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Short-Term Failure Analysis Of Aluminum Conducting Composite Core Transmission Lines

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SHORT-TERM FAILURE ANALYSIS OF ALUMINUM CONDUCTING COMPOSITE CORE TRANSMISSION LINES

A Thesis
Presented to
the Faculty of Engineering and Computer Science
University of Denver

In Partial Fulfillment
of the Requirements for the Degree
Master of Science

by
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August 2009
Adviser: Dr. Maciej Kumosa
ABSTRACT

Recently the inefficiency and lack of capabilities of electric energy transmission have been put in the global spotlight; a transmission line design known as the Aluminum Conducting Composite Core Trapezoidal Wire (ACCC/TW™) attempts to eliminate the deficiencies. The ACCC core is comprised of a new hybrid composite material. The most probable causes of short-term damage to the ACCC core were investigated through a series of finite element models. It was found that excessive bending was the most likely cause of short-term damage to the core. During bending of the ACCC core, stresses are concentrated at the interface in the carbon fiber composite region. The composite materials’ compressive strength was measured, and found to be significantly lower than similar composite materials based upon the same reinforcing fiber. An excessive bending finite element model was experimentally validated through a series of four point bend experiments in which acoustic emissions were monitored. From this work, it has been determined the extent to which the ACCC core can be bent without creating significant damage.
ACKNOWLEDGEMENTS

The author would first like to thank his family, for all of their love and support throughout this process. Thanks must also be given to Dr. Maciej Kumosa for the endless effort that he has devoted to helping further this research. His guidance and advice have been greatly appreciated. Without his help, this research could not have been completed. Thanks also to Dr. Daniel Armentrout, who has been an excellent resource for the experimental work performed in this research. Also, thanks must be given to Jon Buckley whose knowledge in machining practices, and design concepts were instrumental in completing this work.

Finally, the author would like to thank the Western Area Power Administration (WAPA) and Tri-State P&G. Their financial support is much appreciated, and it is the author’s hopes that this research has been, and continues to be, of significant value to them.
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1 – Introduction

1.1 – State of current electric transmission lines

In recent years the electrical energy demand in the United States and around the world has increased, and is on the verge of reaching a point in which current electrical transmission lines cannot meet the consumers’ demand [1-4]. By the year 2017, the North American Electric Reliability Corporation (NERC) predicts that the United States’ energy demand will be up ~17% from the year 2007, to an astounding 870,693 GW-h [1]. Unfortunately, the current technology of electrical transmission lines is not capable of distributing this amount of energy, especially in periods of excessive demand. When the power demands of consumers cannot be met, it is known as a brown-out.

The current technology of electrical transmission lines is based upon a stranded steel core surrounded by 1350 H-19 aluminum strands (Figure 1.1); this technology is known as the Aluminum Conducting Steel Reinforced (ACSR) design. The downfall of the ACSR design is two-fold. First, the 1350 H-19 aluminum strands have a significant resistivity, which causes current drop over the line. This power loss manifests itself in the form of dissipated heat, which feeds into the second major problem of the ACSR design. Because steel and aluminum have relatively large coefficients of thermal expansion, the ACSR line is subject to a significant amount of sagging. When a transmission line sags, it is possible for the line to touch a nearby ground obstruction (such as an un-trimmed tree limb) creating a
direct path to ground for the circuit. This short-circuit can have catastrophic effects, and cause a cascading failure, as was the case in the black-out of the US Northeast in August 2003. This black-out affected an estimated 50 million people, contributed to the deaths of 11 people, and its’ immediate effects lasted well over a week [5].

**Figure 1.1 – Current design of standard electric transmission lines (ACSR) with the structural core based upon stranded steel**

1.2 – Solutions to problems facing the ACSR conductor

A few solutions to resolve the limitations of the ACSR design have been proposed. The most obvious solution would be to add transmission lines to the grid in order to transmit more power, and effectively lessen the power load on existing transmission lines. This solution is not very viable for multiple reasons; one of which being obtaining the land necessary to build new transmission towers is often very hard to come by. In rural areas, land is often protected under various land protection acts, making obtaining the land either impossible, or cost-prohibitive. Likewise, in urban areas, the cost of obtaining land to build
right-of-ways is prohibitive, or building of new transmission lines meets severe opposition from the public. Everyone wants the electric power, but few are willing to have an electric transmission tower in their back-yard. The second major problem with adding new transmission lines to the grid is the capital required to add new towers, and associated infrastructure is currently not lucrative to grid owners [2].

Thus, an alternative solution would appear to be necessary. A few companies have turned to composite materials to solve the power transmission problem. In recent decades, composite materials have shown an ability to solve design problems facing engineers because of the ability to tailor the physical properties of the composite material to the specific design requirements [6]. One company that has introduced a new design of transmission line is 3M©. The 3M© corporation introduced its’ Aluminum Conducting Composite Reinforced (ACCR) design; the ACCR design is based upon a core comprised of a stranded metal matrix composite (MMC), surrounded by zirconium alloyed aluminum strands (Figure 1.2). The composite core is made up of a pure aluminum matrix, reinforced with alumina ($\text{Al}_2\text{O}_3$) fibers. Benefits of the ACCR design are numerous. First, the ACCR design is compatible with the existing transmission towers, with no modifications to the existing structures necessary [7]. Also, the ACCR design is capable of transmitting 2-3 times the amount of current as traditional ACSR lines [7]. Finally, the ACCR lines can be operated up to 240 °C, far out-performing traditional ACSR lines [7]. The one drawback of the ACCR design is that it costs approximately 10 times more per line foot than ACSR.
Figure 1.2 – 3M® high temperature transmission line design (ACCR) with the structural core based on a metal matrix composite

An alternative to the design of the ACCR conductor is the Aluminum Conducting Composite Core Trapezoidal Wire (ACCC/TW™) design of Composite Technology Corporation (CTC) (Figure 1.3). The ACCC/TW™ core design is also based upon composite materials, but rather than using a metal matrix composite, a polymer matrix composite (PMC) is employed. The ACCC core is comprised of a hybrid composite material where the inner part of the core is made up of a proprietary high temperature epoxy matrix reinforced with Toray T700S carbon fibers [2]. Carbon fiber composites have gained wide spread popularity because of their high specific properties [8]. However, because carbon and aluminum will undergo a galvanic reaction when they come in electrical contact with one another, an insulating sheath had to be put between the two [9-10].
The insulating sheath is comprised of ECR glass fibers within the same high temperature epoxy. ECR glass fibers (i.e., boron free fibers) were used due to their resistance to stress corrosion cracking [11-15]. ECR glass fibers were selected in the design of the ACCC conductor as stress corrosion cracking has been shown to be a failure mode that is present in electric power transmission (in particular, non-ceramic insulators) [11-15]. The stress corrosion cracking in electrical transmission is often an issue because of the alternating current present from the transmission lines, in conjunction with moisture from the environment. This is a formulation for nitric acid, which has been shown to significantly reduce the life of composites reinforced with E-glass fibers through an ion leaching mechanism that weakens the composite [11-15]. However, when boron free glass fibers (i.e. – ECR glass fibers) are used, the stress corrosion cracking issue is mitigated.
Advantages that the ACCC/TW design provides are numerous. First, incorporating carbon fibers solves the problem of line sag at high temperatures. Due to the bond structure, carbon fibers have an axial coefficient of thermal expansion that is negative [9]. Additionally, carbon fiber composites are almost always a perfectly elastic material. Thus, the transmission line will retain its’ initial tensioning sag after thermal cycling to a far greater extent than the ACSR design, which will undergo plastic deformation if the expansion becomes too great and not fully recover its’ initial tensioning. Another advantage of utilizing carbon fibers is the superior tensile strength that is offered. The ACCC core offers a tensile strength of 2400 MPa [2], which allows for greater tensioning (while still maintaining a factor of safety) when a particular span calls for it (e.g. – long span, clearance issues, etc.); this makes the ACCC design far more versatile than the traditional ACSR conductors.

Since the composite core in the ACCC design is less dense than the traditional steel core [2], more aluminum per line foot can be included while still keeping the same weight as in-service conductors. This manifested itself in the form of the trapezoidal wire design, as opposed to the traditional circular wire design. Using trapezoidal wires results in more current being carried by the line, as well as less losses of the line. Also, because the ACCC core has such a high ultimate tensile strength it can bear the weight of the aluminum without the aluminum providing much in the way of structural support. Hence, a high purity 1350 O’-tempered fully annealed aluminum is used, which has a lower resistivity value than the 1350 H-19 aluminum of the conventional ACSR design [2], allowing for better conduction with less line loss.
The final advantage that the ACCC conductor offers is the capability of being operated at higher temperatures than the ACSR design. Because of the polymer matrix, most designers limit the operating temperature of ACCC lines to that of the glass transition temperature, $T_g$, of the polymer (typically with some factor of safety). This is a valid approach, as polymers will act rubbery and begin to significantly lose mechanical properties at temperatures above the glass transition temperature [8]. It should be noted here, that the true value of $T_g$ for the ACCC core is somewhat of a debate between the manufacturer’s published value of 200 °C [16], and independent evaluations of $T_g$ that determined a value of 185 °C [17]. In the context of this work, it is of minor significance, but should be pointed out as an issue that is in need of resolution.

Thus, the ACCC/TW conductor resolves the limitations facing the current technology, and is able to do it at only approximately 2.5 times the cost of the ACSR design, making the ACCC/TW conductor an attractive product to utility companies. However, as is the case with every new technology, a significant amount of research must be performed in order to understand all of the possible problems that could arise with the new technology and gain confidence in the design.

In this work the most probable short-term failure mode was determined through a series of finite element models, coupled with experimental validation. It must be pointed out that long-term failure modes and mechanisms of this hybrid polymer based composite material are still a relatively untouched area of research. Due to the reactive nature of polymers, degradation in extreme environmental conditions remains a real concern that is not very well understood.
1.3 – Possible failure modes of ACCC cores

Short-term failure modes for the ACCC core could possibly arise from three main domains. First, manufacturing of polymer matrix composite materials has been shown to be a process that can cause failures due to residual stresses present in the composite [18-22]. These residual stresses arise from the severe mismatch in the coefficients of thermal expansion between the polymer and fiber. By adding a second type of reinforcing fiber an additional layer of complexity is added that could be causing a meso-stress that could affect in-service performance.

A second possible failure mechanism of the ACCC core is axial tension. This failure mode is unlikely, as lines are always designed with an inherent factor of safety with respect to the rated tensile strength (RTS). However, axial tension needed to be investigated in order to insure that no unforeseen interaction was occurring between the laminas that could be causing failure.

