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Multi-physical Study of MEMS Resonators and Oscillators

Xiaobo Guo
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MULTI-PHYSICAL STUDY OF MEMS RESONATORS
AND OSCILLATORS

A Dissertation

Presented to

Faculty of the Daniel Felix Ritchie School
of Engineering and Computer Science

University of Denver

In Partial Fulfillment
of the Requirements for the Degree

Doctor of Philosophy

by

Xiaobo Guo

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Abstract

First, this research presents experimental and theoretical investigation of the response of micro scale dual-plate thermal-piezoresistive resonators (TPR) and self-sustained oscillators (TPO) to different gases and pressures. It is demonstrated that the resonant frequency of such devices follows particular trends in response to the changes in the surrounding gas and its pressure. A mathematical model has been derived to explain the damping dependent frequency shift characteristic of TPO. The solution of the model indicates that the stiffness of the actuator beam decreases as the value of damping coefficient drops at lower gas density caused by the change in the gas molecular mass or pressure. When operated in the TPR mode of the same device, however, the frequency shift of the same silicon structure is mainly a function of gas thermal conductivity. The two different sensing mechanisms are confirmed by the measurement results showing opposite frequency shift for the TPR and TPO in helium-nitrogen mixtures. In pressure tests, frequency shifts as high as -2300ppm were measured for a TPO by changing the air pressure from 84kPa to 43kPa.

Second, the effect of geometry on thermoelastic damping (TED) in micro beam resonators is evaluated using an eigenvalue finite element formulation and its corresponding customized MATLAB program. The vented clamped-clamped (CC) and clamped-free (CF) beams with square-shaped vents along their center lines, are both analyzed. The quality factor and resonant frequency are obtained as functions of
various geometrical parameters including the location, number and size of the vents. The numerical results reveal that the addition of vent sections in the clamped end region can significantly enhance the quality factor under TED. The maximum improved quality factor as high as 3,801 and 2,257 times as those of the solid CC and CF beams are realized. The methodology presented in this work provides a useful tool in the design optimization of micro beam resonators against TED.

Third, a new method to compensate the TED by taking the advantage of piezoresistive effect is proposed. Such method is implemented by applying an electrostatic field through the MEMS beam resonator with negative piezoresistive coefficient. In the case of vibration, the stretched part of the beam generates higher electrical power thus higher temperature and vice versa. Such temperature distribution can compensate the opposite thermoelastic temperature to suppress TED. The work principle is described by a set of coupled differential equations and then solved by an eigenvalue finite element method. The numerical result indicates that the TED in beam resonators can be completely suppressed when the strength of electrical field reaches a critical value, namely CEF. The value of the CEF is further analyzed by parametric studies on various material properties and geometric factors.
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CHAPTER ONE: INTRODUCTION
1.1 MEMS and Their Application
1.1.1 MEMS Technology

Microelectromechanical systems (MEMS) are the technology of very small devices. It merges at the nano-scale into nanoelectromechanical systems (NEMS) and nanotechnology. Micro means the MEMS are usually made up of components between 1 to 100 micrometers or micron, one one-millionth of a meter, in size (i.e. 0.001 to 0.1 mm). Electromechanical system indicates the system is a integration of the electrical and mechanical components. The typical MEMS use its electrical parts to actuate and/or monitor the movement of the mechanical components. Those MEMS devices are also named as "transducer" that converts power from one form to another for the purpose of measurement or control. Most commercialized MEMS transducer is electro-mechanical that converts mechanical stimuli, such as pressure and force into electrical signals, current or voltage for example. However, as the development of the technology, thermo-electrical, thermo-mechanical, chemical, optical and biological MEMS transducers are also in various stages of research and application [1].

The MEMS transducers can be categorized as actuator or sensors or both of them based on their working mechanism and application. Actuators are used to move something, such as capacitive actuator [2-4], piezoelectric actuator [5-7],
thermal-mechanical actuator [8-10]. The mechanical movement is driven by an electrical force. On the other side, MEMS sensors are widely used to measure the physical quality, such as force, pressure, temperature, gas, pressure et al. The related sensing mechanism of MEMS sensors include thermal resistive sensing [11,12], capacitive sensing [13,14], piezoelectric sensing [15] piezoresistive sensing [16], optical sensing [17], resonance sensing [18-20] or a combination with two or more different categories of them, even merged with chemical reaction.

1.1.2 Gas and Pressure Sensing

Gas and pressure sensing are one of the most popular applications of MEMS transducers. Gas sensors have a variety of applications in different industries as well as for environmental research and safety monitoring. For example, highly sensitive, selective, robust and cost-efficient gas sensors are required for leakage detection of explosive gases such as hydrogen and methane, and for real-time sensing of pathogenic or toxic gases. Such sensors can be divided into two categories based on their sensing mechanisms: chemical and physical.

Chemical gas sensors are based on the chemical reaction between the target gases and sensors. Chemical hydrogen sensors for example, utilize palladium (Pd) [21-26] and platinum (Pt) [11,27-30] as catalyst to absorb hydrogen, inducing volume expansion [23-26] of the Pt/Pd layer. In an alternative approach, the Pd/Pt breaks hydrogen gas into hydrogen ions leading to a chemical reaction between the hydrogen ions and the oxide layer in the sensors thus changing the optical refractive index [21,22], temperature difference [27] or the electrical properties [28-30] of the sensing layer. On the other hand, physical gas sensors operate based on the specific physical
properties of target gases, such as thermal conductivity and density. For instance, thermal conductivity gas sensors [11] and [12] have been successfully used for hydrogen sensing, as hydrogen has a much higher thermal conductivity (0.183W/(m·K)) compared with air (0.0263W/(m·K)). Once a thermal conductivity sensor is exposed to hydrogen, the warmer part of the sensor dissipates heat to the surroundings becomes faster leading to a drop in its temperature, which results in the reduction of electrical resistance of the sensor. In general, physical gas sensors can be much cheaper because of their much simpler sensing mechanism and more reliable and durable over long term due to the elimination of chemical contamination/degradation problems.

In pressure sensing, MEMS technology has led to high performance and commercially viable solutions. Most of such sensors work based upon the measurement of static deformation of a diaphragm caused by pressure difference between its two sides. Typically, a constant pressure is fed to one side of diaphragm using a pressure chamber while the other side encounters the variable pressure to be measured. The diaphragm deformation can be measured using different mechanisms, including capacitive [14], piezoresistive [16], optical [17] and resonant frequency shift [19] methods. Among such, resonant pressure sensors have the advantages of simplifying readout system by providing an output frequency that can be directly fed into a computer (no need for A/D conversion), long term stability and higher resolution. In diaphragm-based resonant pressure sensors [19], a resonant structure is attached to the diaphragm which is the preliminary sensing element. The change in the surrounding pressure deforms the diaphragm and alters the internal stress of the resonator.
Consequently, the natural frequency of the resonator shifts. Alternatively, resonators [18] can also be used as the primary pressure sensing element. In such approach, the change in the ambient pressure alters the damping of the resonant system and results in a resonance frequency shift. The resonant element in such pressure sensors should be in direct contact with the environment. To maximize the damping effect, such sensors take the advantage of squeezed film effect by trapping a thin film of air between the resonator and the electrostatic transduction electrodes. The excessive air damping, however, could lead to very low quality factors compromising frequency stability and measurement resolution in such devices.

1.2 MEMS Resonator

As one of the most important categories of MEMS, micro mechanical resonator are being developed aggressively for a variety of applications nowadays, such as sensing [31-33], time application and frequency controls [34,35] due to their advantages of high frequency and high quality factor. Literally, MEMS mechanical resonators are just resonators in micro scale. Resonator is a structure or system that naturally oscillates/vibrates at some frequencies, named its resonant frequencies or

![Figure 1.1 An one-dimensional mass-spring-damper system.](image-url)
natural frequency, with greater amplitude than at other frequencies. The simplest mechanical resonator is a mass, spring and damper system in mechanics as shown in Fig. 1.1, where \( m \) is the mass of the block; \( k \) is stiffness of the spring and \( c \) is the damping coefficient used to presents the ambient and internal damping. The natural frequency of the \( m-k-c \) system is

\[
f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}
\]

(1.1)

where \( \omega_0 \) is the natural angular frequency. To maintain the system in a stable vibration state, a harmonic driving force \( F \) is needed.

The displacement of the block, \( x \), is described by the motion equation [36] that,

\[
mx'' + cx' + kx = F \cos(\omega t)
\]

(1.2)

where \( \omega \) is the angular frequency of the driven force. The displacement is also harmonic and its amplitude is \( X \), while the static displace under a static force \( F \) is defined as \( X_0 \), and

![Figure 1.2 The variation of the amplitude ratio under different frequencies of the driving force, while \( c=0.4, k \) and \( m \) have unit values.](image)
\[ X_0 = \frac{F}{k} \]  

(1.3)

Figure 1.2 shows \( X/X_0 \) as a function of \( \omega/\omega_0 \). The system will achieve its maximum vibration amplitude or vibration energy, when the frequency of the driven force is equal to the natural frequency of the system.

