Surface modeling topsoil distribution on a reclaimed coal-mine site at Blackmesa Mine Complex, Kayenta, Arizona

Janine Ferarese
Surface modeling topsoil distribution on a reclaimed coal-mine site at Black Mesa Mine Complex, Kayenta, Arizona

Janine Ferarese
University of Denver Department of Geography
Capstone Project
for
Master of Science in Geographic Information Science
October 31, 2011

_____________________
Steven R. Hick
Capstone Advisor and Academic Director
Upon the Recommendation of the Department

_____________________
Andrew Goetz, Ph.D.
Department Chair
Abstract

A necessary precursor to ensure proper vegetation growth on reclaimed coal-mine areas is even distribution of topsoil. This thesis discusses development of surface models to describe the accurate determination and visualization of the distribution of topsoil. Prediction surfaces from topsoil-depth point samples were created using the surface interpolation methods, Inverse Distance Weighted and Kriging. The validity and accuracy of each method was assessed to determine the best method to evaluate actual topsoil distribution at a coal mining site. The model can be utilized by non-spatially trained personnel working in the mining arena as an aid in assessing how appropriately the topsoil has been distributed over newly reclaimed mine areas.
Table of Contents

Abstract ......................................................................................................................... ii
Table of Contents ........................................................................................................ iii
List of Figures ................................................................................................................ iv
Introduction .................................................................................................................. 2
Background ................................................................................................................... 4
Thesis Statement .......................................................................................................... 7
Literature Review ......................................................................................................... 9
General Discussion on Spatial Modeling ...................................................................... 13
Methodology ................................................................................................................. 30
Results .......................................................................................................................... 45
Conclusion .................................................................................................................... 51
Future Opportunities .................................................................................................. 52
References ..................................................................................................................... 54
Appendix A – List of Acronyms ................................................................................... 56
Appendix B – IDW Model Parameters ........................................................................ 57
Appendix C – Kriging Model Parameters .................................................................... 58
List of Figures

Figure 1. Reclamation Specialists Monitoring Topsoil-Depth. Courtesy OSM. 8
Figure 2. The Normal Distribution. Graphic from Wikipedia (Mhtoews) …… 17
Figure 3. QQ Plot. Graphic from Wikipedia (Sbkbedas)………………………… 19
Figure 4. Boxplot To Normal Probability Function. Graphic from Wikipedia (Jhuch)………………………………………………………….. 20
Figure 5. Voronoi Cluster Map Example. ................................................…… 23
Figure 6. Voronoi Entropy Map Example. ................................................… 24
Figure 7. Polynomial Trend Example ........................................................... 25
Figure 8. General Semivariogram. Graphic from Wikipedia …………………… 28
Figure 9. Black Mesa Mine Complex Area of Interest Map. …………………….. 31
Figure 10. Spatial Statistics of Sampled Points. ……………………………… 33
Figure 11. Geostatistical Modeling Summary Flowchart. ……………………… 34
Figure 12. Boxplot of Topsoil-depth. ............................................................ 35
Figure 13. Histogram and QQ Plot of Raw Data Values. ……………………… 36
Figure 14. Voronoi Cluster Map of Topsoil-depth. ....................................... 37
Figure 15. Examination of Potential Outlier. ………………………………….. 38
Figure 16. Rational for Not Excluding Potential Outlier. ……………………… 39
Figure 17. Voronoi Entropy Map of Topsoil-depth. …………………………… 40
Figure 18. Trend Analysis of topsoil-depth. …………………………………….. 41
Figure 19. Voronoi Standard Deviation Map of Topsoil-depth. ……………… 42
Figure 20. Semivariogram of Topsoil-depth. …………………………………… 44
Figure 21. IDW and Simple Kriging Prediction Surfaces. ……………………… 46
Figure 22. Standard Error of Prediction Surface for the Kriging Derived Prediction Surface. …………………………………………………… 47
Figure 23. Probability Surface of 24-inch Minimum Requirement. …………… 49
Figure 24. Cross-validation Statistics for IDW and Simple Kriging Comparison. ………………………………………………………………… 50
Introduction

Coal is a combustible rock that contains more than 50 percent by weight carbonaceous material. The precursor to coal is peat; the unconsolidated deposit of large amounts of plant remains which have accumulated in widespread wetland environments such as bogs and swamps. Over long periods of time (millions of years) deeply buried peat is transformed physically and chemically into coal via extreme heat and pressure from overlying sediments.

Coal contains an abundant amount of carbon and when carbon, a naturally occurring element in living matter, combines with hydrogen a compound called a hydrocarbon is produced. Such natural hydrocarbons, including coal, are referred to as fossil fuels because they originate from the accumulation and transformation of plants and in some cases, other organisms.

Fossil fuels are burned to produce heat which in turn can be used to produce electricity. Coal is the most abundant fossil fuel on earth and the most dominate fuel for producing electricity. In the United States, coal is the leading energy resource, accounting for almost one third of the country’s total energy production and more than 51% of the nation’s electrical power production. (Greb et al., 2006).

As our most abundant domestic source of energy, coal fills an essential part of the nation’s energy needs. Unfortunately, coal production is linked
with adverse environmental issues and concerns particularly related to
disturbance of hydrologic systems, ground subsidence, and post-mining land
use. In addition, mitigating the effects of past mining practices, and
increasing the health of and decreasing safety risks to the public pose
difficult challenges for government’s regulatory control.

Beginning in the 1740’s, commercial coal mining has been occurring
in the United States without regard to environmental consequences.
Prompted by major environmental impacts from coal mining during the
1960’s and 1970’s, the U.S. Congress enacted The Surface Mining Control
and Reclamation Act (SMCRA) in 1977. These federal laws and regulations
define minimum requirements for the performance of specific activities
during the mining operation and have set standards for environmental
protection that must be met. The SMCRA defined minimum standards to
ensure that the lands affected by coal mining operations are returned to
productive use.

The Office of Surface Mining Reclamation and Enforcement (OSM), a
bureau under the U.S. Department of Interior, is charged with carrying out
the requirements of SMCRA.

OSM carries out the requirements of SMCRA in cooperation with States
and Indian tribes. OSM’s objectives are to “ensure that coal mining activities
are conducted in a manner that protects citizens and the environment during
mining, to ensure that the land is restored to beneficial use after mining,”
and to mitigate the effects of past mining by aggressively pursuing reclamation of abandoned coal mines.”