Finally, bending is a possible failure mode of the ACCC core. ACCC cores must be wrapped around mandrels for transportation, and installation purposes. It is well known that carbon fibers have relatively poor flexural properties due to their high stiffness [9]. Use of the Toray T700S fiber was a wise decision in the current design; i.e. - using a high strength fiber as opposed to a high modulus fiber, but it may not have been enough. Once the most probable failure mode(s) could be predicted, parameters affecting the failure mode(s) needed to be investigated and thoroughly understood in order to improve the design. Also, the predicted failure mode(s) from the finite element models needed to be experimentally validated.
2 – Introduction to Polymer Matrix Composites utilized in the ACCC core

2.1 – Uni-directional composite materials

While the ACCC core is comprised of two different types of composite materials, the two materials have several attributes in common. Both the carbon fiber/epoxy core and the glass fiber/epoxy sheath are uni-directional composites. In the case of the ACCC design, although there are two types of reinforcing fibers, and the purpose that each fiber serves is different, the fundamentals for both types of material are similar.

A uni-directional composite material is a system of materials typically composed of aligned strong and stiff fibers reinforcing a relatively compliant matrix. Both types of reinforcing fibers in the ACCC design are long fibers, and can theoretically be infinite in length [9]. Due to the long fibers, a difficulty is presented to the engineer or designer working with the material. Because of the aligned nature of the composite, even a fiber consisting of an isotropic material will exhibit marked anisotropy in the transverse directions of the composite system. This inherent transverse anisotropy in material properties severely complicates the modeling of uni-
directional composite materials, as well as the design process due to the differing material properties in different directions. Uni-directional composite materials are a specialized category of materials that are ideally suited for use when large tensile loads are present; the superior axial tensile strength of the reinforcing fibers is very attractive to designers in these types of applications, with a significant savings in weight.

2.2 – Reinforcing fibers in the ACCC core

Toray T700S fibers are the reinforcing fiber of choice for the inner-part of the ACCC core [2]. The fiber is a high strength fiber with a moderate elastic modulus. T700S fibers are formed from polyacrylonitrile (PAN) polymer fibers. The final thermal processing of PAN based carbon fibers offers control over the Young’s modulus, and ultimate tensile strength [9]. High-strength fibers (as is the case with T700S fibers) typically have a final heat-treatment of around 1550 °C. Due to the carbon-ring type bond structure of PAN based carbon fibers the fiber itself is transversely isotropic. Determination of the transverse modulus of a carbon fiber is not an easy task, but is a necessary for accurate modeling to be possible. The highly ordered bond structure of carbon fibers also results in transversely isotropic thermal conductivity and expansion values. Material properties of the Toray T700S carbon fibers used in the modeling in this work are presented in Table 2.1 [9, 22-24].
Table 2. 1 - List of all constituent material properties used in this work

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial Modulus (GPa)</td>
<td>230</td>
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<td>3.6</td>
</tr>
<tr>
<td>Transverse Modulus (GPa)</td>
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<td>76</td>
<td>3.6</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
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<td>.22</td>
<td>.2</td>
</tr>
<tr>
<td>Longitudinal Shear Modulus (GPa)</td>
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<td>6.9</td>
<td>1.2</td>
</tr>
<tr>
<td>Transverse Shear Modulus (GPa)</td>
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<td>6.9</td>
<td>1.2</td>
</tr>
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<td>4.90</td>
<td>60.00</td>
</tr>
<tr>
<td>$\alpha_{transverse}$ ($10^{-6}$/°C)</td>
<td>10.00</td>
<td>4.90</td>
<td>60.00</td>
</tr>
</tbody>
</table>

Electrical Corrosion Resistant (ECR) glass fibers are a special type of silica (SiO$_2$) based glass fiber that offers superior corrosion resistance while still maintaining good dielectric properties. In the case of electric transmission ECR glass fibers are a wise choice due to presence of nitric acid in the operating environment [11-15]. ECR glass fibers differ from normal E glass fibers in the fact that they have no boron, which eliminates an ion leaching mechanism that acts in the presence of hydronium (H$_3$O$^+$) rich acids. Glass fibers are commonly amorphous with localized regions of crystallinity; for this reason glass fibers are generally isotropic in nature.

Glass fibers are produced by drawing molten glass stock through platinum bushings. The diameter of the glass fibers is dependent upon several variables, namely, drawing speed, viscosity of the glass, and the hole diameter in the bushings [9]. Several studies have been performed on glass fibers, and have concluded that the surface condition of the fiber is the dominant factor in determining the glass fibers composites’ strength [25-26]. For this reason a sizing agent is applied to the glass
fibers [9]. The sizing agent applied to glass fibers is generally water mixed with an emulsifying polymer and a small molecule lubricant. The lubricant serves to lubricate the fibers, protecting the surface of the glass fibers from abrasive wear as they rub against one another during processing and handling. Sizing agents play a key role in the quality of the interface between the fiber and the matrix in the composite system, and give the fibers anti-static properties [9]. All material properties for ECR glass fibers used are given in Table 2.1.

2.3 – The matrix in the hybrid composite used in the ACCC core

Polymers are often used as the matrix material in composite systems due to their compliant nature. Typical polymers are composed of several chains each containing numerous covalently bonded atoms. Most polymers can be divided into two fairly broad categories: thermosets and thermoplastics. The difference in the two categories arises in the interaction between chains. Thermoplastics (e.g. – PEEK or Polypropylene) are a class of polymer where the chain interaction is primarily van der Waal’s bonds, and chain entanglement (in the case of amorphous thermoplastics). The primary interaction between chains in thermosets (e.g. – epoxies or vinyl ester) is cross-linking. Cross-links are covalent bonds between monomer chains. Due to the fact that van der Waal’s bonds melt at a temperature of ~300 K, and covalent bonds melt at a temperature of ~2500K [8], thermosets generally have a higher glass transition temperature, \( T_g \), than do thermoplastics. The glass transition temperature is the temperature at which a polymer will begin to soften and lose mechanical
properties; \( T_g \) is often the governing parameter in operating temperature of components made out of polymers. For this reason, thermosets are generally utilized in higher temperature applications than are thermoplastics.

The ACCC composite’s matrix is a two-phase high temperature thermosetting epoxy [2]. Unfortunately, its chemical composition is proprietary, and little is known about the exact mechanical or thermal properties of the epoxy itself. However, for modeling purposes of composite systems, the exactness of material properties for the matrix is of minor significance in comparison to the exactness of fiber properties. This is because the matrix materials’ properties are significantly less than that of the reinforcing fiber, and thus play a smaller role in the overall mechanical properties of the composite. Material property determination of composites is covered in Chapter 3.1. Values for the matrix material properties used in this work are presented in Table 2.1.

2.4 – Manufacturing process of ACCC cores

Uni-directional composite materials can be manufactured through several processes, including pultrusion, pressurized consolidation of pre-preg, and injection transfer molding [9]. For the ACCC core the pultrusion process is ideal. Pultrusion is a process in which tows of aligned fibers are dipped in a molten resin bath, and then pulled through a die to give the structure its shape. After being pulled through the die, the composite is kept in an autoclave where the epoxy cures at an elevated temperature. Pultrusion is advantageous for producing the ACCC core due to the
lengths that can be manufactured in a relatively short amount of time. No published information exists on the exact manufacturing process of the ACCC rods; however, it is known that the composite lines are wrapped around a mandrel in order to be transported to a facility that wraps the aluminum around the composite core.
3 – Review of Numerical Methods Used

3.1 – Material Property Predictions

3.1.1 – Mechanical material property predictions

Material property prediction techniques for uni-directional composite materials range from fairly simplistic to incredibly complex. Interestingly the accuracy of these predictions is not always a reflection of the complexity of the technique. As was mentioned in Chapter 2, uni-directional materials are inherently transversely isotropic, thus, expressions for determining the axial as well as transverse properties must be derived.

The simplest approach to determining the mechanical material properties of a uni-directional composite material is based on the “slab-model” [9]. In this method, the fibers and matrix are represented by two equivalent slabs with the thicknesses of each slab proportional to the respective volume fractions. An underlying assumption of this method is that the fiber slab is perfectly bonded to the matrix slab. To determine an expression for the axial modulus, the two slabs are subjected to an amount of strain that is equivalent in both slabs, i.e. -

\[ \varepsilon_c = \varepsilon_f = \varepsilon_m. \] (3.1)

The iso-strain assumption remains valid as long as there is no fiber pull-out, which relates to the critical length that a fiber must be. Often the iso-strain model is referred to as the “Voigt model.”
The applied strain results in a stress state in the composite which is given by

\[ \sigma_c = \nu_f \sigma_f + (1 - \nu_f)\sigma_m \]  \hspace{1cm} (3.2)

Using basic Hooke’s law definitions for all materials, and using the iso-strain assumption, an expression for the composite’s Young’s modulus can be derived

\[ E_{ca} = \nu_f E_f + (1 - \nu_f)E_m. \]  \hspace{1cm} (3.3)

Where \( \nu_f \) represents the volume fraction of fibers, \( a \) stands for axial and the subscripts \( c, f, \) and \( m \) represent the composite, fiber and matrix respectively. Hence, the axial modulus is essentially a weighted average of the constituent moduli. Equation (3.3) can be expected to agree very well with other methods, as well as experimental data [9]. This is due in large part to the fact that provided a good interface exists between the fiber and the matrix, and that the fibers are long enough, fiber pull-out should not occur.

Prediction of transverse material properties is quite a bit more difficult, but as a rough first pass, the slab-model can be used. An obvious error with the slab-model is evident from the geometry. In the slab-model, there is only one transverse direction, and two axial directions; the converse is true for a real uni-directional composite. Transverse material property predictions are based upon the iso-stress assumption

\[ \sigma_c = \sigma_f = \sigma_m. \]  \hspace{1cm} (3.4)

The iso-stress model is also referred to as the “Ruess model,” in the literature. When the two slabs are stressed transversely, the total strain of the composite is a weighted average of the constituent strains experienced, i.e. –

\[ \varepsilon_c = \nu_f \varepsilon_f + (1 - \nu_f)\varepsilon_m. \]  \hspace{1cm} (3.5)
Assuming linear-elasticity holds, and applying Hooke’s law in conjunction with the iso-stress assumption yields an expression for the composite’s transverse Young’s modulus

\[ E_{ct} = \left[ \frac{v_t}{E_{ft}} + \frac{1 - v_t}{E_{mt}} \right]^{-1}. \]  \hspace{1cm} (3.6)

In equation (3.6) the subscript \( t \) refers to the transverse direction. Experimental data has shown to significantly vary from the predictions made by equation (3.6) [9]. These deviations can be explained by the actual deformation that occurs in the matrix. In transverse loading, each fiber serves as a geometric stress concentration, and raises the local stress level in the matrix which leads to local plastic deformation [9, 27]. Thus, the linear elastic assumption is violated, the matrix experiences an amount of plastic deformation, and transverse material property predictions are expected to not be very accurate.