1.2.1 Micro Capacitive Resonator

There are many different kinds of MEMS mechanical resonators including capacitive beam resonator [2-4], piezoelectrical resonator [5,6], thermally actuated resonator [10] et al. No matter how complicated the resonator is, it can be simplified as a mass-spring-damping system. One of the classical MEMS mechanical resonators, the micro silicon beam resonator [2], is indicated in Fig. 1.3. The driving force in the device is the Coulomb force of the capacitor formed by the movable beam and the fixed electrode. The resonant frequency of the beam resonator is found by sweeping the

![Figure 1.3 Schematic view of a micro capacitive clamped–clamped beam resonator.](image)
frequency of voltage to find its maximum vibration amplitude. In the case of such resonator, it is usually analyzed as two dimensional plane-strain problem, since the thickness is larger than its width.

1.2.2 Thermal Actuated Piezoresistive Resonator

Another important category of MEMS mechanical resonator is thermally actuated resonator [10]. The thermally actuated resonator fabricated in DU NEMS Lab uses piezoresistive effect to sense its vibration. Therefore, such device is also named thermally actuated piezoresistive resonator (TPR). The structure and its work principle are shown in Fig. 1.4. The device is made of single crystal silicon after N-type doping. To excite its vibration, the current applied through the structure, $I$, is a combination of direct current (DC) and alternating current (AC). Due to the higher resistance of the actuator beams, most of the resulting Joule heating (electrical ohmic

![Image of a thermally actuated piezoresistive resonator]

Figure 1.4 The structure and working principle of the thermally actuated piezoresistive resonator.
loss) will occur in the actuator beams that serve as both thermal actuators and piezoresistive sensing elements. When the beam is heated by a high current, the two plates will move away from each other. On the other side, the plate will move to each other when the beam is cooled by a low current. In other words, the thermal force generated in the beams resulted from the change in its temperature performs as a driving force. Meanwhile, the actuator beams work as a spring and the plates operated as a resonance mass indicated by Fig. 1.1. The support beams is used to support the plate floating from the silicon substrate (handle layer) and make sure the plates can move into or away from each other. In addition, the stiffness of support beams in their flexural direction is negligible for their smaller value compared with the longitudinal stiffness of actuator beams during the vibration.

On the other hand, the vibration of the system is measured by the actuator beams operated as piezoresistive sensing elements. The fluctuating current causes the actuator beams to expand and contract periodically. Meanwhile, the periodic stress on the actuator beam leads to fluctuations in its electrical resistance because of the piezoresistive effect, as defined that,

$$R_{ac} = R_0(1 + \pi_l \sigma_{ac})$$

(1.4)

where $R_0$ is the resistance without stress load; $\pi_l$ and $\sigma_{ac}$ are the piezoresistive coefficient and mechanical stress, respectively. The applied DC bias current $I_{DC}$ is therefore modulated by the resistance fluctuations, producing an measurable AC output voltage $U_{AC}$ component that indicates the vibration amplitude of the resonator because $U_{AC} = I_{DC}R_{ac}$. The resonance frequency of the TPR can be found by sweeping
the frequency of the input AC current when the output $U_{AC}$ achieved its maximum value.

1.2.3 Self-sustained Thermally Actuated Piezoresistive Oscillator

The TRP can also be operated in its self-excited vibration mode by input a large DC bias instead of the combination of DC and AC. To distinguish it from the forced vibration mode, the MEMS device is named thermally actuated piezoresistive oscillator (TPO) who works in the mode of self-excited vibration. Self-excited vibration is one important category of nonlinear vibrations. The self-excited vibration is maintained by an internal driven force obtained from an uniform source of power, such as the constant input electrical current (DC) of TPO. The internal force leads to the increasing vibration amplitude until some nonlinear effect limits any further increase [37]. The alternating internal driven force to sustain the vibration is created by the motion itself. The alternating force also disappears when the motion stops spontaneous. However, in a forced vibration the periodical driving force to maintain the vibration exists independently of the motion and persists even the vibration is stopped, such as the AC input current of TPR. There are many different sources of self-excited vibrations, such as friction [38], wind [39] and so on. Usually, the self-sustained vibration mentioned above needed to be suppressed for their harmful effect. However, as a resonant MEMS device, the TPO is preferred since its vibration is excited by a simple DC bias without the need for a sustaining electronic amplifier, comparing with other MEMS oscillators [40].

As one kind of self-excited vibrations, the internal driven force also exists in TPO. Such internal force is generated starting from inputting a large DC current
through the structure as shown in Fig. 1.4. If the DC bias is large enough, any random mechanical noise in the structure can be amplified by the following sequence leading to sustained oscillations at the mechanical resonant frequency of the structure. The sequence is denoted in Fig. 1.5 [41] and it happens as following: if mechanical noise causes the plates to move slightly further from each other, the resulting tensile stress in the actuator beams along with the negative piezoresistive coefficient of N-type single crystal silicon reduces the electrical resistance of the actuator beams. Under a constant DC bias, the reduced resistance is translated into lowered ohmic loss which cools the actuator beams down thus forces them to contract. Upon contraction, the inertia of the plates causes an over-contraction in the actuator beams. The resulting compressive stress increases the electrical resistance of the actuator beams due to piezoresistive
effect. The increase in the resistance leads to higher ohmic loss and eventually the re-expansion of the actuator beams. Provided that the overall energy gain in the whole cycle of the events equals the energy loss caused by the total damping of the vibrating system, self-sustained oscillation can be initiated and maintained in a stable state.

1.3. Thermoeelastic Damping

In real application, the resonator with lower damping is always preferred, as it is essential that the resonator vibrates consistently at the desired frequency with a sharp peak, as indicated by Fig. 1.6, and requires as little energy as possible to maintain its vibration. In addition to the damping coefficient \( c \), the quality factor or \( Q \)-value is another parameter frequently used to describe how under-damped a resonator is. The dimensionless quality factor is defined as,

\[
\frac{X}{X_0} = \frac{k}{\omega_0 m c}
\]

Figure 1.6 The variation of amplitude ratio under different frequencies of the driving force, where \( k \) and \( m \) have unit values, while \( c \) equals 0.4 and 0.1 for low \( Q \) and high \( Q \), respectively.
From a physical perspective, the quality factor is also defined as,

\[ Q = \frac{1}{2\xi} = \frac{\sqrt{km}}{c} \quad (1.5) \]

where \( W \) denotes the stored vibration energy in a vibration system and \( \Delta W \) indicates the energy dissipated per cycle of vibration caused by damping.

There are many different mechanisms for energy dissipation inside microbeam resonators, including thermoelastic damping, fluid damping [42-45], support loss [46-49] and surface loss [50-52]. Air damping [44] can be eliminated by packaging the resonators in vacuum because of the small size of the devices. Among these energy loss mechanisms, thermoelastic damping (TED) has been an active research area for a long term since it imposes an upper limit on the attainable quality factor.

1.3.1 The Cause of TED

Thermoelastic damping (TED) arises from thermal currents generated due to the temperature difference during the deforming of the resonator. It is well known that the contraction/expansion of elastic structures leads to high/low thermoelastic temperature. The bending of micro beams causes dilations of opposite signs on the upper and lower halves. Consequently, there is a transverse temperature gradient of finite thermal expansion in the structure, generating local heat currents. From the viewpoint of thermodynamics, the irreversible thermal current causes increased entropy and leads to energy dissipation. Figure 1.7 schematically shows the fundamental resonance mode of the displacement and its corresponding thermoelastic temperature of a clamped-clamped beam resonator. The TED is caused by the heat
1.3.2 The Study of TED

The research on TED is not a newly arising area anymore. Long time ago, Zener [53,54] have already studied the TED in reeds and wires. However, as the development of MEMS resonator, the study of TED has attract a wide attention again. The modern application of the theory to micro/nano electromechanical systems (MEMS/NEMS) should be attributed to Lifshitz and Roukes [55] whose work predicated the TED limited quality factor of micro and nano beam resonators based on the thermal conduction along the beam's width direction. By applying the Green’s function, Prabhakar [56] analyzed the TED by solving two-dimensional heat transfer equation to obtain the thermoelastic temperature gradients both along the beam's length and width simultaneously. Prabhakar [57] also analyzed the frequency shifts due to
TED. Wong [58] presented an analytical solution for TED in a ring gyroscope based on Zener’s method. However, these works were restricted to the flexural mode vibrations. Hao [59] derived the solution of quality factor for disk resonators vibrating in a contour mode by calculating the entropy increase per cycle of vibration. Sun [60,61] derived the analytical solution for out-of-plane vibrations of circular plate resonators through the thin plate theory in cylindrical coordinates. In more complex conditions, the TED under residual stresses is investigated by Zamanian [62] and Vahdat [63] for beam resonators. Kim [64] applied the simplified shell equations and utilized iterative schemes to analyze the TED of nano-mechanical tube resonators with initial stresses. Tunvir [65] studied the nonlinear effect induced by large vibration amplitude and found that there are opposite trends of the change in TED under adiabatic and isothermal surface thermal conditions. Vengallatore [66] and Prabhakar [67] presented analytical solutions to calculate the frequency dependence of TED in bilayered beam resonators.

In addition to the analytical methods, the finite element formulation [45,68-72] used to solve the TED was found to be a more efficient tool for those systems with complex geometrical shapes, mixed boundary conditions or non-homogeneous material properties. Yi’s group improved this method for vented beam [71] axisymmetric rings [68], disk and elliptical [69] resonators by deriving a generalized eigenvalue scheme of the problem for both two-dimensional and three-dimensional structures, even in the presence of fluid viscous damping [45]. Moreover, the Fourier reduction [68] method based on the eigenvalue scheme has shown a high efficiency on computation. The commercial software COMSOL© can also solve the thermal
elastic damping by finite element method [71], but the analysis procedure needs the customer to solve the undamped frequency $f_0$ firstly, and the accuracy also depends on the manual scaling setting [73]. What's more, the finite element method is also successfully applied to solve the support loss [47-49] by using perfect match layer element to model the infinite boundary condition.