Background

In adherence with the rules of SMCRA, prior to the start of coal mining activities, a permit must be obtained from the OSM. The permit application must describe several requirements: (1) the existing conditions at the proposed mine site, (2) the procedures and equipment to be used in the operation, (3) the potential environmental impacts resulting from the operation, (4) the measures to be taken to protect the environment, (5) the intended post-mining land use to which the area will be returned upon completion of mining, and (6) the specific techniques to be used to reclaim the area to the intended use. The process includes restoring the land to its approximate original appearance by restoring topsoil and planting native vegetation and ground covers. (U.S. Department of Interior, 2007).

Throughout the life of the mine (LOM), i.e. the timeframe when earth is first broken to begin extraction of coal through when final reclamation is completed, compliance inspections of the progress of reclamation and observance to regulations of SMCRA are made by the OSM either directly or via oversight of State or Tribal programs. At each step along the way the mine operator under the enforcement of OSM must comply with Federal and State laws and with SMCRA. The LOM can span decades, especially in the

1 OSM Mission Statement from http://www.osmre.gov/aboutus/Mission.htm
exceedingly large coal mine operations in the west, and many regulatory inspectors and other earth scientists and engineers work in concert over this time frame.

Reclamation occurs contemporaneously at the mine site in that reclamation activities will be underway in one area while coal removal continues nearby. To ensure compliance at each step, an inspection is conducted monthly for specific areas deemed problematic and one inspection per quarter to include the entire mine site. Upon completion of the inspection, a written report is produced for documentation purposes. The inspection report is an important document that may become a factor if legal action ensues.

Surface coal mining is more complex than simply uncovering the coal and removing it. Attention must be paid to environmental concerns, especially returning the mined land to productive use, and in actuality surface mining entails a sequence of activities.

Although the characteristics of each mine site, including the geology, hydrology, and topography will affect the mining method used, the following activities in sequential order are common to all surface coal mine operations: (1) erosion and sedimentation control, (2) road construction, (3) clearing and grubbing, (4) topsoil removal (salvage) and handling, (5) overburden removal and handling, (6) coal removal and handling, (7) reclamation.
The first steps in preparing an area for coal removal is to clear and grub. Clearing is the act of cutting and/or removing all trees and brush from an area. Grubbing is removing any remaining roots and stumps, low growing vegetation, and grass. These are necessary steps to facilitate topsoil salvage.

Topsoil is the uppermost layer of soil in which plant growth is best achieved. This layer must be removed and properly stored in order to enhance soil productivity, as it is desirable to re-lay this same topsoil during the reclamation phase. However, in areas where the topsoil is of poor quality or unavailable in sufficient quantities the permit may allow the use of topsoil substitutes in reclaiming the land. In either case, there are regulations addressing the redistribution of topsoil. One of which is that topsoil must be redistributed to a depth specified in the mining permit and in a manner that achieves an approximate, uniform thickness.

Sound science is the foundation for effectively implementing SM CRA and use of geospatial technology can be an invaluable tool in the application of SM CRA towards improving public safety and decreasing detriment to the environment.

For the greatest part the coal industry has not fully adopted the power of Geographic Information Systems (GIS). Reliance on antiquated mapping techniques is less than a best solution to reclamation. Many in the mining industry have adopted Computer Aided Design software, which is a step in
the right direction, but it is not without its difficulties. The solution is adoption of modern geospatial standards and techniques.” (Evans, 2008).

**Thesis Statement**

The aim of this thesis is to describe a process that will create a cell-based (raster\(^2\)) analysis prediction surface from sampled topsoil-depth point values to aid in determining how evenly a mine operator has spread the required minimum depth of topsoil over newly reclaimed areas on surface coal mines.

An important objective of the laws enforced by SMCRA is to ensure that mined lands are returned to productive use. The first precursor to this end is the necessity that topsoil be evenly distributed and to a specified depth over the area to be reclaimed. To ensure this requirement is met, OSM or the affiliated State or Tribal SMCRA regulatory body conducts compliance inspections of the area.

During a SMCRA inspection, Regulatory Specialists trained in the use of Global Positioning System (GPS) technology monitor the depth of topsoil that has been redistributed by collecting soil-depth data using a Topsoil Probe and portable GPS unit. (Figure 1).

Coordinates of sample locations along with the attribute depth of the topsoil at that location are downloaded from the GPS receiver and converted

\(^2\) A raster is a spatial image where the data is expressed as a matrix of cells or pixels, with spatial position implicit in the ordering of the pixels.
into a GIS point feature class or shapefile\textsuperscript{1}. The Reclamation Specialist visually reviews the shapefile onscreen in a GIS or creates a hard-copy graphic of the points with their respective topsoil depths annotated in order to determine areas where minimum topsoil depth has not been achieved or has been over-placed too thickly.

Figure 1. Reclamation Specialists Monitoring Topsoil-Depth. Courtesy OSM.

This operation has potential to be greatly enhanced to provide greater interaction of the data among mining professionals and expedite the timeliness, efficiency, and accuracy in decisions required to be made following inspection and production of the written inspection report.

SMCRA requires a minimum average top-soil depth evenly distributed over a reclaimed area. In the absence of more stringent analysis of the

\textsuperscript{1} A shapefile (referred to as a feature class when implemented in a geodatabase) is digital vector storage format for storing geometric location and associated attribute information.
overall distribution of depth the possibility exists that an area could be
described to be within requirements by simple virtue of the arithmetic average
of all points sampled over the entirety of the area being within that
minimum average. This does not reflect the spirit of the law and could
provide potential for a greater chance of improper vegetative growth
resulting in subpar reclamation.

It is possible to more accurately determine and visualize the
distribution between measured topsoil depth and the minimum depth called
for in the mining permit. Further, there is need to automate the process to
aid non-spatially trained mining personnel in making assessment.

Literature Review

Several studies related to the wide range of complex problems
associated with coal mine permitting and abandoned mine land problems are
introduced followed by reviews of studies examining surface modeling
applications.