Similar approaches for determining the shear moduli of uni-directional composite systems and the Poisson’s ratios are applicable. Their derivations are not provided here, however, their results are stated for completeness.

\[ G_{ca} = v_f G_{fa} + (1 - v_f) G_{ma}. \]  \hspace{1cm} (3.7)

\[ G_{ct} = \left[ \frac{v_t}{G_{ft}} + \frac{1 - v_t}{G_{mt}} \right]^{-1}. \]  \hspace{1cm} (3.8)

\[ \nu_{ca} = v_f \nu_{fa} + (1 - v_f) \nu_{ma}. \]  \hspace{1cm} (3.9)

\[ \frac{\nu_{ca}}{E_{ca}} = \frac{\nu_{ct}}{E_{ct}}. \]  \hspace{1cm} (3.10)

Notice that in equation (3.10) the transverse Poisson’s ratio is being determined for the direction being stressed; the off-component Poisson’s ratio is expected to be relatively
large in magnitude with transverse stressing; however, this discussion is not presented here due to the poor agreement with experimental data that is provided by simple models.

A more complex and accurate approach to determining material properties was developed by Eshelby [28-29]. In his approach, Eshelby considered the problem of an ellipsoidal mis-fit inclusion contained within an infinite-homogeneous matrix. The Eshelby method is based upon removing an ellipsoidal inclusion from a homogeneous matrix, and applying a stress-free transformation strain to the inclusion that deforms the inclusion a given amount. Traction are then applied to the inclusion to return it to its’ original shape. Hence, no strain is present in the inclusion, but there is now a resultant stress due to the applied tractions. The inclusion is then placed back in the matrix, and the tractions removed. The inclusion and matrix equilibrate leaving the inclusion strained an amount known as the constrained strain. Since the inclusion is ellipsoidal, the strains (and resultant stresses) are uniform throughout the inclusion.
For the Eshelby approach to be valid a few requirements must be satisfied:

1. The matrix material is homogeneous and isotropic
2. Fibers are uniformly distributed throughout the matrix
3. The interfaces between the fibers and matrix are perfect
4. The inclusion must be ellipsoidal.

In order for the approach to be applicable to composite materials, the inclusion needs to no longer be a piece of the matrix removed, but rather an embedded elliptical inclusion in the matrix that has different material properties than the matrix. The same process is applied; however, a different transformation strain (originally referred to as the mis-fit strain by Eshelby; several authors have since begun calling it an eigen-strain) must be applied to the mis-fit inclusion which results in the same constrained strain. Thus, the essence of the Eshelby problem is to determine the mis-fit strain.

The Eshelby method was a novel approach, however, it failed to work accurately as the volume fraction of fibers became more than ~10%. Thus, work was done in the area by several authors [30-33] in order to extend the Eshelby method to composites containing a realistic volume fraction of fibers. All of these approaches centered on employing the Eshelby method, but also considering the back-ground (or volume averaged) stress present in the composite due to the greater amount of fibers present. The greater amount of fibers leads to stress-field interactions, and is where the term “back-ground stress,” originated. Using this approach, an expression for the composite stiffness matrix was determined.

\[
C_c = \left[ C_m^{-1} - v_f \left\{(C_f-C_m)\left[S-v_f(S-I)\right] + C_m^{-1}(C_f-C_m)C_m^{-1}\right\}^{-1} \right].
\]  

(3.11)
In expression (3.11) all upper-case C’s stand for the respective stiffness matrices, $S$ is the Eshelby matrix (which is defined in [30] for infinite ellipsoidal inclusions), and $I$ is the identity matrix. The elastic compliance matrix for a transversely isotropic material is defined by the 6x6 matrix

\[
S = \begin{bmatrix}
\frac{1}{E_z} & \frac{\nu_a}{E_z} & \frac{\nu_a}{E_z} & 0 & 0 & 0 \\
\frac{\nu_a}{E_z} & \frac{1}{E_z} & \frac{\nu_a}{E_z} & 0 & 0 & 0 \\
\frac{\nu_a}{E_z} & \frac{\nu_a}{E_z} & \frac{1}{E_z} & 0 & 0 & 0 \\
\frac{1}{G_a} & 0 & 0 & \frac{1}{G_a} & 0 & 0 \\
\frac{1}{G_a} & 0 & 0 & \frac{1}{G_a} & 0 & 0 \\
\frac{1}{G_a} & 0 & 0 & \frac{1}{G_a} & 0 & 0
\end{bmatrix}.
\] (3.12)

Note that the elastic compliance matrix of a material is related to the elastic stiffness matrix of a material by

\[S = C^{-1}.\] (3.13)

In the case of an isotropic material $E_a = E_t$, $\nu_a = \nu_t$, and $G_a = G_t$ which allows equation (3.12) to be simplified. The purpose of defining the compliance matrix as opposed to the stiffness matrix is due to the computational tools that have become available recently. If the stiffness matrix of a composite material is calculated, one is left with a set of linearly-dependent equations that then must be solved to determine the pertinent material properties. However, if the Eshelby approach is implemented in a numerical tool (e.g. – Maple or Matlab) definition of the respective compliance matrices is straight forward, and extraction of results is greatly simplified; the only difficulty is inverting the various 6x6 matrices, however any sufficient numerical tool contains its own matrix inversion algorithm.
Using the material properties for the fibers and the matrix given in Table 2.1, the composite material properties were determined in this work for the carbon fiber composite and the glass fiber composite that comprise the ACCC core as a function of the volume fraction of fibers. Results for the axial and transverse modulus are presented in Figures 3.2 a and b. The ACCC composite contains a volume fraction of fibers of 60% [2-4], thus, results for material properties ($v_f = 60\%$) were fed into the finite element code to perform the modeling work (Table 3.1).

![Figure 3.2a- Comparison of the Eshelby method and Rule of Mixture axial modulus predictions](image-url)
Table 3.1 – Material properties \( (v_f = 60\%) \) for ACCC core used in subsequent finite element computations

<table>
<thead>
<tr>
<th>Property</th>
<th>ECR/Epoxy Composite</th>
<th>Carbon Fiber/Epoxy Composite</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial Modulus (GPa)</td>
<td>47.0</td>
<td>139.4</td>
</tr>
<tr>
<td>Transverse Modulus (GPa)</td>
<td>10.8</td>
<td>7.0</td>
</tr>
<tr>
<td>Axial Poisson’s Ratio</td>
<td>.214</td>
<td>.270</td>
</tr>
<tr>
<td>Transverse Poisson’s Ratio</td>
<td>.050</td>
<td>.010</td>
</tr>
<tr>
<td>Axial Shear Modulus (GPa)</td>
<td>6.3</td>
<td>2.9</td>
</tr>
</tbody>
</table>

3.1.2 – Thermal material property predictions

Determination of the thermal properties for uni-directional composite materials follows in a similar fashion to the mechanical properties. Expressions for the axial and transverse coefficient of thermal expansion are derived in [9, 34-35]; only the final equations are presented in this work

\[
\alpha_c^{\text{axial}} = \frac{\alpha_m (1 - v_f)E_m + \alpha_f v_f}{(1 - v_f)E_m + v_f E_f}. \quad (3.14)
\]

\[
\alpha_c^{\text{tr}} = \alpha_m (1 - v_f)(1 + v_m) + \alpha_f (1 + v_f) - \alpha_c v_c. \quad (3.15)
\]

\[
\alpha_c = \alpha_m - v_f \left[ \begin{bmatrix} C_m - C_f \end{bmatrix} \left[ S - v_f (S - I) \right] - C_m \right]^{-1} C_f (\alpha_f - \alpha_m). \quad (3.16)
\]
The axial coefficient of thermal expansion (equation (3.14)) is again based upon the slab model, and a force balance is used to derive the expression. Schapery developed equation (3.15) based on an energy consideration approach [34]. It should be noted that equation (3.15) is a function of the axial coefficient of thermal expansion calculated using equation (3.14). Finally, an approach using the Eshelby method can be used to determine the thermal expansivity of a composite (equation (3.16)). In equation (3.16) the C’s represent the respective 6x6 stiffness matrices, S is the 6x6 Eshelby matrix, I is the 6x6 identity matrix, and α’s are the respective 6x1 thermal expansion vectors [35]. Thermal expansion is a symmetric rank two tensor, which can be expressed as a 3x3 matrix; additionally, from linear algebra [36], a symmetric 3x3 matrix can be represented as a 6x1 vector quantity with no loss of information.

A comparison of the Eshelby method with the simple axial and transverse predictions was again made (Figure 3.2 a and b) using the constituent thermal properties given in Table 2.1.

![Figure 3.3a - Comparison of the Eshelby method and slab model axial CTE predictions](image-url)
Both of the simplistic methods show similar trends as the Eshelby method. However, for the purposes of material property definitions for the finite element models, the Eshelby method was used. Also, an interesting observation in Figure 3.2a is that at a volume fraction of 48%, the Eshelby method predicts that the carbon fiber composites’ CTE will be negative. Values for the axial and transverse coefficients of thermal expansion used in the finite element calculations are found in Table 3.2.

### Table 3.2 – Coefficients of thermal expansion (vf = 60%) used in subsequent finite element calculations

<table>
<thead>
<tr>
<th>Material</th>
<th>$\alpha_{axial}$ ($10^{-6}$)</th>
<th>$\alpha_{transverse}$ ($10^{-6}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon Fiber/Epoxy</td>
<td>-0.35</td>
<td>28.20</td>
</tr>
<tr>
<td>ECR Fiber/Epoxy</td>
<td>6.56</td>
<td>24.40</td>
</tr>
</tbody>
</table>

### 3.2 – Review of the Finite Element Method

The finite element method has been shown to be an invaluable tool for modeling field distribution problems since the 1960’s; particular favor was gained in stress field modeling applications [37]. However, a sufficient understanding of the analysis method must be
possessed by the analyst, or egregious errors may be made. Linear elastic finite element analyses are based in the general Hooke’s law, where the constitutive equation is

\[ [K]\{D\} = \{R\}. \]  \hspace{1cm} (3.17)

In which \([K]\) is known as the global stiffness matrix, and \([D]\) and \([R]\) are the nodal displacement and reactionary force vectors, respectively. The nodal displacement and reactionary force vector components are a combination of known quantities (from prescribed boundary conditions), and unknown quantities that are to be solved for. The global stiffness matrix is a bit more complicated to describe, as it is a function of material properties, as well as the shape function matrix, \([N]\). Shape function matrices are a function of the interpolation matrix, \([X]\), as well as the coefficient matrix, \([A]\) [37]. However, most importantly in the context of this work, shape function matrices are also a function of the global coordinates. For non-linear geometries where displacements are large, this causes the solution method to no longer be direct, but rather requires an iterative solution technique, which significantly increases required computation time [37].

There are various iterative solution methods available, e.g. – the secant method, the Newton-Raphson Method, and the modified Newton-Raphson Method [37-38]. Common commercial FEA packages will typically utilize the Modified Newton-Raphson method due to the savings in computational time offered as compared to other iterative solution methods [37-39]. The Modified Newton-Raphson method can be used to generate a load versus displacement curve for a structural member whose response is defined by the stiffness of the member. The way the method works, is an initial load, \([P_1]\), is applied and based upon the
initial tangent stiffness matrix, \([K_{t0}]\), a value for the displacement vector is calculated, call it \(\{u_a\}\) [39]. From these values, an estimate of the error, \(e\), in the solution can be made from

\[
\{e\} = \{P_1\} - [K_{t0}]\{u_a\}
\] (3.18)

One then calculates a change in displacement, \(\Delta u\), while keeping the load applied at \(\{P_1\}\) and the initial tangent stiffness matrix the same. The displacement and corresponding error are then updated, and the process continues, until the error is below a value specified for convergence, yielding the displacement \(\{u_1\}\) of the member for an applied load \(\{P_1\}\). The process is continued by increasing the applied load to a value of \(\{P_2\}\) and updating the value of the tangent stiffness matrix to \([K_{t1}]\). In this manner, one can define the entire load displacement curve for a member. Clearly the rate of convergence of the solution will be a function of the size of the increments in applied load.