To increase the quality factor, one way to is applying the fabrication with high precision. For example, Pourkamali and Ayazi [2,3] utilized HARPSS process to decrease the gap between the MEMS capacitive resonator and the electrodes. However, the TED still showed an upper limit to the total energy loss. On the other hand, to decrease TED, Candler and Duwel [74] applied a geometry design that applies slot cuts in the beam resonator to disrupt the heat flow and then decrease the TED based on a series of experimental studies.

1.4 Overview of the Dissertation

The structural robustness and simplicity along with small size and low cost fabrication process of the TPRs make them promising candidates for various sensing applications [75,76]. In chapter two of this dissertation, the performance and behavior of such devices operated as both TPRs and TPOs for gas and pressure sensing have been studied. The theoretical analysis for the structures is also carried out to not only explain and justify the different response behavior of TPOs and TPRs but also explained the mechanism of self-excited vibration of the TPO in mathematical level.

In chapter three and four, two different methods to decrease the TED are studied by finite element method. In chapter three, a predictive modeling tool that can be used to quantitatively evaluate the effect of beam geometry on the mitigation of
thermoelastic damping is provided. The methodology is used to analyze the TED limited quality factor of clamped-clamped (CC) and clamped-free (CF) beams with vents along their centerlines and to find the optimal design of the vents, including the number, location and size, for maximizing the quality factor under TED.

However, the method to decrease the thermoelastic damping by adding vents to the micro beam resonator has a fabrication limit for nanoscale resonators. In chapter four, a new method by utilizing the piezoresistivity to compensate the TED of micro beam resonator is therefore proposed. The corresponding differential equations to describe the coupling among the electrical, mechanical and thermal processes are derived and solved by an eigenvalue finite element method. A series of numerical studies based on such finite element method are performed to investigate the correlation between the strength of applied electrical field and the TED limited quality factor under different material and geometry parameters.
CHAPTER TWO: GAS AND PRESSURE SENSING

2.1 Introduction

University of Denver (DU) NEMS Lab has studied the potential sensing application of the TPRs such as air-born particle [75] and organic gas [76] detection. In this chapter, the performance and behavior of such silicon structure operated as both TPRs and TPOs for gas and pressure sensing [20] will be investigated.

Figure 2.1 The scanning electron microscope (SEM) view of two dual-plate silicon resonant structures: (a) the structure with small plates (60μm×60μm) and (b) the zoomed-in view of its actuator beam. (c) the structure with large plates (99μm×99μm) and (d) the zoomed-in view of its actuator beam.
Figure 2.1 shows the scanning electron microscope (SEM) views of a dual-plate silicon structure which can be operated in both TPR and TPO modes. The structures were fabricated using a single mask process [41] on a N-type silicon on insulator (SOI) substrate with device layer thickness of 10μm and buried oxide layer thickness of 4μm. The fabrication process is indicated in Fig. 2.2. Restricted by photolithography limits, structures with minimum actuator beam width of 2μm were first fabricated. Consecutive thermal oxidation and oxide etch steps (in hydrofluoric acid) were then used to narrow down the thermal actuator beams. The submicron width of the thermal actuator beams (shown in Fig. 2.1), achieved by such technique, is used to reduce the required bias current and therefore the power consumption for reaching self-oscillation.

Figure 2.2 Fabrication process used to implement the resonant silicon structures on SOI substrates.
2.2 Sensing Mechanism

Although both the TPO and the TPR operation modes are based on thermal actuation and can be realized by the same structure, their different operation mechanisms (forced/self-excited vibration) considerably differentiates their response to the change in the surrounding gases and ambient pressure. The behavior of the thermal-piezoresistive resonant structures under both operating configurations is analyzed in this chapter. Taking the symmetrical geometry of the resonant structures (shown in Fig. 2.1) into account, the dual-plate silicon structures can be simplified as a one degree of freedom spring-mass system as indicated in Fig. 2.3. The actuator beams work as spring, while the plate is operated as a resonant mass. The flexural stiffness of the support beams are negligible for their much lower value than the longitudinal stiffness of the actuator beams.

![Figure 2.3](image)

(a) The structure of the TPR/TPO and (b) its simplified mass-spring model.

2.2.1 Thermal Conductivity Based Analysis of TPR/TPO

The temperature difference between the TPR/TPO and their surroundings is mainly maintained with Joule heating caused by the DC bias current. The generated
heat is dissipated through conduction to the ambient gas and the substrate of the resonator, convection and radiation. The convection and radiation are usually negligible for the temperature range of the resonators and their small dimensions [77,78]. When the dual plate silicon structure is heated by the DC bias, the temperature change in the resonators can be directly related to the change in their resonance frequencies \( f \) which is:

\[
f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}
\]

where \( m \) is the mass of the resonance plates and \( k \) is the stiffness of the actuator beams (spring). For most solids including silicon, the young’s modulus and therefore structural stiffness \( k \) decreases as temperature increases [79]:

\[
k = k_0 + c_T T_{beam}
\]

where \( c_T, k_0 \) and \( T_{beam} \) are the temperature coefficient of the young’s modulus, the actuator beam stiffness at 0K and the temperature of the actuator beam, respectively. When the surrounding gas changes from one gas to another gas with higher thermal conductivity, more heat will be dissipated via gas conduction from the plate into the surroundings and therefore the temperature of the TPR/TPO drops. This causes an increase in the stiffness of the actuator beams which in turn results in a higher resonance frequency.

The change in heat dissipation \( \Delta P_{dis} \) through the ambient gas is [80]:

\[
\Delta P_{dis} = S_{plat} A_{plat} \Delta \kappa_{gas} (T_{plat} - T_{amb})
\]

where \( S_{plat} \) is the shape factor, \( A_{plat} \) is the surface area of the plate, \( \Delta \kappa_{gas} \) is the change in thermal conductivity of the surrounding gas, \( T_{plat} \) and \( T_{amb} \) denote the temperature of the
plate and the ambient gas, respectively. Based on Eq. (2.3), the TPR/TPO with larger plate area $A_{plat}$ and higher temperature $T_{plat}$ will lead to a higher frequency shift.

### 2.2.2 Damping Based Analysis of TPR

In the case of TPR operation mode (under forced linear vibration), the resonator should satisfy the following equation of motion:

$$m\ddot{x} + c\dot{x} + kx = F_{th} \cos(\omega t)$$  \hspace{1cm} (2.4)

where $F_{th}$ is the harmonic thermal force generated by the combination of a AC and DC currents; $c$ is the damping coefficient and $\omega$ is the angular frequency of the AC input. This is a classic damped-resonance problem where the damped resonance frequency is given by:

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m(1 - 2\zeta^2)}}$$  \hspace{1cm} (2.5)

where the damping ratio $\zeta$ is defined as

$$\zeta = \frac{c}{2\sqrt{km}}$$  \hspace{1cm} (2.6)

According to Eqs. (2.5) and (2.6), the frequency of the TPR has a higher value when damping imposed by the surrounding gas is smaller. Generally lower gas density [81], either due to lower gas molecular mass or lower pressure results in lower damping coefficient.

### 2.2.3 Damping Based Analysis of TPO

The effect of damping by surrounding gas on a TPO can however be in the opposite direction compared with that of a TPR. During the mechanical vibration (oscillation) of the TPO, the fluctuation amplitude of the Joule heating $\Delta P$ in the actuator beams is given by the product of its periodic resistance fluctuation amplitude, $\Delta R$, and the DC current $I$ through it:
\[ \Delta P = I^2 \Delta R \] (2.7)

and the output peak to peak oscillation voltage \( V_{p-p} \) monitored by an oscilloscope [41] is given by \( V_{p-p} = I \Delta R \).

The temperature fluctuation amplitude, \( \Delta T \), in the actuator beams caused by the periodic change in Joule heating \( \Delta P \) can be described by the following heat equation [80]:

\[ \Delta \dot{T} = \Delta P / c_t - A_{sur} h \Delta T \] (2.8)

where \( A_{sur} \) is the thermal dissipation area, \( h \) is the thermal dissipation coefficient and \( c_t \) is the thermal capacity of the actuator beams. Here, the term \( A_{sur} h \Delta T \) represents not only the heat conduction from the actuator beam to the ambient gas, but also from the actuator beam to the substrate. The value of the thermal capacity \( c_t \) is calculated using \( c_t = \rho c_p V \), where \( \rho \), \( c_p \) and \( V \) are the density, the specific heat capacity of silicon and the total volume of the actuator beams, respectively.

By adding the periodic change in the length of the actuator beams, \( x \), driven by the periodic change in its temperature \( \Delta T \), the equation of motion for the spring-mass-damper system turns into:

\[ m \ddot{x} + c \dot{x} + k(x - \alpha l \Delta T) = 0 \] (2.9)

where \( \alpha \) is the thermal expansion coefficient of silicon and \( l \) is the initial length of the actuator beams. The stiffness of the actuator beams is calculated by \( k = E A_{sec} / l \), where \( E \) is the young’s modulus of silicon and \( A_{sec} \) is the total cross-sectional area of the actuator beams.
The normal stress $\sigma$ generated in the actuator beams is defined as:

$$\sigma = k(x - \alpha l \Delta T)/A_{sec} \quad (2.10)$$

The change in the resistance $\Delta R$ of the actuator beams due to the piezoresistive effect is:

$$\Delta R = \pi_l \sigma R_0 \quad (2.11)$$

where $\pi_l$ is the piezoresistive coefficient and $R_0$ is the resistance of the actuator beams under no stress.