The danger posed by old or poorly designed maps is very real; a
missing map or a map that is incomplete or in error can and has cost human
lives, damage to homes and property, and prove detrimental to the
environment. "A reclamation program needs a systematic, logical process
able to discriminate among many similar project contenders to meet the
needs of SM CRA and to be legally defensible." (Rohrer, et al, 2008).
One facet of the SMCRA rules define the requirement that a
Cumulative Hydrologic Impact Assessment (CHIA) be completed before
proposed coal mine permits may be accepted. The West Virginia University
(WVU), with support from OSM developed a suite of software tools to
support CHIA’s of proposed mine activities in West Virginia. Incorporating a
GIS interface, they added the Environmental Protection Agency’s (EPA)
watershed model, and Hydrologic Simulation Program-Fortran (HSPF) to
their own Watershed Characterization and Modeling System (WCMS) to
predict changes in water-quality and quantity caused by surface mining. The
model contains over 20 parameters and uses a joint calibration approach,
using historical stream flow records from five watersheds and four
verification watersheds throughout West Virginia. (Lamont et al, 2008).

The marriage of GIS and Remote Sensing is one not destined for
separation. Many GIS practitioners didn’t appreciate the contributions of
remote sensing to GIS in the past and its tremendous potential
contributions.” (Green, 2009). The Pennsylvania Department of
Environmental Protection (PA-DEP) and OSM cooperated on a project that
created 3-dimensional models of large anthracite open-pit mining operations
in Pennsylvania. (Anthracite is the highest ranking form of coal which
produces more heat per ton when burned because it is a more concentrated
form of carbon due to increased time and pressure during the alteration
from peat.)
It is necessary to accurately determine the volume of open pits that result from certain methods of surface coal mining. Accurate estimates of backfill material are needed to calculate bond liability to the mine operator. For very large affected areas such as the case in areas in east-central Pennsylvania, the use of remote sensing technology is critical to abate the cost and difficulties, and simple inability to accurately ground measure such large areas. Using color high-resolution aerial photographic imagery, the PA-DEP and OSM generated digital photogrammetric data with the goal of three-dimensionally modeling mine sites for volumetric calculations (Hill, 2004).

Analytical approaches to GIS have evolved over the few decades since its inception culminating in the science of spatial statistics and spatial analysis whereby defining geographic relationships leads to solutions that aid in modeling landscape diversity and pattern. Spatial statistics and analysis can augment traditional statistics by providing means to map variations in data by translating discrete point data into a continuous surface representing the geographic distribution of the data. Many studies have been done using geostatistical techniques that benefit from the use of interpolation techniques.

Spatial variability of soil physical properties was carried out using geostatistical analysis to produce productivity rating systems for use in precision farming. Rating maps of parameters were prepared as a series of colored contours using Kriging interpolation and semivariogram models to
support findings that good soil physical health is essential for optimum sustained crop production (Amirinejad 2011).

In a similar study (Liu, 2006), the relationship between soil organic carbon (SOC) and landscape aspects in a region of Northeast China was conducted which explored using geostatistical Kriging interpolation techniques to distinguish the spatial distribution pattern of SOC (Liu, 2006).

The estimation of spatial variability of precipitation has been shown to be crucial for accurate distributed hydrologic modeling (Zhang, 2009). The study by Zhang used a GIS system incorporating Inverse-Distance-Weighting (IDW) and Kriging as well as other methods to facilitate automatic spatial precipitation estimation. The study reports that spatial precipitation maps estimated by different interpolation methods have similar areal mean precipitation depth but significantly different values of maximum and minimum precipitation. Zhang suggests implementing multiple spatial interpolation methods and evaluating their estimates using a correlation coefficient.

This exemplifies the point that proper use and interpretation of created surfaces cannot go understated. A common problem in geographic data analysis is incorrectly creating and interpreting, or failing to interpret, resultant maps (Wingle, 1992). A study evaluating the accuracy of several interpolation techniques including IDW and Kriging found that irrespective of the surface area, few differences existed between the employed techniques.
if the sampling density was high, but performance tended to vary at lower sampling densities (Chaplot, et al., 1999). A handicap in representing surfaces is the existence of noise (Demirhan, 2003) wherein with environmental investigations noise can be introduced by sampling errors in the form of duplicate samples taken from the same location resulting in different observation values.

An interpolated surface can produce an appealing picture, but remain only that in the absence of proper construction and interpretation. It is important to be knowledgeable about the uncertainty of the predictions. Using statistical theory and the functionality of GIS software, the analysis, modeling, and interpretation of data with location coordinates can be performed using techniques termed spatial data analysis. Specifically, continuous data such as that representing topsoil-depth and location use a spatial analytical technique called geostatistics.

**General Discussion on Spatial Modeling**

Analysis of data precedes the modeling of it. Analysis must begin with exploration of the data so that insight is gained towards the proper application of the methods to be used in the modeling process.

The Environmental Systems Research Institute (ESRI©) ArcGIS™ Desktop extension, Geostatistical Analyst, and the Spatial Statistics toolbox can be used to aid in analyzing and modeling the distribution of topsoil in newly reclaimed areas on surface coal mine sites.
The ArcGIS™ Geostatistical Analyst extension is a collection of tools used for spatial data exploration and includes the ability to produce histograms, scatter plots, and semivariograms of the data. Geostatistical Analyst can be used in the identification of anomalies or outliers in the dataset as well as serving to help determine the optimal interpolation prediction method. Additionally, it can be used to create the interpolated prediction surface, including the prediction uncertainty.

The ArcGIS™ Spatial Statistics toolbox contains statistical tools for analyzing spatial distributions, patterns, processes, and relationships.

Products resulting from utilizing GIS tools are visually appealing and impressive, and it should be kept in mind that “it is easy to lose sight of something that everyone knows at some level, namely, that most datasets have errors both in attributes and in locations, and that processing data propagates errors.” (Krivoruchko, 2011) Further, the best way to be confident in the output from a statistical model is to understand the key ideas behind the model. (Krivoruchko, 2011)

The approach to applying statistical analysis techniques to continuous, terrestrial geographically based (i.e. location) data is known as geostatistical analysis or geostatistics. Geographic data do not usually conform to the requirements of standard statistical procedures due to spatial autocorrelation.
Spatial autocorrelation is the relationship of how a variable correlates with itself over space. It is the assumption, as stated by Tobler's First Law of Geography, that points closer together have smaller differences between their values than points farther apart from each other. Random patterns do not exhibit spatial autocorrelation so if there is any systematic pattern in the spatial distribution of a variable, it is said to be spatially autocorrelated; measuring the extent to which the occurrence of an event in one area unit constrains, or makes more probable, the occurrence of an event in a neighboring area unit. The term is usually applied to ordered datasets where the correlation depends on the distance and direction separating occurrences.