Once a mesh has been generated, and the stiffness matrix defined, the final piece to obtaining the finite element solution is to integrate through the given load-displacement history. One could integrate through the entire volume in a three-dimensional problem at each individual node, or one could utilize Gaussian quadrature integration [37], also known as reduced integration [39]. The general method involves using fewer integration points than nodes in the element, weighted by a given amount dependent upon where the integration point is with respect to its position in the element. The weighting factors account for the changes in the field distribution throughout the element. Thus, in the case of a reduced integration 8-node hexahedral element (known as C3D8R elements in Abaqus), an integration point is applied at the center of mass of the element. This reduces the size of the number of integrations, which consequently reduces the required computational time, with a
minimal effect on the accuracy of the solution [39]. An added benefit of using reduced integration elements over full integration elements is that convergence of non-linear solutions is obtained quicker [39].
4 Finite Element Models of the ACCC Core subjected to in-service loading

The finite element method has proven to be an invaluable tool in predicting a component’s response to mechanical stresses [37]. In this work, several finite element analyses were performed in order to determine a most probable cause for short term failures of the ACCC core. The various analyses were performed to simulate conditions that the ACCC core would experience either during the manufacturing process, installation, or while in-service.

Three loading cases were examined in detail, namely, stresses due to thermal changes, uni-axial tension, and bending. The ACCC cores are produced via a unique pultrusion process, which simultaneously pulls both of the reinforcing fibers through the liquid epoxy resin. Hence, the constituent composites of the ACCC cores are subjected to an appreciable amount of stress from the cooling process due to the mismatch in thermal expansion coefficients (Figure 3.3). These stresses due to cooling needed to be evaluated to determine if they could be inducing any internal damage to the composite that would affect its short-term performance.

The second loading case considered was uni-axial tension. As the ACCC cores are the primary structural member of the conductor and are loaded in axial tension in-service,
uni-axial tension was investigated. It is a standard design practice to tension the lines to one-quarter of the rated tensile strength of the conductor [2-4], thus, one could assume that the lines should never fail in-tension as the safety factor is approximately 4. However, since the ACCC rods are transversely isotropic in nature a uni-axial tension model was developed to investigate if there were any unforeseen transverse or shear stress effects that could be causing structural damage.

Finally, a finite element model was developed to determine if bending of the ACCC rods could be causing structural damage to the load bearing core. It has been reported that uni-directional carbon fiber composite materials perform relatively poorly in flexure [9]. This poor performance in flexure is directly related to the large axial Young’s modulus of carbon fibers. Thus, using the finite element method, bending of ACCC cores was numerically simulated.

4.1 ACCC Thermal Model

4.1.1 – Thermal model methodology

A finite element model of the ACCC core was developed (Figure 4.1) to investigate the internal stress distribution of the composite core due to cooling. In the pultrusion process, continuous fibers are dipped in a molten resin, and then pulled through a die in order to obtain the desired shape. Cure of the pultruded composite occurs from an elevated temperature to approximately room temperature. It has been shown that pultruded composites are susceptible to fiber-matrix debonding upon cooling due to the mismatch in thermal expansion coefficients of the fiber and matrix [18-22, 40]. These stresses are often referred to as micro-stresses, and are well understood. However,
in the case of the ACCC core, it is foreseeable that a meso-stress could be developed at the composite interface that could result in a delamination.

To investigate the mechanical stresses induced in the ACCC composites due to cooling the core (Figure 4.1) was cooled from 250 °C to 25 °C. The ACCC core was modeled as having an outer diameter of 9.53 mm and the glass fiber/epoxy composite layer was modeled as being 1 mm thick; the rod had a length of 50 mm. The finite element mesh consisted of 1,000 8-noded hexahedral elements (C3D8R). Planes of symmetry were utilized, thus only one-fourth of the ACCC rod was modeled. Composite material properties were calculated using the modified Eshelby method, and are summarized in Tables (3.1 and 3.2); a volume fraction of fibers of 60% was used to determine composite material properties.
In order to establish an upper-bound on the stress state in the ACCC rods, all calculations in the model were governed by an implicit, linear-elastic constitutive model. Time dependent deformation of the matrix was not considered, nor was the stress relaxation effects’ of the matrix upon cooling. Thus, the thermal model presented here was truly an upper bound on the stress state of the ACCC rod subjected to a 225 °C temperature range.

4.1.2 Thermal model results

The thermal analysis confirmed that axially the glass fiber/epoxy composite should undergo a significantly greater amount of strain than the carbon fiber composite. This is most evident when viewing the free end of the rod on a relatively large deformation scale after cooling has occurred (Figure 4.2).

![Figure 4.2 – End effect of the ACCC rod cooled from 250 °C to 25 °C](image)

Because the glass fiber/epoxy composite is subjected to a greater amount of axial strain, one would expect the glass fiber composite to be in a state of residual axial tensile stress due to cooling, while the carbon fiber composite would be expected to be in a state
of residual axial compressive stress. Figure (4.3a) confirms the aforementioned assertion; the stress linearization plot depicts the axial stress state of the ACCC rod along the diameter. The stress linearization plot was taken along the rod radius in the mid-section of the rod.

![Figure 4.3 - Axial (a) and Transverse Radial and Tangential (b) Stresses along the Rod Diameter from the Thermal Analysis](image)

As for the transverse stress state, results are again in good agreement with the theory of elasticity. Figure (4.3b) shows that the radial stress is continuous across the composite material interface, while the hoop stress is markedly discontinuous. Additionally, it is observed that the radial stress decreases to zero at the edge of the ACCC rod. This agrees with the elasticity principle that radial stresses go to zero at free surfaces. All shear stresses were found to be smaller than 2 MPa; thus, they were deemed negligible.

It has been found that by optimizing the cooling cycle, these stresses (both micro- and meso) can be reduced via visco-elasticity [18-22]. Depending on the visco-elastic-plastic properties of the polymer matrix and the cooling rate, the residual stresses will be
significantly reduced in comparison with the data presented in Figures 4.3 a and b. Therefore, it can be safely estimated that the residual stresses inside the ACCC rods due to cooling calculated in this study governed by a linear elastic constitutive relationship are in fact much larger, by approximately 50%, than what is physically observed in typical polymer matrix composites [18-22].

Table 4.1 - Strengths of typical uni-directional composites based on carbon and glass fibers in epoxy resins [24, 41-44]

<table>
<thead>
<tr>
<th>Strength Component</th>
<th>Carbon Fiber/Epoxy</th>
<th>Glass Fiber/Epoxy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial Tension (MPa)</td>
<td>2450</td>
<td>850</td>
</tr>
<tr>
<td>Axial Compression (MPa)</td>
<td>1570</td>
<td>710</td>
</tr>
<tr>
<td>Transverse Tension (MPa)</td>
<td>70</td>
<td>40</td>
</tr>
<tr>
<td>Shear (MPa)</td>
<td>98</td>
<td>60</td>
</tr>
</tbody>
</table>

Typical strength values for long fiber glass/epoxy and carbon composites are presented in Table 4.1. Clearly, the axial, transverse and shear stresses obtained from the thermal analysis are far too small to cause any type of meso-cracking during pultrusion even while ignoring the visco-elasto-plastic effects. Hence, as long as due-diligence is taken during the post-cure cycle of manufacturing, mechanical stresses due to a mismatch in thermal expansion coefficients in the ACCC conductor should not be affecting the short-term performance of the core.

4.2 – Uni-axial tension model

4.2.1 – Uni-axial tension methodology

Using the FE model shown in Figure 4.1, the ACCC rod was also subjected to uni-axial tension. A load of 11.4 kN was applied to simulate the actual tensioning process
that the conductors experience in-service. Similar to the thermal case, planes of symmetry were utilized. Results were taken in the axial mid-section of the rod.

4.2.2 - Uni-axial tension model results

The uni-axial tension analysis was performed for comparison with the other load cases (thermal and bending). The axial and transverse stresses in the rod subjected to tension are shown in Figures 4.4a and b, respectively.

Figure 4.4 - Axial (a) and Transverse Radial and Tangential (b) Stresses along the Rod Diameter from the Uni-axial Tension Analysis

Obviously, if the ACCC rods are subjected to extensive tensile loads, micro-cracking and fiber failure will develop and the rods will fail catastrophically in-service. For the typical in-service loading considered in this study (11.4 kN) the tensile stresses in the carbon section of the rod are approximately 900MPa, which is approximately 37% of the ultimate tensile strength of a typical uni-directional carbon fiber composite (Table 4.1). In the glass part of the rod the axial stresses are slightly higher than 200MPa, which is less than 30% of the typical ultimate tensile strength for this type of composite. The
transverse and shear stresses were found to be negligible in comparison to the dominant axial stress state. It should be pointed out that the onset of significant damage, associated with matrix cracking, in pultruded uni-directional composite materials subjected to uni-axial tension occurs on average at approximately 70% of the load at failure [44-45]. Therefore it can be concluded that the tensile stresses that the ACCC rods are subjected to in-service are not of great enough magnitude to cause significant damage to the composite.

4.3 Small Bending Model

4.3.1-Small bending model methodology

In order to consider bending of the ACCC rods, another finite element model was created. The rod was modeled as having a length of 785.4 mm, with an outer diameter of 9.53 mm. Again, the glass layer was modeled as having a uniform thickness of 1 mm. Due to geometrical symmetry only one-half of the ACCC rod needed to be modeled. The model was comprised of 5,250 C3D8R elements. One end of the rod (subsequently referred to as the bottom of the rod) was constrained from moving in the axial direction. Additionally, the middle node on the bottom of the rod was constrained in all active degrees of freedom. This constraint was included to add stability to the system, and did not inhibit the Poisson Effect. All of the nodes at the other end of the rod (subsequently referred to as the top of the rod) were prescribed a displacement of 50 mm in the x-axis to simulate bending of the ACCC rod.
4.3.2 Small bending model results

The distribution of internal stresses of the ACCC rod subjected to bending proved to useful in determining a most probable failure mechanism. As an example, the von Mises stress contour plot is shown in Figure (4.5). From Figure (4.5) it is observed that there are large stress concentrations generated at the carbon fiber composite/ glass fiber composite interface.

![Figure 4.5 - von Mises stress contour plot of ACCC rod subjected to linear-elastic bending](image)

Examination of the stress linearization plot (Figure 4.6) along the diameter of the ACCC rod subjected to bending provided more insight into the nature of the stress distribution.

