
E3 - Math	25.17***		2.67	
E4 - Math	25.17***		2.67	
E1 - DCCS	6.82***		.589	
E2 - DCCS	6.82***		.589	
E3 - DCCS	6.82***		.589	
E4 - DCCS	6.82***		.589	
Correlation				
ly1 with g2 (math constant change)	-0.536***		.151	
ly1 with ld1	.723***		.047	
ly1 with d2	.385*		.179	
ld1 with d2 (DCCS constant change)	.357*		.151	
d2 with g2	.352		.209	
Math T1 with DCCS T1	.131*		.062	
Math T2 with DCCS T2	.131*		.062	

Math T3 with DCCS T3	.131*	.062
Math T4 with DCCS T4	.131*	.062
Covariance		
ly1 with g2 (math constant change)	-30.73*	14.69
ly1 with ld1	42.98***	5.89
ly1 with d2	6.71*	3.39
ld1 with d2 (DCCS constant change)	1.83*	.928
d2 with g2	1.75	1.33
Math T1 with DCCS T1	1.72*	.874
Math T2 with DCCS T2	1.72*	.874
Math T3 with DCCS T3	1.72*	.874
Math T4 with DCCS T4	1.72*	.874
R^2		
- Math T1	.854***	.021
- Math T2	.868***	.017
- Math T3	.888***	.014
- Math T4	.914***	.012
- DCCS T1	.720***	.036
- DCCS T2	.257***	.061
- DCCS T3	.269***	.061
- DCCS T4	.287***	.060

Table 35

Parameter Estimates - Number Reverse & Math LCS Bivariate Dual Change Coupling Models

	Number Reverse & Math		S.E.	
Means				
-Intercept	30.98*** (LY1)	416.58*** (LN1)	.826	3.97
-Slope	298.26** (G2)	730.33*** (N2)	96.73	111.79
Variance				
-Intercept	256.35***	1423.13***	29.86	251.35
-Slope	61.12	296.76**	34.62	86.78
MATH Proportional Effects				
dy2 → ly1	1.36**		.416	
dy3 → ly2	1.36**		.416	
dy4 → ly3	1.36**		.416	
NR Proportional Effects				
dn2 → ln1	-1.854***		.310	
dn3 → ln2	-1.854***		.310	
dn4 → ln3	-1.854***		.310	
Coupling: MATH changes in NR				
dn2 → ly1	2.40***		.589	
dn3 → ly2	2.40***		.589	
dn4 → ly3	2.40***		.589	
Coupling: NR changes in MATH				
dy2 → ln1	-0.784**		.258	
dy3 → ln2	-0.784**		.258	
dy4 → ln3	-0.784**		.258	
Errors				
E1 - Math	-83.10**		25.77	
E2 - Math	18.86*		7.47	
E3 - Math	26.97***		5.53	
E4 - Math	36.55**		13.74	
E1 - NR	4337.39***		455.88	
E2 - NR	1142.25***		118.45	
E3 - NR	318.49***		62.82	
E4 - NR	242.74***		39.39	
Correlation				
ly1 with g2 (math constant change)	-0.283***		.079	
ly1 with ln1	.908***		.029	
ly1 with n2	.094		.151	
ln1 with n2 (NR constant change)	.231**		.074	

n2 with g2	.873***	.078
Math T1 with NR T1	Fixed @1	
Math T2 with NR T2	-0.019	.067
Math T3 with NR T3	-0.030	.103
Math T4 with NR T4	-0.029	.105
Covariance		
ly1 with g2 (math constant change)	-35.39*	18.00
ly1 with ln1	548.21***	54.94
ly1 with n2	25.98	39.71
ln1 with n2 (NR constant change)	149.83**	58.08
n2 with g2	117.51*	51.23
Math T1 with NR T1	-2.74	9.61
Math T2 with NR T2	-2.74	9.61
Math T3 with NR T3	-2.74	9.61
Math T4 with NR T4	-2.74	9.61
R^2		
- Math T1	Undefined	
- Math T2	.899***	.041
- Math T3	.880***	.024
- Math T4	.878***	.046
- NR T1	.247***	.037
- NR T2	.275***	.040
- NR T3	.599***	.064
- NR T4	.706***	.045