Consideration of spatial autocorrelation necessitates the application of geostatistics in modeling distributed topsoil-depth because it employs analytical techniques and methods using spatial software including geographic information systems. The results of geostatistical analysis typically are primarily derived from observation rather than experimentation but most advanced techniques require knowledge of the discipline they are to be applied to as well as professional judgment in their use and interpretation.

Prior to the start of any spatial analysis and before the actual modeling of a surface, it is advisable to visually inspect the data by examining its location and distribution to determine any spatial relationships. The mean or
center location can be determined by calculating the average \( x \)- and \( y \)-coordinates of the dataset. Because the average value can be affected by extreme values, comparison of the mean center with the middle, or median center location, determined by ranking the \( x \)- and \( y \)-coordinates in numeric order and choosing the middle value in the list, can yield information on whether some locations in the data are dispersed or tightly clustered. The directional distribution of the data lends information about the spread of the data based upon the standard deviation of the \( x \)- and \( y \)-coordinate locations.

Equally important is exploration of the data values themselves (in this case topsoil-depth) for investigation into any variation and trends inherent within the spatial data. Some geostatistical analysis tools assume the data is normally distributed and operate correctly only if it is the case.

The Normal distribution is a continuous probability distribution of a measurement, \( x \), that is subject to a large number of independent, random, additive errors. Mathematically it is the Gaussian function,

\[
f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}
\]

where, \( \mu \) is simultaneously the mean, median, and mode (i.e. the location of the center of the peak), \( \sigma^2 \) is called the variance and describes how concentrated the distribution is about the center. The square-root of the
variance is the standard deviation, \( \sigma \), which measures the width of the density function.

If the data exhibit characteristics of the Normal distribution, then graphically the distribution is represented by a bell-shaped curve where there is a clustering of values near the mean. There will be equal numbers of values on either side of the center of the curve. In Figure 2 each colored band has a width of one standard deviation. The numbers written as percentages represent the percent of the dataset accounted for. (e.g., 68.2% of the set have values within one standard deviation of the mean, 95.4% have values within two standard deviations, and 99.6% within three standard deviations.)

Figure 2. The Normal Distribution. Graphic from Wikipedia (Mwteows.)
If the frequency distribution of a dataset is normal the data will be symmetric about the mean and the data is said to have skewness, $\alpha_3$, of zero. When the mean and median are not similar in value, asymmetry occurs in the data and will be reflected in the distribution curve being skewed to the left for skewness values $\alpha_3 < 0$, and to the right for skewness values $\alpha_3 > 0$. Mathematically, the skewness, $\alpha_3$, may be calculated by:

$$\hat{\alpha}_3 = \frac{n}{(n - 1)(n - 2)} \sum_{i=1}^{n} (x_i - \bar{x})^3$$

where, $\sigma$-hat is standard deviation, $n$ is the number of sample values, $x_i$ is the $i^{th}$ value in the set, and $x$-bar is the mean of the sample values.

Additional data exploration methods available that allow inspection into the properties of datasets include the normal QQ plot. It graphically compares the distribution of a given variable to the Normal distribution. The QQ plot is a scatter plot where the quantiles of a dataset are plotted against the quantiles of the Normal distribution. Figure 3 illustrates that in the QQ plot, perfect normality is represented by a straight line and the actual data are plotted as points along the line. The closer the points are to the Normal line, the more the data has many of the characteristics of a normally distributed set of data. QQ plots are also helpful in determining outliers. The values at the tails of the QQ plot can potentially be outliers and can point to
the necessity to further investigate such points before continuing with an analysis.

![Figure 3. QQ Plot. Graphic from Wikipedia (Skbkedas.)](image)

Another quick and convenient descriptive statistical aid to graphically explore datasets such as topsoil-depth sample points is through use of the Boxplot, sometimes called the box-and-whisker diagram. A boxplot summarizes five descriptive statistical parameters: the minimum sample value, the lower quartile (Q1), the median (Q2), the upper quartile (Q3), and the maximum sample value. A boxplot generally can also indicate observations that may be outliers. Figure 4 illustrates the comparison between the boxplot and the Normal distribution. In Figure 4, IQR is the "interquartile range," being the mid-spread which contain 50% of the data.
and equal to the difference between the upper and lower quartiles. These first and third quartiles correspond to a cumulative proportion of 0.25 and 0.75 of the data points, respectively. By arranging the data in increasing order, 25% of the values will lie below the first quartile, and 25% of the values will lie above the third quarter. The quartiles are special cases of the general quantile that is calculated by

\[ \text{Quantile} = (i) - 0.5/n \]

where \( n \) is the number of data points and \( (i) \) is the \( i^{th} \) rank of the ordered dataset values.

Figure 4. Boxplot To Normal Probability Function. Graphic from Wikipedia (Jhguch.)
The "peaked-ness" of a frequency distribution is measured by a parameter known as kurtosis. Similar to the idea of skewness which is a measure of the symmetry of a distribution, kurtosis, $\alpha_4$, describes the shape of the frequency curve; more pointy distributions tend to have higher kurtosis values. Kurtosis serves to describe the size of the tails of a distribution, the measure of which can provide the likeliness that the distribution will produce outliers. Normal distributions will exhibit the value of 3 for the kurtosis measure. Mathematically, kurtosis can be calculated by

$$\hat{\alpha}_4 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^4}{\sigma^4} - 3$$

where, $\sigma$-hat is standard deviation, $n$ is the number of sample values, $x_i$ is the $i^{th}$ value in the set, $x$-bar is the mean of the sample values, and

$$\sigma = \sqrt{\frac{n(n+1)}{(n-1)(n-2)(n-3)} \cdot \frac{b - \frac{3(n-1)^2}{(n-2)(n-3)}}{n-3}}$$

It is important in the data exploration phase that it is determined how the data is distributed, because some Geostatistical Analyst interpolation tools assume the data is normally distributed. In instances where this is not the case it may be possible to apply transformations to the data that can
sometimes make the datasets more Normal. Data transforms used in Geostatistical Analyst includes the Box-Cox transformation that is a family of transforms defined for positive values and described by,

$$z = \frac{x^k - 1}{k}, k > 0, x > 0$$

where $k$ is an integer parameter and $x$ is the value being transformed.