![Figure 4.6 - Axial (a) and Transverse Radial and Tangential (b) Stresses along the rod diameter from the small bending analysis](image)
From Figure 4.6 it is observed that the axial stress state is again dominant. Transverse stresses were found to be symmetric and on the order of a few MPa; likewise, all shear stresses were found to be less than 1 MPa in magnitude. Examination of Figure (4.6a) explains the von Mises stress contour presented in Figure (4.5). A large jump discontinuity at the interface of the carbon fiber composite and glass fiber composite interface is evident. The jump discontinuity may be counterintuitive as bending stress in beams is typically greatest at the outer fibers of specimens. However, the stress concentration at the composite interface can be explained by the fact that the carbon fiber composite has a significantly larger axial Young’s modulus than does the ECR glass fiber composite. Hence, it will carry a greater amount of the applied bending moment.

For a displacement of 50 mm the maximum axial tensile and compressive stresses in the carbon fiber composite were found to be 300 MPa. This amount of stress is not of the correct magnitude to be causing structural damage to the composite. However, these stress levels indicated that bending could be of concern, and needed to be considered in-depth. For the linear-elastic bending model presented here, the assumptions are made that the mesh deformation is small, and that the loading direction does not change. In order to investigate the stress distribution within the ACCC rod while it is being wrapped around a mandrel a far more sophisticated model needed to be developed.

4.4 - Non-Linear Bending Model

4.4.1- Non-linear bending model methodology

In order to transport the ACCC conductor to the job sites, the manufacturers wrap the hybrid composite material around mandrels. These mandrels typically have a
Thus, to model the ACCC subjected to a realistic bending scenario the finite element model developed in §4.3 was modified. A rigid mandrel that had a diameter of 1 m was added to the model; due to planes of symmetry only one-quarter of the rigid mandrel needed to be modeled. The rod length was again 785.4 mm (this was done such that the length of the rod matched the arc length of the rigid mandrel). The outer diameter of the rod was again 9.53 mm, while the glass sheath remained 1 mm thick. The entire model, including the rigid mandrel consisted of a total of 6525 C3D8R elements.

To achieve wrapping of the ACCC rod around the rigid mandrel a penalty based contact definition was defined between the outer surface of the glass layer and the rigid mandrel with a penetration tolerance of .005 mm. No frictional sliding between the rod and the mandrel was considered in this study. The end of the rod initially in contact with the rigid mandrel was constrained from moving in the axial direction. Additionally, the node that was initially in contact with the mandrel was constrained from moving in all translational degrees of freedom. Nodes at the opposite end of the rod had rigid connecting beam elements attached to a node off of the rod in order to apply the load to the rod. This loading mechanism allowed for loads to be transferred effectively to the rod, without inducing severe hour-glassing in the elements [39]. The axial fiber direction was initially aligned with the global z-axis in the model.

An iterative solution method (described in §3) was used to solve the geometrically non-linear model. This solution method is computationally expensive, but is a very powerful solution method. In the iterative solution method, a load is prescribed and the
corresponding displacements of the nodes are calculated; the displacement solution is iteratively refined until convergence is obtained. Another complication introduced by a geometrically non-linear model is that the mesh undergoes a significant amount of deformation. Recall that the global stiffness matrix, \([K]\), is a function of the nodal coordinates. Hence, as the nodal coordinates changed with the excessive deformation, the global stiffness matrix must be recalculated at each step, thus the analyses become quite computationally expensive.

### 4.4.2 - Non-linear bending model results

The rod was fully wrapped around the mandrel in one second which was taken as an arbitrary time unit, as the solution method was implicit. The finite element model of the ACCC rod subjected to large bending is shown in Figures 4.7 (a) before and (b) after bending.

![Figure 4.7 - Large bending model; (a) Before and (b) After deformation](image)

Loads were applied in the global x and z axes in order to fully deform the ACCC rod around the mandrel (Figure 4.8). Loads in the x-direction were linearly ramped,
while loads in the z-direction were weighted toward the end of the analysis in order to fully deform the ACCC rod around the mandrel.

![Graph showing load applied as a function of time for the 1 m drum analysis.](image)

**Figure 4.8 - Applied load as a function of time for the 1 m drum analysis**

The maximum axial compressive stress in the middle of the rod as a function of the total applied load is presented in Figure 4.9. A plateau region is observed in which the axial compressive stress remains constant as more load is applied. This plateau occurs because the stress state is governed by the deformation that the rod undergoes (particularly the diameter and curvature of the mandrel). Once a rod has conformed to the mandrel, the applied bending moment is no longer changing in that section of the structure, resulting in a constant stress state.
The stress distributions in the rod subjected to large bending were similar to the small bending case (Figure 4.6). However, significant deviations were observed. The axial and transverse (radial and tangential) stresses along the rod diameter are illustrated in Figures 4.10 a and b, respectively. Results were taken from the mid-section of the rod upon completion of the wrapping process. Notice that the maximum axial stresses are again located at the glass composite/carbon composite interface. For the 1 m diameter mandrel the maximum axial tensile and compressive stress state in the rod was found to be approximately 1GPa (Figure 4.10 a). The transverse stresses were found to be very small (Figure 4.10 b), and similar in nature to the small bending cases. It was noted that the transverse stress distributions deviated slightly from the linear bending case (Figure 4.6 b), due to the rod contacting the mandrel, and the penalty based contact definition.
The axial stress state (Figure 4.10 a) is again similar to the case of the small bending analysis; however, the stress gradient is not perfectly symmetric, in good agreement with [46].

The region that is put in the greatest amount of tensile stress is not likely to fail as the tensile strength of uni-directional carbon fiber composites is far greater than the induced stress state. However, the region that is put into the largest compressive stress has a far more considerable chance of incurring damage. The compressive strength reported in Table 4.1 is reported for Toray T700S fibers in an aerospace grade composite material [24]. Aerospace grade composites are manufactured via vacuum bag injection molding (or a similar process) which allows for incredibly precise fiber alignment. Thus aerospace grade composites perform well both in tension and compression. Conversely, the pultrusion process does not offer as great a control over the fiber alignment. Several studies on the effects of fiber misalignment on material properties have been carried out,
and have concluded that tensile strength should be relatively unaffected [9], while compressive strength decreases significantly with increasing fiber misalignment [43]. The compressive strength is reduced because as the fiber is misaligned further from the axial direction, the amount of load that it can bear is reduced. Hence, bending around mandrels was determined to be the most probable cause of short-term damage to the ACCC core. It is hypothesized that if bent too excessively, damage will be initiated in the compression zone of the carbon fiber composite and propagate throughout the composite.
5 – Excessive Bending of ACCC Cores

From the results obtained in §4 it was hypothesized that bending of ACCC rods around mandrels for transportation and installation purposes was the most damaging loading condition. Thus, the parameters that govern the compressive axial stress state in the ACCC rod during bending needed to be quantified. Two parameters were investigated in depth; first, the effect that the thickness of the glass composite layer had on the axial stress state of the hybrid composite was quantified. The second parameter investigated was the effect of the size of mandrel upon which the rod was wrapped around.

Finally, it was hypothesized that the compressive strength of the ACCC composite (in particular the carbon fiber composite section) is significantly lower than the compressive strength specified by the manufacturer due to the misalignment of the fibers with the axial direction. This hypothesis was evaluated through a series of compression tests performed on both entire ACCC specimens and specimens that were comprised of only the carbon fiber composite core. Additionally, acoustic emission events were monitored during the compression tests to qualitatively determine if a different failure process existed between the hybrid composite and the uni-directional carbon fiber composite.
5.1 Effect of the thickness of the ECR glass fiber composite

5.1.1 – ECR glass fiber composite thickness effect model methodology

The non-linear bending model of the ACCC rod (Figure 4.7) was modified in order to determine what the effect of the thickness of the glass fiber composite layer was on the axial stress state of the ACCC rod. In the original model, the glass fiber composite was modeled as having a uniform thickness of 1 mm. Four additional models were created with varying glass thicknesses to see how the axial stress state would be affected; the glass thicknesses are summarized in Table 5.1.

<table>
<thead>
<tr>
<th>ECR Glass Fiber Composite Thickness (mm)</th>
<th>Carbon Fiber Composite Diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>7.13</td>
</tr>
<tr>
<td>1.1</td>
<td>7.33</td>
</tr>
<tr>
<td>1.0</td>
<td>7.53</td>
</tr>
<tr>
<td>0.9</td>
<td>7.73</td>
</tr>
<tr>
<td>0.8</td>
<td>7.93</td>
</tr>
</tbody>
</table>

In all of the models the outer diameter of the entire ACCC rod remained a constant 9.53 mm. Hence, increasing the thickness of the glass fiber composite layer meant that the ratio of ECR glass fiber composite to carbon fiber composite was also increasing.

5.1.2 – ECR glass fiber composite thickness effect model results

Again, all models were run in ABAQUS v.6.6.1™. An iterative solution method was used to solve the geometrically non-linear problem. From the work done in §4.4.2 it
was determined that the axial compressive stress generated in the carbon fiber composite section of the ACCC composite would be the limiting performance metric of the ACCC composite in bending. Thus, the maximum compressive stress from each analysis was extracted and used to quantify the effect that the thickness of the ECR glass fiber composite had on the stress state within the rod.

It was observed that the thickness of the glass layer had a profound effect on the stress state within the rod. A linear relationship between the maximum axial compressive stress and the thickness of the glass layer was found, with the observed compressive stress decreasing with increasing glass sheath thickness (Figure 5.1).

\[ y = -262.03x + 1227.4 \]

\[ R^2 = 0.9998 \]

**Figure 5.1 - Maximum compressive stress as a function of the ECR glass fiber composite thickness**

With the outer diameter of the rod fixed, and the thickness of the glass layer increasing, the distance of the interface of the composite materials to the neutral axis was decreasing. A decrease in the distance from the neutral axis to the composite interface explains the linear dependence that the axial compressive state had on the glass thickness;
recall, that bending stress is directly proportional to the distance away from the neutral axis. It was noted that the maximum stress observed in the glass remained unaltered because the outer diameter remained constant. The observed stresses in the glass composite layer were not large enough to cause failure of the composite in either tension or compression.

5.2 – Mandrel size effect

5.2.1 – Mandrel size effect model

To investigate the effect of the size of the mandrel upon which the ACCC rod is wrapped six different mandrel diameters were considered. The six mandrel diameters considered were 500mm, 750mm, 1000mm, 1500mm, 2000mm, and 3000mm. The rods themselves were modeled as having an outer-diameter of 9.53 mm, and the ECR glass composite sheath was modeled as having a uniform thickness of 1mm.

For all analyses in this section, a constant mesh density was maintained both transversely and axially. This was done due to the varying lengths of the rod for the different mandrel sizes. Transversely the mesh density remained 20 elements on the major diameters of the rod, while the axial mesh density was maintained at 75 elements for the varying rod lengths.