Substituting Eqs. (2.7), (2.10) and (2.11) into (2.8), results in:

$$\Delta T = N k(x - \alpha l \Delta T) - A_{sur} h \Delta T \quad (2.12)$$

where $N$ is defined as $N = T^2 \pi_l R_0/(c_t A_{sec})$.

Substituting Eqs. (2.12) into (2.9), results in:

$$\Delta \tilde{T} + (N k \alpha l + A_{sur} h + c/m) \Delta \tilde{T} + (c N k \alpha l + c A_{sur} h + k) \Delta \tilde{T} / m + k A_{sur} h \Delta T / m = 0 \quad (2.13)$$

Under periodic dynamic conditions, the temperature change $\Delta T$ has the form $\Delta T = \Delta T_0 e^{i\omega t}$. Substituting the temperature change $\Delta T$ in the phasor form into Eq. (2.13), leads to:

$$-i\omega^3 - (N k \alpha l + A_{sur} h + c/m) \omega^2 + (c N k \alpha l + c A_{sur} h + k) i \omega / m + k A_{sur} h / m = 0 \quad (2.14)$$

For Eq. (2.14) to be satisfied, both the real and imaginary parts should each be equal to zero which results in:

$$(N k \alpha l + A_{sur} h + c/m) \omega^2 = k A_{sur} h / m \quad (2.15)$$

and
\[ \omega = \sqrt{(cNkal + cA_{\text{sur}}h + k)/m} \]  

(2.16)

Substituting Eqs. (2.16) into (2.15), the following equation is obtained:

\[ (mNkal + mA_{\text{sur}}h + c)(cNkal + cA_{\text{sur}}h + k) = mkA_{\text{sur}}h \]  

(2.17)

where

\[ c = -(k \pm \sqrt{3})/(2(A_{\text{sur}}h + Nkal)) - (mA_{\text{sur}}h + mNkal)/2 \]  

(2.18)

and

\[ S = A_{\text{sur}}^4h^4m^2 + 4A_{\text{sur}}^3h^3m^2Nkal + 6A_{\text{sur}}^2h^2m^2N^2k^2a^2l^2 + 2A_{\text{sur}}h^2km + \]

\[ 4A_{\text{sur}}hm^2N^3k^3a^3l^3 + m^2N^4k^4a^4l^4 + 2mN^2k^3a^2l^2 + k^2 \]  

(2.19)

However, the power dissipation caused by the term \( A_{\text{sur}}h\Delta T \) in Eq. (2.8) is negligible compared to the total input electrical power [10]. Neglecting the term \( A_{\text{sur}}h\Delta T \) simplifies Eq. (2.17) to:

\[ (mNkal + c)(cNkal + k) = 0 \]  

(2.20)

which results in:

\[ c = -mNkal. \]  

(2.21)

In Eq. (2.21), the energy loss caused by the damping effect is represented by the damping coefficient \( c \), while the energy gain from the thermal-piezoresistive-mechanical coupling is indicated by the term \(-mNkal\). To initiate and maintain oscillation, it is important to generate enough energy to counteract the energy loss.

Since the damping coefficient \( c \) is positive, the device should necessarily have a negative piezoresistive coefficient (N-type single crystal silicon). Assuming the fixed geometry of a TPO, there has to be a minimum input current (or DC power \( I^2R_0 \)) through the TPO to meet Eq. (2.21) for initiating and maintaining its oscillation.
If the energy loss is higher than the energy gain, the oscillation will vanish, while the vibration energy will keep increasing if the energy gain is higher than the energy loss. The increased vibration energy comes from the excessive energy gain after the losses of energy are compensated in every cycle. The amplitude increase will be accumulated in successive cycles, and theoretically, in a completely linear system, the vibration energy will keep increasing until a mechanical failure happens in the system. However, in reality, the stiffness of the actuator beams shows significant nonlinearity [82]. The nonlinear stiffness term reduces the overall stiffness as the vibration amplitude increases.

The changes in the density of gas, such as the change in the gas molecular mass or pressure, disrupts the balance between the two sides of Eq. (2.21) by altering the damping coefficient. If the energy loss is reduced because of the lower damping under lower gas molecular mass or lower gas pressure, the energy gain in each oscillation cycle becomes higher than the energy loss. Therefore, the vibration amplitude starts to increase until the actuator beams' mechanical nonlinearity decreases the stiffness of the actuator beams \( k \), eventually putting the oscillation in another stable state as Eq. (2.21) is satisfied.

Based on Eq. (2.21), the changes in the stiffness of the actuator beams \( dk \) is:

\[
dk = -\frac{dc}{(mN\alpha l)} \tag{2.22}
\]

where \( dc \) is the change in damping coefficient. Equation (2.22) shows that for smaller values of input currents (or DC power) and therefore smaller value of \( N \), larger
changes in stiffness $dk$ will occur for the same amount of the change in the damping $dc$ for the same TPO.

On the other side, Eq. (2.16) can be simplified to:

$$\omega^2 = (cN\alpha l + 1)k/m$$

(2.23)

Since $cN\alpha l << 1$, Eq. (2.23) can be further simplified to

$$\omega^2 \approx k/m$$

(2.24)

Comparing Eq. (2.24) with Eq. (2.1) indicates the frequency of the TPO is just the resonance frequency when it operated as TPRs since $f=\omega/(2\pi)$ . Equation (2.24) also denotes a lower stiffness of the actuator beams results in a lower oscillation frequency. As it was discussed for Eq. (2.21), it could be inferred that the TPO will have a lower oscillation frequency when the damping is decreased by a drop in the density of the surrounding gas and vice versa.

2.3 Measurements

In order to perform gas and pressure sensing tests using the TPRs and TPOs, the measurements are conducted in two parts. Due to the hazards associated with handling and working with hydrogen, helium that has somewhat similar physical properties (e.g. thermal conductivity and density) was used in the experiments. Nitrogen was used to dilute helium to different concentrations because the physical properties of nitrogen are close to those of air. The physical property of hydrogen, helium, nitrogen and air is listed in Tab. 2.1 [80]. The TPOs and TPRs are then operated in different concentration of the helium. Whereas for pressure sensing, the TPRs and TPOs are run under both low pressures of air (in a vacuum setup) and high pressures of nitrogen (in a pressure chamber).
2.3.1 Gas Sensing Behavior of TPR/TPO

Figure 2.4 shows (a) the camera picture and (b) the schematic diagram of the experimental setup used to characterize the sensory behavior of the TPRs/TPOs. The DC is provided by the power supply while one port of the network analyzer is used to supply AC current and another port of it is used to measure the frequency shifts. To test the same devices operated as TPOs, the network analyzer should be replaced by an oscilloscope for measuring $V_{pp}$ or with a frequency counter to measure the frequency. At the same time, the DC should be increased to the threshold value to excite the self-sustained oscillation. The pressure in the gas chamber was kept constant during the measurements. The silicon chip containing the dual-plate resonant structures were mounted on a printed circuit board (PCB) and placed in a gas chamber. For testing the devices in their TPR mode, the concentration range of helium in nitrogen was changed from zero to a 1:2 ratio. Since the structures showed higher sensitivities in their TPO mode, a smaller range of the helium concentrations in nitrogen (from zero to a 1:5 mixture of helium-nitrogen) was used. Four different geometries of the MEMS devices were used in the experiments, labeled as S10, S20, L10 and L20, in which ‘S’ and ‘L’
stand for small plate (60μm×60μm) and large plate (99μm×99μm), respectively (Fig. 2.1). Also, '10' and '20' stand for the actuator beam lengths which are 10μm and 20μm, respectively. Measurements were performed under different input DC powers.

The DC power consumption and corresponding frequencies measured for different dual-plate silicon structures in nitrogen are shown in the legend of Fig. 2.5.

Figure 2.4 (a) The camera picture and (b) the schematic diagram of the experimental setup used to characterize the frequency behavior of the MEMS devices operated as TPRs in different gas mixtures.
Figure 2.5a shows the measured frequency shifts for different TPRs under different helium concentrations in nitrogen. As expected, the resonance frequency of all TPRs increase as the gas thermal conductivity increases (higher helium percentage). The change in the resonance frequency is almost proportional to the helium concentration. The observed maximum frequency shifts are 624ppm in a 1:2 mixture of helium-nitrogen compared with pure nitrogen. The minimum detectable limit of the helium concentration that can be measured by this particular TPR is 0.022% (220ppm) assuming that the minimum detectable frequency shift is 1Hz. Comparing the frequency shifts for the same device at different DC power levels reveals that when biased at a higher DC power, the resonators show higher sensitivity. Moreover, the frequency shifts of the device with larger plate, even at lower power level (L20-17.5mW) are higher than that of the smaller device biased at higher DC power (S10-25.5mW). Such conclusions confirm the results derived from the theoretical analysis. More sensitive TPRs could be realized by using the device can withstand higher input power and have larger plates.