If the frequency distribution for a dataset is broadly uni-modal (one major peak) and left-skewed, the natural log transform (logarithms base $e$) can adjust the pattern to make it more symmetric and similar to a Normal distribution.

The process of ranking the data and then mapping each rank to the corresponding data rank of a normal distribution is known as the normal score transformation, another method to help normalize data.

Other exploratory analysis of spatial data includes the geostatistical methods of Voronoi Mapping and Global Trend Analysis.

A Voronoi map is constructed by forming a series of polygons around the location of topsoil-depth sample points such that any location within an individual polygon boundary is closer to the sampled point inside that polygon than to any other sampled point. Sampled points whose surrounding polygon shares a border with other sampled points’ polygons will be considered neighbors, allowing a variety of local statistics to be computed. It
is possible to use the ESRI® Geostatistical Analyst extension Voronoi Mapping tool in a topsoil-depth study to aid in identifying potential local outliers. The cluster Voronoi method involves placing the data into five class intervals. If the class interval of a polygon is different from its neighbors the software colors the polygon grey to distinguish it. Figure 5 illustrates an example of a Voronoi Cluster Map where the grey cells reflect potential outliers.

Figure 5. Voronoi Cluster Map Example.

Stationarity, a property in which the relationship between two points depends not upon their exact location but only the distance between them,
is another assumption of many geostatistical analysis techniques. An entropy Voronoi map can be used to examine this local variation between data points. It is calculated based on the class of each surrounding polygon.

Figure 6 illustrates an example of a Voronoi Entropy Map. The red and orange areas reflect areas with higher variability and are thereby less stationary than the light greener colors.

If data do not display appropriate stationarity it could be due to a trend being present in the data. A trend is analyzed by fitting a polynomial function to describe the variation in the data,

\[ f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 \]
where \( n \) is a non-negative integer and where each value \( \{a_n, \ldots, a_0\} \) is a constant. The order of the polynomial refers to the exponent in the leading term. A first-order polynomial is linear and displays as a straight line, a second-order as a parabola, and a third-order as a cubic function.

The ESRI© Geostatistical Analyst extension Trend Analysis tool provides a three-dimensional perspective of data where sample locations are plotted on the \( x,y \) plane with the height (topsoil depth value) of the data projected onto the \( x,z \) and \( y,z \) planes as scatter plots. Figure 7 illustrates an example of how a cubic trend (3\(^{rd}\) order polynomial) and a weak, parabolic trend (2\(^{nd}\) order) could present. Trend analysis can indicate if there is large-scale data variation and in what direction; the blue line is North-South, the green line is East-West.
After completion of the exploration of data comes creation of the interpolated topsoil-depth surface. Interpolation involves the prediction of values at locations where no sample has been taken and is derived from functions on the known values at actual sampled locations.

Two of the most common interpolation techniques include Inverse Distance Weighting (IDW) and the family of methods called Kriging. IDW is a deterministic method, meaning the interpolation involves non-statistical, mathematical methods that predict surface cell values for unmeasured x,y locations using the actual measured topsoil-depth point values surrounding the unknown, predicted locations. It explicitly implements Tobler’s First Law of Geography by assuming that each measured topsoil-depth point has a local influence that diminishes with distance. It weights points closer to the unknown, predicted x,y location to a greater amount than locations further away. As the distance approaches zero (i.e. the location of the predicted point equals that of the measured point) the relative weight approaches 1. Therefore, IDW is an exact interpolator, meaning the predictions will equal the measured values at the location of the measured value. That is, the interpolated surface can never exceed the measured values. A power function, $p$, determines the rate at which the weights decrease with distance from the measured point. As such, the weights are proportional to the inverse distance between the actual measured point and the unknown, predicted point raised to the power, $p$. To determine the optimal $p$ value it is
necessary to minimize the root-mean-square predicted error (RMSPE) which is a summary statistic that quantifies the error of the prediction surface. The Geostatistical Analyst extension will iteratively calculate different power values to determine the lowest RMSPE and hence, the optimum power.

While the deterministic technique of IDW uses only the existing configuration of measured sample points and depends solely on the distance to the prediction location to create the interpolated surface, the Kriging methods use geostatistical techniques that incorporate the statistical properties of the measured sample points. Kriging is a multi-step process that necessitates exploratory data analysis as described above. Additionally, because it is based on stochastic principles it can produce error or uncertainty surfaces along with the prediction surface, as well as surfaces that define the probability of predicted values exceeding (or failing to exceed as in the case of minimum topsoil-depth) a critical value. The kriging method involves quantifying the underlying spatial structure of the data through a process known as variography. It is an advanced geostatistical procedure based in fitting models that include autocorrelation.

Autocorrelation is assessed and quantified using a semivariogram function (gamma) which plots one-half the square of the difference between two values against the distance separating them.

\[ \gamma = \frac{1}{2} [Z(s_i) - Z(s_j)]^2 \]
where $s_i$ and $s_j$ are the two points in the pair and $Z(s)$ is the value at each point. As the distance separating the two points increases so does the difference between their values increase, if spatial autocorrelation exists. Figure 8 is an illustration of the general semivariogram plot.

The height of the semivariogram is called the Sill and defines the variation between pairs of data points. The variation increases with distance and the Range of the semivariogram is that distance between pairs of points where the function flattens out; past this point the values are considered independent and no longer spatially dependent.
Underpinning the geostatistical modeling technique of Kriging is fitting the data to the optimum empirical semivariogram function. It is this model that will be used by the Geostatistical Analyst software to produce the interpolated surface.

Prior knowledge of what is desired from the outcome of a predicted surface model along with exploratory data analysis points the way to which interpolation methods are best suited for any particular dataset under study, and accounts into consideration any assumptions that may be required of the method. After the surfaces are created it is necessary to determine how well they predict values at unknown locations by study of calculated statistics that can serve as diagnostic indicators. The process of cross-validation can be used to determine whether the model values are reasonable. Cross-validation is a robust technique that uses all of the data by removing, one by one, each data point and its associated value, then calculating the value at this now "unknown" location using the remaining points. The procedure is repeated for every point in the dataset. Then, cross-validation compares all of the measured and predicted values and creates scatter-plot graphs and summaries of the predicted versus measured values statistics.

In addition, using the cross-validation summary statistics and plots makes it possible to compare different interpolation models relative to one another even if the surfaces have been created by different methods.
The details of the exploratory data analysis and interpolation processes used to analyze topsoil-depth data at a mining site are addressed at length in the following methodology and result sections.