5.2.2 – Mandrel size effect results

The analyses showed that for all of the mandrel diameters the axial compressive stress state in the middle of the rod became constant when that section of the rod had fully conformed to the curvature of the mandrel. Plateau regions (similar to Figure 4.9) for each mandrel diameter were observed (Figure 5.2). Once a rod conformed to the
curvature of the mandrel, the applied bending moment was no longer changing, thus nor was the bending stress.

![Graph showing the relationship between applied load and axial compressive stress for different mandrel radii.](image)

**Figure 5.2 - Maximum axial compressive stress as a function of the mandrel radius**

Mandrels with smaller radii required a greater applied load in order to fully deform the rod around their respective mandrel. This makes intuitive sense, as greater deformation is undertaken when rods are bent around smaller radii mandrels. These analyses illustrated that the observed stress states in the ACCC rods were controlled by the geometry of the model, in particular the radius of the mandrel, and not controlled by the applied load once the rods became fully deformed.

After the rods were fully deformed around their respective mandrels, the maximum compressive stress was again observed at the carbon composite/glass composite interface (Figure 5.3). When values of the axial stress components in the middle of the rod were plotted against the mandrel radius, an inverse relationship was
found (Figure 5.4). This is in good agreement with an analytical solution of a homogeneous, isotropic beam deformed in a circular manner.

![Diagram](image)

**Figure 5.3 - Concentration of axial tensile and compressive stresses at the interface between the two different composites**

![Graph](image)

**Figure 5.4 - Maximum axial compressive stress as a function of the mandrel radius**

\[ y = 418448x + 89.365 \]

\[ R^2 = 0.9873 \]
5.3 – Compression Testing

5.3.1 – Compression testing procedure

Since carbon fiber composites are known to have a much lower compressive strength than tensile strength [9], the presently available ACCC rods needed to have their compressive strength evaluated. It was not obvious how the glass layer on the composite would affect the ultimate compressive strength of the rod, thus, two sets of compression experiments were run per ASTM standard D695 [47] on as received lines of the ACCC conductor.

All specimens were prepared per the guidelines set out in [47]. The first set of compression experiments were run with the ACCC rod completely intact (glass fiber composite encompassing the carbon fiber composite). Specimens had an outer diameter of 9.53 mm, thus specimens were made to a length of 19.05 mm (ASTM D695 requirement for cylindrical samples). While preparing the specimens, a few techniques proved to produce excellent results. Specimen flatness was achieved on a Bridgeport Series 1 – 2J end mill. A carbide cutting tool was used at a speed of 65.8 smpm. It was found that by climb-milling the perimeter of the specimen that fraying of the edges could be eliminated. After flatness was achieved, specimens were wet-polished with 600 grit Tri-M-Itte Emory cloth. Before specimens were tested they were viewed under an optical microscope to insure that no tool marks or preparation induced damage was evident.

The second set of experiments was performed on specimens that had the glass layer removed. Removal of the glass layer was performed on a swing lathe at a speed of 11.9 smpm, with a feed rate on 1.02 mm/s. The machined rod was then wet polished
while still in the lathe with 600 grit Tri-M-It Emery cloth. While doing this, it was found that the maximum achievable outer diameter for the carbon fiber composite specimens was 6.10 mm; because of this constraint, the gage length of the carbon fiber composite samples was altered to 12.20 mm in order to comply with [47]. All other specimen preparation steps remained identical.

Compression tests were performed on an MTS 880 servo-hydraulic test machine following [47] (Figure 5.5).

![Figure 5.5 - An ACCC specimen in the compression test fixtures with the AE sensor mounted on the top of one of the grips.](image)

Specimens were tested in stroke control. Displacement and load outputs were measured using proprietary software produced by Digital Wave. Additionally, acoustic emission was monitored using the Digital Wave software in conjunction with a single Digital Wave B-1025 transducer. A Digital Wave PA2040G/A pre-amplifier with a 40 dB gain was used to amplify signals for collection. Signals were passed through a 20 – 4000 kHz band pass filter. The trigger was filtered with a band pass filter of 50 – 500 kHz. A threshold value of .1 V was used to gather the AE events. Finally, the signal had a gain of 6 dB, while the trigger had a gain of 12 dB. Load, displacement, and acoustic emission data were collected and stored on a personal computer for further analysis.
Once the ultimate compressive strength of both sample preparations was determined, proof tests on each type of specimen were performed. Specimens were loaded to 80% of their respective ultimate compressive strength, and then immediately unloaded. The proof test samples were then split axially, polished, and examined using Scanning Electron Microscopy (SEM). The micro-structure of the proof tested specimens was then compared to the micro-structure of the failed specimens to gain insight into the failure process.

5.3.2 – Compression testing results

Seven replicates were tested for each type of specimen. The average ultimate compressive strength and standard deviation for both tested composites are summarized in Table 5.2. Representative stress-displacement curves for the carbon fiber only and ACCC composites are shown in Figures (5.6) and (5.7). Well defined Hookean regions were observed, followed by a catastrophic failure characterized by the significant drop in stress level. The acoustic emission events curve is superimposed on the stress-displacement curve. It is seen that the entire ACCC composite exhibits a greater number of acoustic emission events when compared to the carbon fiber composite.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Mean (MPa)</th>
<th>Standard Deviation (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon Fiber Composite</td>
<td>724</td>
<td>44</td>
</tr>
<tr>
<td>ACCC</td>
<td>615</td>
<td>62</td>
</tr>
</tbody>
</table>

Table 5.2 - Ultimate Compressive Strength for the Carbon Fiber composite and ACCC rod
It appears that the average ultimate compressive strength for the carbon fiber composite (724 ± 44 MPa) is greater than that of the entire ACCC composite (615 ± 62 MPa).
MPa). Two statistical tests were performed to test this hypothesis. First, an F-test for equality of variances was performed. From the test, the null hypothesis is accepted, and it is concluded that there is no statistical difference in variance between the two samples ($F_{\text{TEST}} = 1.974; \text{df} = 6, 6; P = .214$). Since the variance of the two sets of samples was statistically equivalent, a t-test for equality of means assuming equal variances was performed. The null hypothesis is strongly rejected, and it is concluded that mean ultimate compressive strength for the carbon fiber composite is statistically greater than that of the entire ACCC rod ($t_{\text{TEST}} = 3.809; \text{df} = 12; P = .001$).

Additional statistical work was performed on the number of acoustic emission events. Results for the average number of acoustic emission events for the carbon fiber composite as well as the ACCC specimens are shown in Table 4. An F-test for equality of variance concludes that the variances in the two sets were statistically equivalent ($F = 1.599; \text{df} = 4, 3; P = .365$). A t-test for equality of means assuming equal variances showed that the number of acoustic emission events was statistically greater for the ACCC specimens than for the carbon fiber composite specimens ($t = 11.338; \text{df} = 7; P = 4.6 \times 10^{-6}$). The fact that more acoustic emission events were recorded for the ACCC rod than for the carbon fiber composite indicates that there is a different failure process in the hybrid composite.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon Fiber Composite</td>
<td>21</td>
<td>8</td>
</tr>
<tr>
<td>ACCC Composite</td>
<td>95</td>
<td>11</td>
</tr>
</tbody>
</table>

Table 5.3 - Statistics of the Acoustic Emission Events for the carbon fiber composite and ACCC rod
5.3.3 – SEM analysis of compressed specimens

Scanning Electron Microscopy of the two different specimens was able to confirm the difference in failure processes. For the uni-directional carbon fiber composite, compressive failure mechanisms (kinked fibers resulting in a shear type failure) are abundant in the literature and are fairly well understood [9, 43, 48-49]. Kink bands were prevalent in the carbon fiber composite specimens that were tested to failure, as well as in the 80% proof tests (Figure 5.8).

![Figure 5.8 - Representative SEM image of kink-banded carbon fibers due to a compressive stress](image)

For the ACCC specimens, examination of the micro-structure proved very useful in determining the failure process, and explaining the greater number of acoustic emission events observed. Formation of the kink band with several broken fibers and severe matrix damage following the shear failure line was evident in the ACCC specimen that was 80% proof tested (Figure 5.9). However, no damage was observed in the ECR
glass fiber/epoxy composite, nor near the interface in the ACCC specimens that were proof tested. Upon examining the micro-structure of a failed ACCC specimen, the reason for the relatively large number of acoustic emission events became evident. Kink bands were again evident in the carbon fiber composite, similar in nature to the damage in the ACCC specimens that were 80% proof tested (Figure 5.9), but a severe delamination was observed at the interface of the two composites in the specimens that were failed (Figure 5.10). This delamination explains the far greater number of acoustic emission events registered [50].

Figure 5.9 - SEM images of an ACCC specimen exhibiting kinked carbon fibers (evident in both the proof tested specimens and the failed specimens)
Figure 5. 10 - Delamination of the carbon fiber composite from the ECR glass fiber composite (only evident in the failed ACCC specimens)

From the compression tests performed in this work, the hypothesis was confirmed that the ultimate compressive strength of the carbon fiber composite produced via pultrusion for the ACCC composites has a lower compressive strength than what is specified by the manufacturer was confirmed. Lee et al have shown that misalignment of fibers with the axial direction results in a significant decrease in the amount of load that a uni-directional carbon fiber composite can bear in compression [43]. The result is that the ACCC composite will not perform as well in flexure as one would expect if using the manufacturers specified compressive strength as the design criteria.
6 – Probabilistic study of the ACCC core subjected to excessive bending

The previous sections of this work have identified excessive bending as the most likely cause of short term damage to the ACCC core. Several parameters were investigated for their effects on the internal stress state within the ACCC core. However, in all of the models used to investigate the internal stress state of the ACCC core, a simplification in the geometry was made. In all of the models it was assumed that the carbon fiber composite was perfectly circular, and was concentric with the glass fiber composite. When viewing a cross-section of the hybrid composite (Figure 6.1), it is evident that neither of these assumptions are correct.

Figure 6.1 – ACCC core highlighting the non-circular and non-concentric carbon fiber composite
6.1 – Probabilistic FE model of the ACCC core

6.1.1 – Methodology for the probabilistic finite element study

To improve the accuracy and robustness of the finite element model that wraps the ACCC core around a rigid mandrel the Monte Carlo method was employed to randomize the geometry generation of the carbon fiber surface. This was done with knowledge of the underlying distribution of the carbon fiber diameter.

In order to determine the distribution of the carbon fiber composite diameters, seventy-two measurements were made of randomly selected cross-sections of the ACCC core with a set of engineering calipers. The cumulative density function (CDF) of the carbon fiber composite radii can be seen in Figure 6.2. From the seventy-two measurements an average radius of the carbon fiber composite was determined to be 3.55 mm, with a standard deviation of .15 mm. It is seen in Figure 6.2 that the carbon fiber radius distribution is fit well by a normal distribution at the 1% significance level ($K$-TEST = .057; n=72; $K$-CRIT=.192) [51].