Figure 2.5b illustrates the measured frequency shifts for the same resonant structures working as self-sustained TPOs. As expected, such devices show an opposite trend (negative frequency shifts in higher helium concentration) compared with when they were operated as TPRs. The increased helium concentration lowers the gas mixture density leading to a reduced ambient damping and therefore a lower oscillation frequency. The experimental results are in agreement with the discussions in the previous section. Frequency shifts as high as -1340ppm were obtained in a 1:5 mixture
Figure 2.5 Measured frequency shifts for different resonant structures in different gas mixtures, (a) working as TPRs and (b) working as self-sustained TPOs. The frequency shifts (in ppm) are with respect to their natural frequency in pure nitrogen.
Figure 2.6 A self-sustained TPO working in different gases, (a) the frequency shifts (with respect to the frequency in nitrogen) and $V_{p-p}$ versus gas density, (b) the frequency shifts versus the thermal conductivity of different gases.
of helium-nitrogen with respect to pure nitrogen which shows much higher sensitivity in the TPO mode than the TPR mode. The minimum detectable limit of the helium concentration for this particular TPO is 0.004% (40ppm) assuming the minimum detectable frequency shift is 1Hz. Furthermore, the frequency shifts for the same TPO with lower DC bias powers are higher, confirming the conclusion based on the mathematical model indicated in the previous section. Frequency shifts measured for the device with larger plates, even at higher power consumption (L20-29.5mW) is higher than that of the device with smaller plates at lower power consumption (S10-27.1mW) which shows the fact that oscillators with larger plates indicate better sensitivities. This is mainly due to the larger contact area between the plate and the ambient gas and then higher damping. Taking advantage of such rules, more sensitive TPOs can be designed and applied for future research and development.

Figure 2.6 shows the measurement results for a self-sustained oscillator in different gases, including helium (He), 1% hydrogen in nitrogen (HN), nitrogen (N₂), synthetic air (SA) and oxygen (O₂). The frequency shifts of the TPO are clearly dependent on the gas density (higher density→higher frequency) in Fig. 2.6a. This trend agrees with the measurements shown in Fig. 2.5b for different helium mixtures. The lower \( V_{pp} \) at higher oscillation frequency indicates lower vibration amplitude resulting from increased damping caused by the gas with higher density (larger molecular mass). Moreover, Fig. 2.6b shows that the frequency of the oscillator is not a function of the thermal conductivity of the surrounding gases.
2.3.2 Pressure Sensing Behavior of TPR/TPO

The measurements of the pressure sensing are presented in two separate sections: under and above atmospheric pressure test by two different experimental setups.

2.3.2.1 Lower Pressure Sensing

The test for characterizing TPRs/TPOs as pressure sensors were performed in a vacuum setup shown in Fig. 2.7. The $V_{p-p}$ of the TPO was measured by the

![Figure 2.7](image)  
(a) The photo view and (b) the schematic view of the experimental setup used to characterize the frequency behavior of the TPO in different air pressure.
oscilloscope. The frequency was monitored using a frequency counter to replace the oscilloscope. To test the frequency of the device when it operated as TPR, the outside electrical connect should be changed as Fig. 2.2b. The silicon chip containing the devices was placed on the printed circuit board (PCB) and embedded in a sealed bell-jar (glass vessel) [75]. A vacuum pump connected to the outlet in the sealed bell-jar was used to suck the air out and generate partial vacuum around the TPR/TPO. The pressure under the bell-jar was controlled by adjusting the air flow going through the inlet and outlet valves. The pressure inside the bell-jar was measured by a commercial pressure sensor and the data was transferred to a voltage signal that was read by a voltmeter. The frequency shift of the TPO measured by the frequency counter and the pressure read from the voltmeter were recorded automatically by a LabVIEW© program. However, the peak to peak oscillation voltage $V_{p-p}$ was manually measured by using an oscilloscope, as well as the frequency response monitored by a network analyzer when the same silicon structure was operated as TPR.

The modal analysis result in COMSOL for the device 'L10' in the test is demonstrated in Fig. 2.8. The fundamental resonance frequency is 3.468MHz. The lower measured resonance frequency is resulted from the lower dynamic stiffness caused by the steady thermal stress and the thermally softer effect caused by the Joule heating by DC. The experimental results obtained from such device operated as TPR/TPO in low air pressure are shown in Fig. 2.9. Figure 2.9a shows both the frequency shift and the peak to peak oscillation voltage of the TPO as a function of the surrounding air pressure. The frequency of the TPO was measured to be 3.456MHz.
with a power consumption of 9.10mW. A frequency shift of -2300ppm was measured while changing the ambient air pressure from 84kPa to 43kPa. Furthermore, as expected, the TPO had larger vibration amplitudes in lower surrounding air pressures indicated by the larger value of $Vp-p$ which is proportional to the vibration amplitude. As expected from the analysis in previous section, lower damping necessitates lower stiffness to satisfy the oscillation conditions shown by Eq. (2.21). Due to the mechanical nonlinearity of the actuator beams, their stiffness decreases at higher amplitudes resulting in lower oscillation frequencies.

Figure 2.9b shows the measured resonance frequency-pressure behavior of the same resonant micro-structure operated as a TPR. The resonance frequency of the TPR was measured to be 3.465MHz with power consumption of 0.44mW and a 0.21mA DC. Its total frequency shift of 42ppm was observed by changing the surrounding pressure.
Figure 2.9 (a) The experimental result of the TPO 'L10' whose structure is shown in Fig. 2.1c. (b) The experimental result of the TPR in the same silicon structure with the calculated damping coefficient and frequency shift.
from 84kPa to 43kPa which is more than 50X smaller than the absolute shift of -2300ppm measured for the same structure in its TPO configuration. Hence, the device is much more sensitive in the TPO mode than in its TPR mode as a pressure sensor. The opposite trend in the frequency shift between the TPO and the TPR configurations is also in agreement with the conclusions made by the derivations in the previous section.

Figure 2.9b also shows the calculated damping coefficient with respect to the change in the ambient pressure. The value of the damping coefficient $c$ was obtained based on Eq. (2.21) by using the measurement results of the TPO presented in Fig. 2.9a, the piezoresistive coefficient of $-100 \times 10^{-12} \text{Pa}^{-1}$ [83], the measured resistance of 8160ohm for the TPO and the thermal capacitance $c_t$ of $1.1 \times 10^{12} \text{J/K}$ based on the COMSOL model. Equation (2.24) was then used to calculate the stiffness $k$ of the actuator beams.
(spring) of the TPO. Finally, the frequency shifts of TPRs with respect to different damping coefficients in different surrounding pressures are obtained based on Eq. (2.5). The calculated frequency shifts of the TPR are in good agreement with the experimental results as indicated in Fig. 2.9b.

Figure 2.10 shows the cyclic response of the same TPO to several cycles of pressure change in ranges of 44-63kPa, 63-72kPa and 75-79kPa. The change in the pressure was controlled through a valve while all the data were collected by the LabVIEW® program. When the pressure changed in a narrower range, the change in the resonant frequency of the TPO was very quick. However, there was a lag of a few seconds when the pressure change was wider, mainly because it took some time for the pressure under the bell-jar to be stabilized after the valve is closed/opened. Hence, the frequency of the TPO reacts to the changes in pressure in real time.

2.3.2.2 High pressure sensing

High pressure (above atmospheric pressure) measurements showed in Fig. 2.11, were performed by placing the PCB carrying the MEMS device in a sealed pressure chamber. The chamber was connected to a nitrogen gas tank. The pressure chamber through a pressure regulator. The pressure inside the chamber was monitored by the pressure gauge on the pressure regulator.

In the low pressure test, it was observed that some of the TPOs cannot sustain their oscillation if the ambient pressure changes significantly. One reason is that the inside the chamber was increased by feeding nitrogen from the gas tank into the actuator beam is too narrow to withstand higher input current which is needed to compensate the increased external damping in higher pressures. Another reason is due
to the mechanical failure with high vibration amplitude caused by such narrow actuator beam, especially when the actuator beam becomes thermally softer under higher input currents. However, based on the conducted experiments, the device could not operate as a TPO unless the actuator beam is narrowed down to the nanoscale (as shown in Fig.

Figure 2.11 (a) The photo and (b) schematic view of the experimental setup used to characterize the frequency behavior of the TPO in high pressure environment.
2.1). In practice, it is quite challenging to achieve such small sizes repeatedly and reliably. To solve this issue, an improved design was developed to achieve oscillations in broader pressure ranges. The previous device possessed side edge dual actuator beams (SDA) while the new devices employ inner dual actuator beams (IDA) and central single actuator beam (CSA) as shown in Fig. 2.12. The number of openings (used for releasing the structures) is reduced from four large ones to nine small ones for a better mass distribution over the plate. The device with single actuator beam can effectively decrease the power consumption. Figure 2.12d compares the geometry of

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![Figure 2.12](image)

Figure 2.12 The top-view of the devices with (a) the previous designed side edge dual actuator beams (SDA), (b) new design with inner dual actuator beams (IDA) and (c) central single actuator beam (CSA). (d) The top-view and the schematic view of the geometry of the SDA (left side), IDA and CSA (right side) actuator beams.
the narrowed actuator beams in SDA, IDA and CSA. Interestingly, the new devices are capable of operating as self-sustained TPOs with even wider actuator beams. The wider and symmetrical actuator beams guarantee the TPO to work in a broader pressure range for two reasons. First, wider actuator beams can withstand higher input current to compensate more damping in higher ambient pressure. Second, the symmetrical geometry and larger width of the actuator beams result in larger stiffness of the actuator beams. Therefore, there will be a smaller vibration amplitude and hence less tensile strain to reduce the risk of mechanical failure when a higher vibration energy is provided by higher input current.