Methodology

Two interpolation techniques, IDW and Kriging, were investigated in order to describe a process that will create an analysis prediction surface from sampled topsoil-depth point values to aid in determining how evenly a mine operator has spread the required minimum depth of topsoil over newly reclaimed areas on surface coal mines.

Scripts from the ESRI ArcGIS Desktop® 10.0, ArcInfo license, Spatial Statistics and Geostatistical Analyst extension toolkits were used to study the topsoil-depth data.

Study Area and Data

The study area is located on a western U.S. coal mine under the jurisdiction of OSM. It is a large operation named Black Mesa Complex Mine, located on both Hopi Tribe and Navajo Nation lands near the city of Kayenta, in Navajo County, Arizona. The complex covers 65,219 acres.

The red outline on Figure 9 is the boundary of the permitted area of the mine site. The black dashed outline on the figure denotes the location of a portion of the mine in the topsoil distribution stage of reclamation and delineates the region of interest for this study. The topsoil-depth data studied is confined within this approximate 575 acre region.
Data for the location of the topsoil sample points was recorded with a Trimble GeoXT® GPS receiver. The data was downloaded from the GPS receiver and converted into the point shapefile format. A polygon shapefile was created to delineate the outline of the mine area from which the topsoil samples were collected and to act as a mask in geoprocessing operations.

Figure 9. Black mesa Mine Complex Area of Interest Map.
A total of 216 topsoil-depth samples were collected. Eighty-six of them were collected by the mine operator and the remainder 130 samples were collected by personnel from the DSM. Both sets were collected using the same methodology and were in the shapefile format but with slightly different schema. They each contained a common field that contained the topsoil-depth measurement in the same units and the two files were merged into a single file-geodatabase feature class, retaining the common field. To ensure that no two sampled locations were coincident which could introduce noise into the analysis (Demirhan, 2003) all data points were checked against duplication. No duplicate locations were present. The two closest samples were 21-feet apart and the maximum distance between any two samples was 6,700-feet.

Initial Examination of Data

Cursory exploration of the geographic distribution of the sample locations was performed using the distribution measurement scripts in the Spatial Statistics toolbox. Figure 10 illustrates this process. The black outline in the Figure delineates the study area of interest and the background is a hillshade of the area created from a digital elevation model (DEM) derived from high-resolution stereo satellite imagery taken of the mine site.

The statistics depicted in Figure 10 demonstrate that the mean and median centers of the data points are relatively close together suggesting that the data is neither much dispersed nor tightly clustered. The 1-standard
The deviation directional distribution ellipse is oriented 38.5 degrees, and shows the data spread is oriented towards the northeast-southwest direction. Of the 216 samples, 96 (44%), almost half of them lie outside the ellipse, supporting neither dispersion or clustering but there are bare areas that are under-sampled.

The minimum topsoil-depth that was mandated to have been placed, and evenly distributed, on the reclamation area was 24 inches. The actual depths measured had values ranging from 6 to 39 inches. From the range in measured values it may be unlikely that the objective was met.

Figure 10. Spatial Statistics of Sampled Points.
Geostatistical Data Analysis Methodology

The approach taken to analyze the topsoil-depth data to create interpolated prediction surfaces followed a methodical process which began with converting the raw data and representing it in the GIS, as was discussed in the previous section. Following the visual and cursory examination of the data, the formal exploratory data analysis took place, after which interpolation models were fit to the data, diagnostics performed on the surfaces, concluding with comparing the models to determine which was best. Figure 11 provides a flowchart of the steps carried through.
Formal Exploratory Data Analysis

A simple Boxplot, Figure 12, of the raw topsoil-depth data values was constructed and reflects the data may not be normal but skewed and contain potential outliers (green dots.)

Figure 12. Boxplot of Topsoil-depth.

To more appropriately determine whether the topsoil-data exhibit characteristics of a Normal distribution a histogram frequency diagram and a QQ Plot were constructed from the raw data values. Figure 13.
Figure 13. Histogram and QQ Plot of Raw Data Values.

The histogram of the data shows the distribution is asymmetric with left-tail skewness. Normal data is defined with skewness of zero and kurtosis of 3, but the topsoil-depth data presents skewness of 0.93 and kurtosis of 3.4. The mean and median value are not equal, being 17.1 and 16, respectively. The data values do not correlate well with the Normal line in the QQ Plot. The unimodal nature and left-skewed aspect of the data lends potential that a natural logarithmic transformation could adjust the data to more normal characteristics. Additionally, both the histogram and the QQ Plot point to potential outlier data at the high end.
To examine the potential of outliers a Voronoi Cluster map was created. The grey polygons in Figure 14 are the locations for which topsoil-depth data values need to be further investigated before interpolation proceeds.

![Voronoi Cluster Map of Topsoil-depth.](image)

A natural log transformation was performed on the data and it helped to produce more symmetry and to approach more normal characteristics. The QQ Plot of the transformed data is better correlated to the Normal line.
Even with the transformation in place there remained a potential outlier exposed on the QQ Plot. It was investigated in detail using ArcMap and the Voronoi Cluster map. In Figure 15, the selected data point did not map to potential outliers in the Voronoi map (grey polygons) and after the transformation the point fell within the bounds of the upper tail of the histogram. For these reasons the point was not excluded from the dataset.

Figure 15. Examination of Potential Outlier.
Each of the potential data outlier points exposed in the Voronoi Cluster map were selected in ArcMap and examined in detail by comparing their topsoil-depth value with surrounding values. Only one point was deemed an outlier but after further investigation into the topography of where it was located, it was not excluded from the dataset. The point in question is illustrated in Figure 16.
The yellow point in Figure 16 differs enough from its two closest neighbors to possibly be considered an outlier. Because it is located on the opposite side of a ravine from the two differing neighbors it was deemed liable that the value is not out of the ordinary and for this reason it was not deleted.