![Figure 6.2 – Cumulative density function of the average carbon fiber radius](image)
To incorporate the Gaussian behavior of the carbon fiber radius into the finite element model, the perimeter of the carbon fiber surface was discretized into 30 points. A radius was then randomly generated for each point, based upon Monte Carlo sampling of the distribution shown in Figure 6.2. The 30 randomly generated points were then used to create the carbon fiber surface. Next, the glass fiber surface was created, both surfaces were meshed, and the faces of each component were extruded into three-dimensional continuum elements. A schematic of the geometry transformation is shown in Figure 6.3.

![Figure 6.3 – Schematic of the geometric transformation made in the FE model of the ACCC core](image)

Similar to the previous finite element models, the axial mesh density was maintained at a value of seventy-five elements for each analysis. The transverse mesh density was twenty-six elements along the major diameters of the rod. The rigid mandrel was modeled as having a diameter of 1 m. Linear-elastic material properties were calculated via the Eshelby method, for a volume fraction of fibers of 60% (§3). Loads were again applied to the ACCC rod through a concentrated force applied to a node that was rigidly beamed to the nodes on the free end of the rod. Contact was defined through a penalty based contact definition. Reduced integration continuum elements (C3D8R)
were used for computational efficiency, with minimal effect on the accuracy of the solution [37, 39].

It was expected that the geometric inconsistencies introduced into the model would have several effects on the internal stress state. Depending on the magnitude of the carbon fiber radius, the thickness of the glass layer will be varied, and so too then would be the stress state (based upon the results presented in §5.1.2). Additionally, in locations with relatively sharp changes in curvature local geometric stress concentrations will be introduced [46, 52]. In order to develop a substantial distribution of the stress state within the ACCC core 100 geometrical configurations were considered.

As it has been shown previously in this work the limiting performance metric of the ACCC core when subjected to bending will be the compressive strength of the carbon fiber composite, thus, the maximum compressive stress in the carbon fiber section of each finite element analysis was extracted to build the stress distribution, hereafter referred to as \( \sigma_{FEM} \).

6.1.2 – Results of the probabilistic finite element study

Upon completion of the 100 finite element analyses a distribution of the limiting performance metric was acquired. The average stress from the distribution was 926 MPa, with a standard deviation of 25.5 MPa. The cumulative density function of the finite element stress distribution is presented in Figure 6.4. It is seen in Figure 6.4 that the finite element stress distribution is fit very well by a normal distribution at the 1% significance level (K-S\(_{\text{TEST}} = .053\); df = 100; K-S\(_{\text{CRIT}} = .163\)).
Figure 6.4 – Cumulative density function of the finite element stress generated from 100 geometric configurations

Figure 6.5 provides examples of the best and worst case geometrical conditions considered in the study. Figure 6.5a is an example of a relatively safe geometrical condition; on the compression side of the rod the glass fiber composite is relatively thick, and the interface is relatively circular. This combination of parameters results in a significantly lower stress state observed in the ACCC core. Conversely, when viewing Figure 6.5b a relatively dangerous geometrical condition exists. It is observed that on the compression side of the rod, the glass fiber composite is relatively thin, and the carbon fiber composite juts out into the glass fiber composite. This geometrical irregularity results in a rise in observed stress values, and is more dangerous to the structural integrity of the ACCC core.
Figure 6.5 – (a) Lower compressive stress state in the carbon fiber composite due to relatively thick glass layer and smooth geometric changes. (b) Higher compressive stress state in the carbon fiber composite due to sharp geometric changes, and the relatively thin glass fiber composite layer

6.2 – Four point bending tests of the ACCC core

6.2.1 – Experimental methods for the four point bend tests

To measure the strength of the ACCC core in bending, four point bend tests were performed on ACCC specimens using a newly designed test fixture. Four point bend tests are a standard method of measuring the flexural strength, a.k.a. the rupture strength, of a material. The four point bend test evolved from the simpler three point bend test, and is now the preferred method for measuring the flexural properties of materials due to the uniform bending moment that is generated within the gage section.

Several parameters were incorporated into the design of the test fixture. First, since acoustic emission was to be monitored, the loading points of the fixture were designed to be rolling point contact. Designing the loading pins to be rolling was intended to reduce the frictional noise that would occur between the sample and the fixture.
Modifications to the loading pins had to be made as well. Traditional four point bend fixtures use a round geometry for the loading pins; however, traditional fixtures are intended to test flat specimen geometries. Since the ACCC cores are round, using round bars would result in a point contact that would cause an extreme stress concentration at the loading point. Thus, to maintain a line contact of stress, the loading pins were designed with a saddle machined in them to seat the ACCC specimens (Figure 6.6).

The final major design parameter that was altered from standard practice was the span length. Traditional outer span lengths are 16 to 24 times the diameter of the specimen to be tested [53]. However, the outer span length of this fixture was designed to be 32 times that of the outer diameter of the Drake sized ACCC core (i.e. – 304.8 mm). The longer span length was assumed to more realistically represent the process of wrapping the ACCC core around a mandrel. The inner span length was made to be 101.6 mm; the design of the four point bending fixture results in what is known as a one third span set-up. A schematic of the fixture geometry is given in Figure 6.7. Specimens were 365.7 mm in total length per the guidelines of [54], this allowed for 10% specimen overhang on each side. This configuration allowed for convenient coupling of the AE transducers to the specimen.
To calculate the applied bending moment to the specimens, the process suggested in [52] was utilized. The loading equation, \( q(x) \), (all lengths, \( x \), are defined in millimeters, while loads, \( R \) and \( P \), are in Newtons) is defined as

\[
q(x) = R(x - 30.48)^{-1} - P(x - 132.08)^{-1} - P(x - 233.68)^{-1} + R(x - 335.28)^{-1}
\]

(6.1).

Integration of the loading function results in the shear function, \( V(x) \)

\[
V(x) = R(x - 30.48)^0 - P(x - 132.08)^0 - P(x - 233.68)^0 + R(x - 335.28)^0 + C_1
\]

(6.2).

Integration of the shear function yields the moment function, \( M(x) \)

\[
M(x) = R(x - 30.48)^1 - P(x - 132.08)^1 - P(x - 233.68)^1 + R(x - 335.28)^1 + C_1x + C_2
\]

(6.3).

Because the reactionary forces were included in the loading expression, the constants \( C_1 \) and \( C_2 \) reduce to zero. By subjecting equations (6.2) and (6.3) to the boundary conditions \( V(L^+) = M(L^+) = 0 \), it is found that

\[
R = P
\]

(6.4).

Finally, from the geometry of the test fixture it is clear that the load detected at the load cell, \( F_{LC} \), will be twice the load at an individual loading pin, i.e. –

\[
F_{LC} = 2P
\]

(6.5).
The moment diagram for an arbitrary load of 2670 N measured at the load cell is presented in Figure 6.8. Figure 6.6 reiterates the usefulness of the four point bend test, demonstrating the uniform bending moment within the gage section.

The maximum applied bending moment will be uniform throughout the gage section, and can be calculated via equation (6.3). Due to the differences in material properties between the glass fiber composite and the carbon fiber composite, particularly axial stiffness in this case, the portion of the applied bending moment carried by each material will be different. The total applied bending moment, $M_A$, will be equal to the sum of the bending moments carried by the carbon fiber composite and the glass fiber composite

$$M_A = M_C + M_G \quad (6.6)$$

Additionally, from elasticity [55] it is known that
\[ \frac{M_G}{I_G} = \frac{E_G}{R} \quad (6.7) \]

\[ \frac{M_C}{I_C} = \frac{E_C}{R} \quad (6.8) \]

where \( I_G \) is the second area moment inertia of the glass fiber composite, \( E_G \) is the axial modulus of the glass fiber composite, and \( R \) is the radius of curvature. Similarly, \( I_C \) is the second area moment of inertia of the carbon fiber composite, \( E_C \) is the axial modulus of the carbon fiber composite, and \( R \) is the radius of curvature. The second area moments of inertia can be calculated for the carbon fiber composite and the glass fiber composite assuming that the carbon fiber composite is perfectly circular in nature, and has a diameter of \( d_{\text{CARBON}} \) (measured in §6.1.1) via

\[ I_C = \frac{\pi d_{\text{CARBON}}^4}{64} \quad (6.9) \]

\[ I_G = \frac{\pi(d_{\text{ACCC}}^4 - d_{\text{CARBON}}^4)}{64}. \quad (6.10) \]

Rearranging expressions (6.7) and (6.8) and summing the two expressions yields

\[ (M_G + M_C)R = E_G I_G + E_C I_C. \quad (6.11) \]

Substituting in the applied bending moment from equation (6.6) and solving for \( R \) results in

\[ R = \frac{E_G I_G + E_C I_C}{M_A}. \quad (6.12) \]

Finally, the bending stress, \( \sigma_{\text{BEND}} \), in the specimen can be calculated

\[ \sigma_{\text{BEND}} = \frac{E y}{R} \quad (6.13) \]
where $y$ is the distance from the neutral axis. Equation (6.13) implies that the bending stress at the interface will be greater in the carbon fiber composite due to its’ larger axial modulus. The bending stress calculated via equation (6.13) in the carbon fiber composite is valid for $y$ values in the range $[-3.55, 3.55]$. Additionally, the bending stress in the glass fiber composite is valid for $y$ values within the intervals $[-4.76, -3.55)$ and $(3.55, 4.76]$.

Four point bend tests of the ACCC core were performed on an open loop MTS 880 servo-hydraulic test frame. Tests were performed in displacement control at a loading rate of 3 mm/min. A Digital Wave data acquisition unit was used to monitor load, and crosshead displacement. The Digital Wave data acquisition unit was also used for monitoring the acoustic emissions. Two Digital Wave B-1025 transducers were used to acquire acoustic emission signals. A Digital Wave PA2040G/A pre-amplifier with a 40 dB gain was used to amplify signals for collection. Signals were passed through a 20 – 4000 kHz band pass filter. The trigger was filtered with a band pass filter of 50 – 500 kHz. A threshold value of 300 mV was specified. Finally, the signal had a gain of 3 dB, while the trigger had a gain of 6 dB. The transducer was attached to the samples by means of a compliant spring (Figure 6.9). Petroleum jelly was used as a couplant to transmit the acoustic emissions from the specimen to the transducer.
6.2.2 – Results of the four point bend tests of the ACCC core

Five experiments were performed in accordance with the test protocol prescribed in §6.2.1. From the five tests an average load at failure of 2546 N was observed, with a standard deviation of 52 N. A representative load-displacement curve for the ACCC core subjected to four point bending is shown in Figure 6.10. A failed ACCC specimen is shown in Figure 6.11.