The high pressure tests were started from the atmospheric pressure and the frequency shift was recorded until oscillation was stopped as a result of the increased damping. The input current of the TPO was kept at a high level in the beginning to

![Figure 2.13 The result of CSA5 operated as TPO in different nitrogen pressure.](image)

Figure 2.13 The result of CSA5 operated as TPO in different nitrogen pressure.
enlarge the pressure range under which oscillations can be sustained. Figure 2.13 shows the measurement obtained from CSA5 operated as TPO in the high pressure test. The suffix number '5' after the device name represents the length of the actuator beam. The pressure range under which the CSA5/TPO could maintain self-sustained oscillations was from 84kPa to 429kPa. The total frequency shift over such pressure range was measured to be 656ppm that is two times higher than its TPR mode (-315ppm). However, as indicated in Fig. 2.13, the frequency shift of the TPO shows a higher change rate at first but saturates towards the end. The input power of the TPO and the peak to peak output voltage $V_{p-p}$ were measured to be as high as 18.69mW and 546mV, respectively, which are much higher than those in the low pressure test. Similar to the low pressure test, the change in the $V_{p-p}$ shows a linear relationship as the pressure changes indicated in Fig. 2.13.

Table 2.2 High pressure measurement* of IDAs and CSAs.

<table>
<thead>
<tr>
<th>Device</th>
<th>$I$ (mA)</th>
<th>$P$ (mW)</th>
<th>$f$ (MHz)</th>
<th>$\Delta Pr$ (kPa)</th>
<th>$\Delta f$ (ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IDA5</td>
<td>1.41</td>
<td>5.93</td>
<td>8.233</td>
<td>&lt;7</td>
<td>-</td>
</tr>
<tr>
<td>IDA15</td>
<td>4.87</td>
<td>35.52</td>
<td>5.677</td>
<td>138</td>
<td>1671</td>
</tr>
<tr>
<td>CSA3</td>
<td>4.08</td>
<td>13.47</td>
<td>6.284</td>
<td>28</td>
<td>18</td>
</tr>
<tr>
<td>CSA5</td>
<td>3.88</td>
<td>18.69</td>
<td>5.376</td>
<td>345</td>
<td>656</td>
</tr>
<tr>
<td>CSA10</td>
<td>3.24</td>
<td>14.58</td>
<td>4.867</td>
<td>179</td>
<td>50</td>
</tr>
</tbody>
</table>

*I* is the input current, *P* is the input power, *f* is the frequency at atmospheric pressure (84kPa in Denver), $\Delta Pr$ is the workable pressure range and $\Delta f$ is the frequency shift in such pressure range.
All other measurements obtained from the TPOs operated in the high pressure test are briefly listed in Tab. 2.2. As expected, all the TPOs showed higher frequencies as the ambient pressure was increased. In the group of IDA/TPOs, the workable pressure range of IDA15 was higher than IDA5 for its higher input power. In addition, IDA15 also showed the maximum sensitivity among all the TPOs. In the group of CSA/TPOs, CSA5 had the broadest workable pressure range and its working power was also the highest among the CSA/TPOs. In conclusion, the workable pressure ranges of IDA/TPOs or CSA/TPOs are broader if they can withstand a higher input electrical power.

2.4 Conclusion

Gas recognition and pressure sensing capability of MEMS resonant structures operating as both TPRs/TPOs were investigated. The theoretical analysis and the experimental measurements show that TPRs are more sensitive to the thermal conductivity of the surrounding gas while the frequency shift of the TPOs is mainly affected by the damping resulting from the surrounding gas (a function of gas species and pressure). The sensitivity trends of the TPOs and the TPRs derived from mathematic models were also confirmed by the measurements.

A much higher frequency shift was observed in the experiments for the TPOs compared with the TPRs resulting from the same change in the helium concentration. The test of the TPO in different species of gas indicates that its frequency shift has no clear correlation with gas's thermal conductivity but mainly depends on gas's density. In the pressure measurement, the response of the TPOs to the changes in gas pressure were measured to be 50X larger than that of the same micro structure when it was
operated as a TPR. New TPO designs that are more suitable for operation under higher pressure range were also successfully realized and tested.

In addition to the higher sensitivity of the TPOs than the TPRs showed by the measurement, devices operating in their TPOs mode also have a simpler working mechanism as their vibration is only driven by a DC current. The TPRs/TPOs with robust monolithic crystalline silicon structures can be exposed to the environment directly. Batch-fabrication of such devices using simple and well-established MEMS fabrication techniques can decrease the cost significantly. The TPRs can be applied as low cost thermal conductivity gas sensors, while the TPOs have more potential as low cost gas density sensors or pressure sensors.

Acknowledgement

The author is pleased to acknowledge the support from the University of Denver multidisciplinary research program and the National Science Foundation under grant #0923518.
CHAPTER THREE: FEA AND TED OF VENTED BEAM RESONATORS

3.1 Introduction

As mentioned in chapter one, to decrease the thermoelastic damping (TED), Candler et al. [74] experimentally applied a specific addition to the geometry - slots cut in beams. In the case of vibration, the slots act to disrupt heat current across the beam, therefore altering the process of thermoelastic dissipation. This method enables tuning of the quality factor by structure design without the need to scale its size, thus allowing for enhanced design optimization. Inspired by their work, the current study [71] in this chapter is seeking a predictive modeling tool that can be used to quantitatively evaluate the effect of beam geometry on the mitigation of thermoelastic damping for optimal design. The program of the predictive tool developed in the MATLAB platform is based on an eigenvalue finite element analysis (FEA), while the geometries are clamped-clamped (CC) and clamped-free (CF) beams with vents (slot cut) along their centerlines. We aim for finding the optimal geometry design of the vented CC/CF beam to increase the quality factor maximally. Besides TED limited quality factors, the corresponding resonant frequency as another important factor is also investigated via the finite element analysis.
3.2. Methods

One of the common capacitive micro beam resonators [2] has already been shown in Fig. 1.3 along with the definition of the geometry parameters. The micro beam is usually considered as a two-dimensional Euler-Bernoulli beam with clamped-clamped boundary condition. The thermoelastic temperature mode of such beam in its fundamental resonance frequency is indicated in Fig. 1.7. The irreversible heat current is generated from the hot region to the cold region, and indicated by the arrows. Such heat flow leads to thermoelastic damping (TED). The analytical solution [55] and the numerical solution to calculate the TED limited quality factor will be introduced in the following sections.

3.2.1. Analytical Approach of TED

For a two-dimensional Euler-Bernoulli beam, the first governing equation [84] describes the vibration coupled with the thermal stress due to the thermoelastic temperature. Such equation is formulated by adding the thermoelastic strain term $E \alpha I_T$ to the equation of motion:

$$
\rho A \frac{\partial^2 Y}{\partial t^2} + \frac{\partial^2}{\partial x^2} \left( E I \frac{\partial^2 Y}{\partial x^2} + E \alpha I_T \right) = 0
$$

(3.1) where $x$ is the longitudinal position; $Y$ is the out-of-plane displacement in the beam width direction as shown in Fig. 1.3; $E$, $\rho$ and $\alpha$ are the elastic modulus, the density and the thermal expansion coefficient of the solid, respectively; $A$ is the cross sectional area of the beam; $I$ is the mechanical bending moment of inertia and $I_T$ defines the bending moment induced by the thermal deformation where

$$
I_T = \int_A y_T dydz
$$

(3.2) where $T$ is the fluctuating temperature caused by vibration or thermoelastic
temperature. The total temperature $T_t$ of the beam is the sum of two parts $T_t = T_e + T$, where $T_e$ is the environment temperature. When the term $EaI_T$ is absent, Eq. (3.1) would degenerate to a free vibration without TED.

Similarly, the second coupling equation can be obtained by considering the heat generation caused by the alternating compression and stretching of the material. For the corresponding heat transfer problem, by neglecting the component of heat flux in the $x$-direction, the heat diffusion equation can be written in the following form,

$$\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} - \frac{EaT_0}{(1-2\nu)c_v} \frac{\partial \varepsilon}{\partial t}$$  \hspace{1cm} (3.3)

where $k$, $\nu$ and $c_v$ are the thermal diffusivity, Poisson's ratio and the heat capacity of the solid (such as silicon), respectively; $\varepsilon$ is the elastic strain. The last term in Eq. (3.3) is the heat generation due to thermal expansion/contraction.