An entropy Voronoi map was created to obtain information about the local variation in the top soil-depth data. In the Voronoi Entropy map of topsoil-depth shown in Figure 17, the darker red and orange polygons reflect areas with higher variability, while the lighter greens denote less variability. The map for topsoil-depth shows the data somewhat stationary.
Figure 18 shows data exploration of topsoil-depth using trend analysis. Trend analysis shows there is slight variation in the data in both the east-west (green line) and north-south (blue line) direction. Figure 19 is a Voronoi Standard Deviation map that shows the largest variability is in the northeast and southwest area as evidenced by the red polygons. It can be concluded that the data is not totally stationary.
It is unlikely that natural data will present all of the characteristics of normality and stationarity. The Geostatistical Analyst program offers means to transform the data to provide accurate prediction of topsoil-depth at unknown locations by geostatistical interpolation techniques such as Kriging. Prediction surfaces using the deterministic IDW method and the geostatistical Kriging method were carried out.
**Create Surfaces**

The initial surface created was performed using the IDW interpolation method. IDW is a deterministic interpolator that is exact, so measured data points will be predicted with the same value as was measured. It is a quick way to take a first look at an interpolated surface because there are not many decisions to make on model parameters since it does not depend upon assumptions about the data other than it being continuous.

Kriging was used to create the second prediction surface. Kriging is much more flexible than IDW. Because it relies on statistical models it allows for investigation of graphs of autocorrelation such as the semivariogram. Figure 20 shows the optimum semivariogram that was fitted to the data using the Simple Kriging method.

The flexibility achieved through Kriging must be balanced against the requirement that many decisions be made about the parameters involved in the interpolation. Version 10.0 of the ESRI® Geostatistical Analyst extension is equipped with functionality that allows for the software to optimize many parameters based upon the structure of the spatial data. Still, the advanced geostatistical methods involved in producing surfaces via Kriging involve iteration and "trying out" different methods. The Simple Kriging method using the Normal Score Transformation was found to perform the best and was used to create the second prediction surface for topsoil-depth distribution.
Perform Diagnostics

The method parameters for the IDW and the Simple Kriging models that were used to create the predicted surfaces are listed in Appendix B and C, respectively.

Cross-validation summary statistics and scatter plots are produced as part of the output from the Geostatistical Analyst tools. The statistics and plots were examined to determine how the interpolation models performed.

In addition to the summary statistics and plots, geostatistical methods (Kriging) are equipped with the ability to produce a separate surface that displays the standard error on the predicted surface. For the Simple Kriging
model, this diagnostic was also performed and the additional surface created.

Assuming the data is normal, or can be transformed to normality, it is possible to create a probability surface that displays the likelihood of exceeding or failing to exceed a critical value. For the topsoil-depth, the critical value required by the permit is 24 inches of topsoil evenly distributed over the entire area. In order to further assess if this was accomplished, a probability surface was created.

Compare Models

Similar to assessing the validity of a single model, summary statistics and plots were used to compare the IDW and Kriging prediction surfaces. Although the number of statistics produced by the Geostatistical Analyst program for IDW is fewer than for Kriging, it remained possible to make the assessment.

Results

Inspection of the topsoil-depth point values resulted in the knowledge that the data did not conform to the Normal distribution but was skewed to the left. Formal exploratory data analysis resulted in observation of several potential outliers being present but further investigation supported the choice to not exclude them from input into the surface model interpolation procedures.
Two optimized prediction surfaces were created using the deterministic IDW method and the geostatistical Simple Kriging method. Each surface was classified in the same manner. Five classes were chosen in order to provide for more unambiguous interpretation. The class breaks were determined manually to support the best visualization towards the task of determining the distribution of topsoil-depth across an area. Figure 21 shows the two surfaces side by side. Both surfaces represent over distribution of topsoil in similar general areas (green to yellow classes.) The IDW surface, known for its potential to produce "bulls eyes" around data locations, predicted two areas where the topsoil was expressly under applied, at 6-inches (red class), but which the Kriging surface smoothed out.

Figure 21. IDW and Simple Kriging Prediction Surfaces.
Visualization from each surface concurs that overall the topsoil-depth 24-inch minimum requirement evenly distributed across the area was not achieved. The vast majority of the areas represented in both surfaces predict that the topsoil was distributed to a depth of between 6 and 20 inches.

Geostatistical interpolation methods allow for creation of surfaces that quantify the error involved in the prediction surface. Figure 22 is a surface showing the standard error of prediction on the Simple Kriging method derived surface.

Figure 22. Standard Error of Prediction Surface for the Kriging Derived Prediction Surface.
The lighter white-ish areas indicate where the predicted surface performed with less error and the red areas are where the error in the prediction of unknown values is greater. Overall, the Simple Kriging surface performed well with few areas of red and those areas generally tend to be in places where sampling frequency was more sparse.

In addition to producing error surfaces the geostatistical interpolation methods are also capable of producing surfaces that display probabilities in areas where critical values are exceeded or, as in the case of minimum topsoil-depth, not exceeded. Such probability maps operate under the assumption that the data is normally distributed. The simple kriging method that was used in this analysis allows for transformation of the data. The Normal Standard Transformation was utilized on this dataset to produce more normal characteristics in the data prior to surface creation.

A surface describing the probability that the study area contains a topsoil-depth distributed to the required minimum 24-inches is displayed in Figure 23. The surface shows that only a few places (grey areas) are 68% likely to contain the minimum topsoil-depth. The majority of the area has less than 95% likelihood the requirement was met, and a large portion of the area virtually did not meet the requirement (red areas).
The final determination involves quantifying which surface predicted topsoil-depth distribution better, IDW or Kriging. Through utilization of the statistics derived from cross-validation it is possible to compare the performance between surfaces. Figure 24 is the comparison of the cross-validation statistics and scatter plots between the IDW and Kriging surfaces created in this analysis.
The scatter plots of measured versus predicted values in Figure 24 may at first suggest that the IDW method outperformed Kriging by virtue of the tighter correlation observed on the IDW plot to the 1:1 line. The predicted line is the thicker blue one. But the IDW slope is usually less than the slope line in Kriging due to a property of the method that tends to...
under estimate large values and over estimate smaller values. (Johnston, 2001)

The summary statistics are presented below the scatter plots in Figure 24. It is expected that in a good model, the prediction errors will be unbiased and therefore near zero. The mean error in the IDW model is 0.24 and in the Kriging model it is 0.068. Also expected of a good model is that the root-mean-square prediction error (RMSPE) be minimized because the closer the predictions are to true values the smaller it will be. The RMSPE in the IDW model is 4.17 and in the Kriging model it is 4.71.

Conclusion

Coal-mining operations, albeit the predominate means of acquiring the fuel necessary for production of electricity in the U.S., also come with severe adverse consequences to the land and environment.