![Figure 6. 9 – AE transducer mechanically coupled to an ACCC specimen](image)

![Figure 6. 10 – Representative load-displacement curve for the ACCC core subjected to bending](image)
From the load-displacement curve, it is observed that the materials’ response becomes non-linear at relatively large displacements. Interestingly the non-linear response of the material becomes evident at approximately the same time as the onset of significant acoustic emission. The onset of significant acoustic emission begins when the rate of acoustic emission event acquisition rapidly increases; the onset of significant acoustic emission occurs at approximately 31 mm of crosshead displacement in Figure 6.10. The signals that were acquired before the onset of significant acoustic emission could be coming from any number of sources (frictional rubbing between the specimen and the loading pins, matrix cracking, etc.), however, after signal analysis, the events were very low energy, and do not indicate that a significant amount of damage was occurring to the specimen. The gain, threshold, and band-pass filters will all play a role in the look of the signal, but should not affect the onset of significant acoustic emission.
Using equation (6.13) and the loads at failure, the average maximum bending stress in the lot of specimens tested was found to be 2104 MPa with a standard deviation of 43.0 MPa. The bending stress at the onset of significant acoustic emission was also determined using the loads at the onset of significant acoustic emission and equation (6.13). It was found that the average bending stress at the onset of significant acoustic emission in the carbon fiber composite was 1754 MPa with a standard deviation of 154 MPa. The compressive bending stress is the performance metric of interest in this case, as it has been identified as the limiting quantity; thus, all bending stresses were calculated for the maximum compressive region of the carbon fiber composite.

To determine whether or not the non-linear response of the material was permanent, a cyclic test of the ACCC core was performed. An ACCC specimen was loaded in 5 mm increments of crosshead displacement, and then unloaded back to zero displacement. Load and displacement were continually monitored; the cyclic load-displacement curve is presented in Figure 6.12.

Figure 6.12 – Cyclic loading of the ACCC core
From Figure 6.11 it is observed that the non-linear material response begins to occur at approximately 25 mm of crosshead displacement. On each unloading cycle the hybrid composite material exhibits elastic behavior and returns to its’ initial state. Interestingly, during the cyclic load test, the ACCC core failed during unloading of the specimen. On the eighth cycle, the specimen reached a maximum load of 2487 N, but failed at a slightly lower load of 2335 N.

The acoustic emission data lends insight into the failure process of the material; the acoustic emission events curve is superimposed on the load displacement curve of the eighth cycle in Figure 6.12.

![Figure 6.13 – AE events curve superimposed on the load-displacement curve during the failure cycle of the ACCC core](image)

The Felicity Ratio of a material is the percentage of the previous load a specimen has been subjected to at which the onset of significant acoustic emission occurs [56]. A material that has suffered no permanent damage will exhibit a Felicity Ratio of 1, which is known as the Kaiser Effect. It is seen in Figure 6.12 that the Felicity Ratio is relatively
high (88%). This observation indicates that not a great deal of permanent damage had
been done to the hybrid composite during the first seven cycles. Additionally, at the
onset of significant acoustic emission, the rate of signal acquisition is relatively large. On
the seven previous cycles, very few acoustic emission events were registered during the
unloading portion of the cycle. However, on the eighth cycle, the acoustic emission
signals were still being registered during the unloading portion of the cycle at the same
rate as they were being acquired during the loading portion of the cycle.

We propose that significant structural damage had been done in the compression
zone of the carbon fiber region. This damage continued to propagate while the specimen
was still subjected to a relatively large bending moment, until the damage zone reached a
critical size, causing the material to become unstable, and the specimen to fail. Figure
6.13 shows a band of carbon fibers in the compression zone of a failed ACCC specimen
that exhibits failure along the kink-band line. An axial crack is also evident in the image,
as well as matrix cracking.

![Image of ACCC specimen](image_url)

Figure 6.14 – SEM image of ACCC specimen that failed in a four point bend test
6.3 Reliability analysis of the ACCC core subjected to bending

6.3.1 – Reliability analysis methods

Since composite materials exhibit marked scatter in material properties owing to a variety of reasons, it was desirable to develop a probabilistic model of the ACCC core subjected to the wrapping process that considered the randomness in geometrical configurations. Several parameters will affect the local stress state in the carbon fiber composite (e.g. – geometric configuration, fiber strength, fiber distribution, fiber diameter, wetting quality, interface quality, residual stress state, manufacturing technique, etc.). This study, which only considered the randomness in geometrical configuration, is by no means meant to be exhaustive, but was completed to show the impact that one effect will have on the internal stress state.

The previous two sections provide the ground work to formulate a performance function that can then evaluate the probability of damage to the ACCC core subjected to the wrapping process [51]. In classical reliability analyses the resistance of the material is compared to the stress that the part is subjected to. For this analysis, the resistance of the material will be the measured bending strength, $\sigma_{\text{BEND}}$, and the stress that the core is subjected to is the stress distribution from the finite element analyses, $\sigma_{\text{FEM}}$. Hence, the performance function, $Z$, can be defined as

$$ Z() = \sigma_{\text{BEND}} - \sigma_{\text{FEM}} \quad (6.14). $$

Additionally, a performance function that defines the onset of significant acoustic emission subjected to the wrapping process is given by
\[ G() = \sigma_{AE} - \sigma_{BEND} \quad (6.15) \]

The finite element stress is the stress distribution from the 100 geometrical configurations of the ACCC core, and the bending stress is calculated via equation (6.13). Notice, in equation (6.13) that all of the variables were considered to be governed by underlying distributions, and not assumed to be deterministic. All distribution parameters are summarized in Table 6.1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Distribution Type</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{FEM} ) (MPa)</td>
<td>Normal</td>
<td>926</td>
<td>25.5</td>
</tr>
<tr>
<td>( F_{LC\text{MAX}} ) (N)</td>
<td>Log Normal</td>
<td>2546</td>
<td>52.0</td>
</tr>
<tr>
<td>( F_{LC\text{AE}} ) (N)</td>
<td>Log Normal</td>
<td>2123</td>
<td>185.8</td>
</tr>
<tr>
<td>( r_{\text{CARBON}} ) (mm)</td>
<td>Normal</td>
<td>3.55</td>
<td>.15</td>
</tr>
<tr>
<td>D (mm)</td>
<td>Normal</td>
<td>9.513</td>
<td>.003</td>
</tr>
</tbody>
</table>

The reliability index, \( \beta \), of the ACCC core subjected to the wrapping process can be calculated when the performance function becomes negative, i.e. - the strength is exceeded by the stress [51]

\[ \sigma_{BEND} < \sigma_{FEM} \quad (6.15). \]

Similarly, the reliability index for the onset of acoustic can be calculated in the same manner

\[ \sigma_{AE} < \sigma_{FEM} \quad (6.16). \]

Several authors have proposed methods for calculating the reliability index [51, 57-59]. In this work we compared the reliability index prediction made by the First Order Reliability Method (FORM) [57] and the Second Order Reliability Method.
to determine if the limiting state equation is linear. From the reliability index, the probability that the ACCC core will experience irreversible damage due to the wrapping process can be calculated by

$$\text{POF} = \Phi^{-1}(-\beta) \quad (6.17).$$

### 6.3.2 - Reliability analysis results

Using Nessus v.8.4 the reliability index of the ACCC core subjected to wrapping was evaluated [59]. A reliability index of 12.419 for the ACCC rod to fail when wrapped around a 1 m mandrel was calculated from the FORM algorithm. When using the SORM algorithm a reliability index of 12.419 was also calculated. This agreement indicates that the state equation is completely linear. However, this may not be completely accurate as the SORM algorithm is known to fail at very low probabilities [51], as is the current case.

For the onset of significant acoustic emission, a reliability index of 5.116 was calculated from the FORM algorithm. From the SORM algorithm, a reliability index of 5.105 was calculated, indicating the state equation is slightly non-linear. Using equation (6.17) the corresponding probabilities of failure were determined. All reliability indices, and their corresponding probabilities of failure are summarized in Table 6.2.

<table>
<thead>
<tr>
<th>Reliability Analysis</th>
<th>$\beta$</th>
<th>$\text{POF}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\text{BEND}}$ FORM</td>
<td>12.419</td>
<td>$1.0 \times 10^{-35}$</td>
</tr>
<tr>
<td>$\sigma_{\text{BEND}}$ SORM</td>
<td>12.419</td>
<td>$1.0 \times 10^{-35}$</td>
</tr>
<tr>
<td>$\sigma_{\text{AE}}$ FORM</td>
<td>5.116</td>
<td>$1.6 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\sigma_{\text{AE}}$ SORM</td>
<td>5.105</td>
<td>$1.7 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

Examining the respective probabilities of failure, it is evident that when only considering the randomness in geometry, that there is essentially no probability that the
ACCC core will fail when it is wrapped around a 1 m mandrel. When considering the onset of significant acoustic emission, it is observed that the state equation is slightly non-linear. However, minimal disagreement is observed between FORM and SORM, and both algorithms predict that approximately 1.6 in ten million rod geometries will develop a high enough compressive stress that significant acoustic emissions would be generated from the ACCC core.

It must be reiterated that the probabilities calculated in this section only considered randomness in the geometric configuration of the ACCC core. Other factors were identified in §6.3.1 that could play a large role in the observed stress state of the ACCC core subjected to bending. These factors could be negligible, or could be incredibly significant. A complete micro-mechanics study would need to be performed on this hybrid composite to fully understand all of the complexities offered by this seemingly simple technology when subjected to bending.
7 - Conclusions and Future Work

1. Excessive bending is the most damaging short-term loading condition to the ACCC composite core. The compressive stress in the carbon fiber composite was identified as the limiting performance metric when the ACCC core is subjected to bending. Several factors influencing the compressive stress state during bending were investigated, while several others were identified as being in need of quantification.

2. Through finite element modeling it was found that axially the carbon fiber composite will be put in a state of residual compressive stress, while the glass fiber composite will be in a state of residual tensile stress during manufacturing.

3. It was independently confirmed that uni-directional composite materials that are produced through the pultrusion process will have significantly lower compressive strengths. Waas et al have shown that this decrease in compressive strength is due to fiber misalignment with the direction of applied load.

4. The flexural strength of the hybrid composite (ACCC core) was experimentally measured. A first-order expression was derived to
calculate the bending stress at any point in the hybrid-composite rod. Additionally, it was found that the onset of significant acoustic emission during bending occurs in the hybrid composite at approximately 80% of the rupture strength.

5. Through the curve that plots compressive stress as a function of mandrel radius, and the measured flexural properties, a critical bend radius can be calculated.

6. Short-term failure modes should now be relatively well understood, and consequently should be able to be avoided. However, a deeper understanding could be achieved with a micro-mechanics study. Several stress-field interactions are occurring at the microscopic level that could be influencing the failure process. A micro-mechanics study would lend insight into these mechanisms.

7. Aging of polymer based composite materials must be investigated. Little is known about why polymers degrade when subjected to temperature, moisture, ozone, etc. A significant understanding of the aging process would allow for a design life to be estimated.

8. Fatigue of the ACCC core is a long-term failure process that should be investigated. Little is well-understood about fatigue in conventional unidirectional composites, let alone hybrid composite materials.
9. As the ACCC core is to be loaded in tension for its' entire design life, an understanding of the stress rupture characteristics of the ACCC core must be gained.
References


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44. M. Kumosa, D. Armentrout, L. Kumosa, Y. Han, and S.H. Carpenter, “Fracture Analyses of Composite Insulators with Crimped End-Fittings: Part II- Suitable


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