Lifshitz's method [55] is a complex-frequency approach in which a harmonic motion is assumed in the perturbation form as follows:

$$Y(x, t) = Y_0(x)e^{i\omega t}$$
$$T(x, t) = T_0(x)e^{i\omega t}$$  \hspace{1cm} (3.4)

where $Y_0$ and $T_0$ are the alternating amplitude of out plane displacement and the thermoelastic temperature, respectively. The temperature profile can be computed along the cross section of the beam by substituting Eq. (3.4) into Eq. (3.1). The obtained temperature profile can then be used to derive the mode of vibration that is defined as a complex value:

$$\omega = \text{Re}(\omega) + i\text{Im}(\omega) \quad \text{with} \quad i = \sqrt{-1}$$  \hspace{1cm} (3.5)

where the real part $\text{Re}(\omega)$ giving the new eigen frequency (resonant frequency) of the
beam in the present of TED while the imaginary part \( \text{Im}(\omega) \) indicating the attenuation of the vibration as shown in Fig. 3.1. It is obviously that higher damping means a larger value of \( \text{Im}(\omega) \) and vice versa. The amount of TED, expressed in terms of the quality factor \( Q^{-1} \), will then be determined by

\[
Q^{-1} = 2 \left[ \frac{\text{Im}(\omega)}{\text{Re}(\omega)} \right] \quad (3.6)
\]

The final closed form solution [55] for the TED limited quality factor of the beam resonators is:

\[
Q^{-1} = \frac{E \alpha^2 T_0}{c_p} \left( \frac{6}{\xi^2} - \frac{6 \sin \xi \sin \xi}{\xi^2 \cosh \xi + \cos \xi} \right) \quad (3.7)
\]

Figure 3.1 The change of amplitude under damped condition in a free vibration of a mass-spring-damper system, while \( c=0.2 \); initial velocity is \(-0.1\) m/s; \( k, m \) and the initial vibration amplitude all have a unit value.
where

\[ \xi = w \sqrt{\frac{\omega_0}{2k}} \]  

(3.8)

The fundamental undamped resonant frequency \( \omega_0 \) of the beam is

\[ \omega_0 = \beta \sqrt{\frac{Ew^2}{12\rho L^4}} \]  

(3.9)

where \( w \) is the width of the beam; \( L \) is the beam length and \( \beta \) is a coefficient determined by the boundary constraints. The value of \( \beta \) is 22.37 and 3.52 for clamped-clamped and clamped-free conditions, respectively [36].

3.2.2. Finite Element Formulation of TED

As stated in previous section, a micro beam can be simplified as either a two-dimensional plane strain or plane stress problem [84] to save the computational effort. Therefore, the following derivations are based on the two-dimensional problems only. However, three-dimensional problems whose geometries are more complex than a beam can be formulated in a similar way. The equation of motion written in the continuous form is

\[ \rho \frac{\partial^2 u}{\partial t^2} - \nabla (Cu - DT) = \rho \ddot{u} - \nabla (Cu - DT) = 0 \]  

(3.10)

where

\[ u = \begin{bmatrix} u_{xx} \\ u_{yy} \\ u_{xy} \end{bmatrix} \]

\[ \ddot{u} = \begin{bmatrix} \ddot{u}_{xx} \\ \ddot{u}_{yy} \\ \ddot{u}_{xy} \end{bmatrix} \]  

(3.11)

\[ T = \begin{bmatrix} T_{xx} \\ T_{xy} \end{bmatrix} \]

and

\[ C = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & 1/2-\nu \end{bmatrix} \]  

for plane strain, when \( T_b > W_b \)  

(3.12a)
\[ C = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \text{ for plane stress, when } T_b < W_b \]  
\[ (3.12b) \]

In addition,
\[ D = \frac{E\alpha}{(1-2\nu)} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \]  
\[ (3.13) \]

where \( u \) is the displacement tensor; \( T \) is alternating temperature tensor; \( C \) is stiffness matrix; \( D \) is the thermal expansion coefficient matrix; the geometry factor \( T_b \) and \( W_b \) are already indicated in Fig. 1.3. The Galerkin finite element method is then applied to the single elemental domain to approximate the displacement field as below:
\[ u = [N][u_x \quad u_y]^T = [N]u_e \]  
\[ (3.14) \]
\[ \ddot{u} = [N][\ddot{u}_x \quad \ddot{u}_y]^T = [N]\ddot{u}_e \]  
\[ (3.15) \]
\[ T = [N][T_x \quad T_y]^T = [N]T_e \]  
\[ (3.16) \]
\[ \epsilon = [\epsilon_x \quad \epsilon_y \quad \epsilon_{xy}] = [\partial]u = ([\partial][N])u_e = [B]u_e \]  
\[ (3.17) \]

and to the \( i \)th node of the element,
\[ B_i = \begin{bmatrix} \frac{dN_i}{dx} & 0 \\ 0 & \frac{dN_i}{dy} \\ \frac{dN_i}{dy} & \frac{dN_i}{dx} \end{bmatrix} \]  
\[ (3.18) \]

where \( N \) is the shape function, \( \epsilon \) is the elastic strain tensor, the subscript \( e \) represents the nodal value. Based on Eq. (3.10), the Galerkin residual equation \[85\] in the whole physical domain (the beam with all the elements) is
\[ \iint[N]^T[\rho \ddot{u} - \nabla(Cu - DT)] \, dx \, dy = 0 \]  
\[ (3.19) \]

Integration of the second and third term in Eq.(3.19) by parts yields
\[ \iint[N]^T \frac{\partial}{\partial x} (C\epsilon) \, dx \, dy = \iint[N]^T [d(C\epsilon)] \, dy = - \iint[N_x]^T C \, dy + \int[N]^T C \, dy \]  
\[ (3.20) \]
\[ \iint[N]^T \frac{\partial}{\partial y} (C\epsilon) \, dx \, dy = \iint[N]^T [d(C\epsilon)] \, dx = - \iint[N_y]^T C \, dx + \int[N]^T C \, dx \]  
\[ (3.21) \]
\[ \int [N]^T \frac{\partial}{\partial x} (DT) dx dy = \int [N]^T [d(DT)] dy = - \int [N_x]^T [d(DT)] dy + \int [N]^T DT dy \quad (3.22) \]
\[ \int [N]^T \frac{\partial}{\partial y} (DT) dx dy = \int [N]^T [d(DT)] dx = - \int [N_y]^T [d(DT)] dx + \int [N]^T DT dx \quad (3.23) \]

For free vibration, the boundary condition leads to
\[ \int [N]^T C\varepsilon dx = \int [N]^T D\varepsilon dx = \int [N]^T DT dx = 0 \quad (3.24) \]

It follows
\[ \int [N]^T \rho [N] \{\ddot{u}_e\} dx dy + \int [B]^T C[B] \{u_e\} dx dy - \int [N]^T D[N] \{T_e\} dx dy = 0 \quad (3.25) \]

Reducing this equation to the matrix form results in the following condensed expression:

\[ M\{\ddot{u}_e\} + L\{u_e\} - G\{T_e\} = 0 \quad (3.26) \]

where \( M \) is mass matrix, \( L \) is stiffness matrix and \( G \) is the thermal stress induced by thermal deformation.

Likewise, the governing heat diffusion equation, Eq. (3.3), in the differential form can be written as
\[ k\nabla^2 T - \rho c_p \frac{\partial T}{\partial t} - C \frac{\partial \varepsilon^*}{\partial t} = k\nabla^2 T - \rho c_p \frac{\partial T}{\partial t} - C \frac{\partial \varepsilon^*}{\partial t} = 0 \quad (3.27) \]

For the plane strain condition, the strain rate that contributes to thermoelastic temperature only consists of two components \( \varepsilon_x \) and \( \varepsilon_y \), because the shear strain \( \varepsilon_{xy} \) does not generate any heat [84]. Hence,
\[ \dot{\varepsilon}^* = \frac{\partial}{\partial t} \left( \varepsilon_x + \varepsilon_y \right) \quad (3.28) \]

The Galerkin finite element method is then applied to approximate the displacement and temperature fields as:
\[ \dot{\varepsilon}^* = [N_x] \{\ddot{u}_{e_x}\} + [N_y] \{\ddot{u}_{e_y}\} = [B^*] \{\ddot{u}_e\} \quad (3.29) \]

and
\[ B_i^* = \begin{bmatrix} \frac{dN_i}{dx} & 0 \\ 0 & \frac{dN_i}{dy} \end{bmatrix} \]  

(3.30)

Compared with \([B]\) in Eq. (3.18), the term for the shear strain is not included in the matrix \([B^*]\).

The Galerkin residual equation is

\[
\iint [N]^T [k \nabla^2 T - \rho c_p \dot{T} - C \dot{\varepsilon}^*] \, dxdy = 0
\]  

(3.31)

Integration the first term in Eq. (3.31) by parts yields

\[
\iint [N]^T \frac{\partial}{\partial x}(k_x \dot{T}_x) \, dxdy = -\iint (N_x)^T k_x \dot{T}_x \, dxdy + \iint [N]^T k_x \dot{T}_x \, dxdy \]  

(3.32)

\[
\iint [N]^T \frac{\partial}{\partial y}(k_y \dot{T}_y) \, dxdy = -\iint (N_y)^T k_y \dot{T}_y \, dxdy + \iint [N]^T k_y \dot{T}_y \, dxdy \]  

(3.33)

For the insulated thermal boundary condition, i.e.

\[
\int [N]^T k_x \dot{T}_x \, dy = \int [N]^T k_y \dot{T}_y \, dx = 0
\]  

(3.34)

It leads to

\[
\iint [B]^T k[B] \{T_e\} \, dxdy + \iint [N]^T \rho c_p \{T_{\dot{e}}\} \, dxdy + \iint [N]^T \varepsilon [B^*] \{\dot{u}_{\dot{e}}\} \, dxdy = 0
\]  

(3.35)

Rewriting this equation in the matrix form yields:

\[
K \{T_e\} + H \{T_{\dot{e}}\} + F \{\dot{u}_{\dot{e}}\} = 0
\]  

(3.36)

where \(K\) is the thermal conductivity matrix, \(H\) is the heat generation matrix and \(F\) is the coupling matrix representing the heat generation induced by beam deformation.