Creation of cell-based analysis prediction surfaces from sampled topsoil-depth point values can aid in determining how evenly a mine operator has spread the required minimum depth of topsoil over newly reclaimed areas on surface coal mines.

Use of prediction surfaces from sampled topsoil-depth point values could help to more accurately evaluate topsoil replacement commitments defined in surface coal mine permits and could help to ensure better reclamation.
With a topsoil distribution surface layer in hand it will be more apparent which areas require additional topsoil and from which areas excess topsoil may be acquired. With a continuous surface map, it can more readily be determined what is necessary in order to comply with the requirement of an evenly distributed topsoil depth.

The ability to make balanced environmental decisions necessitates taking into consideration interacting factors. Geostatistical tools and methods provide this capacity towards more appropriately and accurately defined environmental analysis, but it must be recognized that models are only approximations of reality and must not be considered exact representations.

**Future Opportunities**

In efforts to further quantify the interpolated surfaces created in this report and apply their result to complementary coal mining scenarios, it could prove valuable to utilize remote sensing technology in concert with them. With the use of high-resolution, multi-spectral satellite imagery collected over several growing seasons at the areas of interest described in this report, it would be possible to classify the imagery using the infra-red band and calculate vegetative indices. It would be possible to compare areas where vegetation growth may not be occurring successfully with the predicted probability maps displaying areas where topsoil-depth was predicted to be less than the optimum 24 inches.
It would prove ideal for non-spatially trained personnel in the mining industry to have access to automatic production of topsoil-depth prediction surfaces without having to be intimately familiar with the intricate decision making involved. This analysis showed there is little difference between deterministic IDW and the many decisions required in geostatistical Kriging, at least for this study site. Similar findings (Chaplot, et al., 1999) determined there is little difference as long as the sampling density is high. But what formulates high sampling density among newly reclaimed topsoil distribution areas remains unknown. If it were determined that the IDW method served to produce a reliable surface model of topsoil distribution then it could be incorporated into an ESRI© ModelBuilder™ model or script for use, perhaps even incorporated as a geoprocessing service in an online web application.
References


Hill, Michael, 2004. 3-D Modeling of Large Anthracite Open Pit Mining Operations to Assist in Pennsylvania’s Conversion to Conventional Bonding. Paper presentation at the 2004 Advanced Integration of Geospatial Technologies in Mining and Reclamation, 2004, Atlanta, GA.


Lamont, Samuel J., Robert, Eli N., Fletcher, Jerald J., and Galya, Thomas, Cumulative Hydrologic Impact Assessments of Surface Coal Mining Using WCM5-HSPF, Paper presentation at the 2008 Geospatial Conference and 2nd National Meeting of SMCRA Geospatial Data Stewards, Atlanta, GA.


**Appendix A - List of Acronyms**

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHIA</td>
<td>Cumulative Hydrologic Impact Assessment</td>
</tr>
<tr>
<td>DEM</td>
<td>Digital Elevation Model</td>
</tr>
<tr>
<td>DOI</td>
<td>Department of Interior</td>
</tr>
<tr>
<td>EPA</td>
<td>Environmental Protection Agency</td>
</tr>
<tr>
<td>ESRI</td>
<td>Environmental Systems Research Institute</td>
</tr>
<tr>
<td>GIS</td>
<td>Geographic Information System</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>HSPF</td>
<td>Hydrologic Simulation Program-Fortran</td>
</tr>
<tr>
<td>IDW</td>
<td>Inverse Distance Weighted</td>
</tr>
<tr>
<td>LOM</td>
<td>Life of Mine</td>
</tr>
<tr>
<td>OSM</td>
<td>Office of Surface Mining and Enforcement</td>
</tr>
<tr>
<td>PA-DEP</td>
<td>Pennsylvania Department of Environmental Protection</td>
</tr>
<tr>
<td>QQ</td>
<td>Quantile-Quantile Plot</td>
</tr>
<tr>
<td>RMSE</td>
<td>Root-mean-square Error</td>
</tr>
<tr>
<td>RMSPE</td>
<td>Root-mean-square Predicted Error</td>
</tr>
<tr>
<td>SMCR</td>
<td>Surface Mining Control and Reclamation Act</td>
</tr>
<tr>
<td>SOC</td>
<td>Soil Organic Carbon</td>
</tr>
<tr>
<td>WCM</td>
<td>Watershed Characterization and Modeling System</td>
</tr>
<tr>
<td>WVU</td>
<td>West Virginia University</td>
</tr>
</tbody>
</table>
Appendix B - IDW Model Parameters
## Appendix C - Kriging Model Parameters

![Kriging Model Parameters](image)

<table>
<thead>
<tr>
<th>Input datasets</th>
<th>Dataset: J16_0SH_PWCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
<td>C:\GEOG_4993_Thesis\Thesis\J16_0SH_PWCC.db</td>
</tr>
<tr>
<td>Type</td>
<td>Feature Class</td>
</tr>
<tr>
<td>Data field</td>
<td>TopSoil_Depth</td>
</tr>
<tr>
<td>Records</td>
<td>216</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Method</th>
<th>Type: Simple</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output type</td>
<td>Prediction</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dataset #</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trend type</td>
<td>None</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Normal Score Transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approximation</td>
<td>Direct</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Searching neighborhood</th>
<th>Smooth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>Smooth</td>
</tr>
<tr>
<td>Smoothing factor</td>
<td>0.2</td>
</tr>
<tr>
<td>Angle</td>
<td>0</td>
</tr>
<tr>
<td>Major semiaxis</td>
<td>521.6597955414371</td>
</tr>
<tr>
<td>Minor semiaxis</td>
<td>521.6597955414371</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variogram</th>
<th>Semivariogram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of lags</td>
<td>12</td>
</tr>
<tr>
<td>Lag size</td>
<td>0.20634044267664</td>
</tr>
<tr>
<td>Nugget</td>
<td>0</td>
</tr>
<tr>
<td>Measurement error %</td>
<td>100</td>
</tr>
<tr>
<td>SHIFTON</td>
<td>No</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model type</th>
<th>Parameter 1.3162109375</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>521.6597955414371</td>
</tr>
<tr>
<td>Anisotropy</td>
<td>No</td>
</tr>
<tr>
<td>Partial sill</td>
<td>1.0080001129302312</td>
</tr>
</tbody>
</table>