Similar to Eq. (3.4), here we also assume a perturbation form of the solution,

\[
T = T_0 e^{i\omega t}
\]  

(3.37)

\[
u = u_0 e^{i\omega t}
\]  

(3.38)

\[
\dot{u} = \dot{u}_0 e^{i\omega t}
\]  

(3.39)

It immediately follows that

\[
\dot{u} = \frac{\partial u}{\partial t} = \frac{\partial}{\partial t} (u_0 e^{i\omega t}) = i\omega (u_0 e^{i\omega t}) = i\omega u
\]  

(3.40)
Substituting Eqs. (3.37)~(3.40) into Eqs. (3.26) and (3.36) results in:

\[ i\omega M\{\dot{u}_e\} + L\{u_e\} - G\{T_e\} = 0 \]  
\[ K\{T_e\} + i\omega H\{T_e\} + i\omega F\{u_e\} = 0 \]  

(3.41) \hspace{2cm} (3.42)

The final matrix equation by combining Eqs. (3.40) ~ (3.42) is

\[
\begin{bmatrix}
-K & 0 & 0 \\
G & -L & 0 \\
0 & 0 & I
\end{bmatrix}
\begin{bmatrix}
T_e \\
u_e \\
\dot{u}_e
\end{bmatrix}
= i\omega
\begin{bmatrix}
H & F & 0 \\
0 & 0 & M \\
0 & I & 0
\end{bmatrix}
\begin{bmatrix}
T_e \\
u_e \\
\dot{u}_e
\end{bmatrix}
\]  

(3.43)

where \( i\omega \) is the eigenvalue of the equation. By solving Eq. (3.43) to calculate its eigenvalue solution \( i\omega \), the thermoelastic damping limited quality factor (\( Q \)-value) can then be obtained from Eq. (3.6).

To the micro beam resonator shown in Fig. 1.3 whose thickness is larger than its width, two-dimensional plain-strain element is used in the following analysis. In addition, the quadratic quadrilateral nine-node (Q9) plain strain elements were performed in the convergence test and compared with the commonly used bilinear quadrilateral four-node (Q4) elements. The four node Q4 and Q9 elements [85] and their mapped element in the \( \xi - \eta \) coordinate are indicated in Fig. 3.2, respectively.

In the case of Q4 element, the shape function is:

\[ N_1 = \frac{1}{4}(1 - \xi)(1 - \eta) \]
\[ N_2 = \frac{1}{4}(1 + \xi)(1 - \eta) \]
\[ N_3 = \frac{1}{4}(1 + \xi)(1 + \eta) \]
\[ N_4 = \frac{1}{4}(1 - \xi)(1 + \eta) \]  

(3.44)

On the other side, the shape function of Q9 element is:

\[ N_1 = \frac{1}{4}(1 - \xi)(1 - \eta) - \frac{1}{2}N_5 - \frac{1}{2}N_8 - \frac{1}{4}N_9 \]
\[ N_2 = \frac{1}{4} (1 + \xi)(1 - \eta) - \frac{1}{2} N_5 - \frac{1}{2} N_6 - \frac{1}{4} N_9 \]
\[ N_3 = \frac{1}{4} (1 + \xi)(1 + \eta) - \frac{1}{2} N_6 - \frac{1}{2} N_7 - \frac{1}{4} N_9 \]
\[ N_5 = \frac{1}{4} (1 - \xi)(1 + \eta) - \frac{1}{2} N_7 - \frac{1}{2} N_8 - \frac{1}{4} N_9 \]
\[ N_5 = \frac{1}{4} (1 - \xi^2)(1 - \eta) - \frac{1}{2} N_9 \]
\[ N_6 = \frac{1}{4} (1 + \xi)(1 - \eta^2) - \frac{1}{2} N_9 \]
\[ N_7 = \frac{1}{4} (1 - \xi^2)(1 + \eta) - \frac{1}{2} N_9 \]
\[ N_8 = \frac{1}{4} (1 - \xi)(1 - \eta^2) - \frac{1}{2} N_9 \]
\[ N_9 = \frac{1}{4} (1 - \xi^2)(1 - \eta^2) \] (3.45)

Since the energy dissipation of vibrating beams is mainly caused by thermoelastic heat generation and propagation the quality factor can be improved by reducing the conduction path. We hypothesize that these goals can be achieved by adding vent sections to the solid beam to disrupt the heat flow across the thickness. To validate the hypothesis, the finite element method mentioned above was applied to investigate the geometric effects on the quality factor and the resonant frequency with the presence of TED. MATLAB® was used as the programming tool in performing the numerical analysis, including generating the mesh, eigenvalue solver and post processing.

3.3. Results and Discussions
3.3.1. Finite Element Convergence Tests

To validate the precision of finite element method, the convergence tests were performed on a clamped-clamped (CC) solid (without vents) beam resonator of 200μm long \((L_b)\) and 10μm wide \((W_b)\). The beam was discretized into rectangle-shaped elements of the same size to ensure that the nodes were evenly distributed along the
length and the width. The nodes along the length are varied from 21 (20 Q4 elements or 10 Q9 elements) to 201 (200 Q4 elements or 100 Q9 elements), while 7 nodes (6 Q4 elements or 3 Q9 elements) are fixed along the width. The silicon material properties
are listed in Tab. 3.1. The value of Poisson’s ratio was set to zero in the convergence test to make a better comparison with the analytical results [55] that leave out the shear effect. The result of the convergence test is shown in Fig. 3.3. It is clearly that the Q9 elements are more preferred for finite element analysis since they result in a quicker convergence than the Q4 elements. Compared with the analytical solution where the quality factor is 14,646 and the frequency is 2.11MHz, the numerical error in the quality factor obtained by the finite element method with Q9 elements is 0.55% by Q9 elements (14,747), meanwhile the error in the frequency is 1.42% by Q9 (2.08MHz) correspondingly, when the maximum mesh density is used in current analysis. It is also noticed that the convergent frequency is slightly lower than the analytical solution, because the latter is based on the Euler-Bernoulli beam model that is less accurate than the elasticity theory. Figure 3.3 also indicates that the Q9 elements can converge quickly to fairly accurate result when the nodes number along the length direction is more than 81 (40 elements). To balance the effort of computation and the accuracy, the results based on Q9 elements with two different mesh densities have been compared in

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus (Pa)</td>
<td>1.57×10^{11}</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.22</td>
</tr>
<tr>
<td>Thermal expansion coefficient (K/L)</td>
<td>2.6 × 10^{-6}</td>
</tr>
<tr>
<td>Thermal conductivity (W/m K)</td>
<td>90</td>
</tr>
<tr>
<td>Specific heat (J/Kg K)</td>
<td>700</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>2,330</td>
</tr>
<tr>
<td>Temperature (K)</td>
<td>300</td>
</tr>
</tbody>
</table>
Figure 3.3 Convergence tests for (a) the quality factor and (b) the fundamental frequency of flexural-mode vibration of CC beam resonator using Q4 and Q9 elements.
Tab. 3.2. It shows that the accuracy based on the mesh with 81 nodes along the beam length is already practicable for analysis. When Poison's ratio is considered, the quality factor and the frequency are 9400 and 2.13MHz, respectively, based on the 7×81 mesh.

Table 3.2 Convergence test.

<table>
<thead>
<tr>
<th>Nodes number (length)</th>
<th>81</th>
<th>201</th>
<th>Err*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality factor</td>
<td>14,727.45</td>
<td>14,726.91</td>
<td>3.667×10^{-05}</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>2,081,657</td>
<td>2,081,584</td>
<td>3.507×10^{-05}</td>
</tr>
</tbody>
</table>

* The error is estimated by comparing the result of 81 nodes to that of 201 nodes along the length of the solid beam.

3.3.2. The Location of the Vents

The same CC beams of 200μm long and 10μm wide are considered. The vented beam is shown in Fig. 3.4 (in x-y plane), as well as the definition of the geometry parameters. To determine the optimal location of the vents, a single vent whose location varies along the beam with a fixed size, 3.33μm wide and 10μm long, has been studied. The vent location is defined by $V_L$ that indicates the distance from the center of the vent to the left clamped end of the CC beam. The results of the

![Figure 3.4 Schematic view of the beam resonator with a single vent.](image-url)
quality factor and the frequency normalized against those of the solid beam are shown in Fig. 3.5. The symmetry in the curves is caused by the symmetry in the vent locations with respect to the mid plane of the beam. The quality factor increases by 16.9% and the frequency decreases by only 0.9% when the vents are located in the clamped region of the beam ($V_L = L_v/2$). Another peak of the result appears when the vent is in the center region of the beam ($V_L = L_b/2$). The quality factor is enhanced by 8.3% while the frequency is 2.0% higher than the solid beam. As shown in Fig. 1.7, the maximum temperature difference is generated in the clamped-end and center of the solid beam along the beam width direction. Therefore, it can be concluded that disrupting the heat flow path across the beam width in such locations can increase the quality

Figure 3.5 Quality factor and frequency of the vented beam as functions of the vent location.
factor efficiently. The numerical result illustrated by Fig. 3.5 has not only proved such conclusion but also shows the vent located in the clamped end region can increase the quality factor more efficiently than the condition when the vent is located in the center region. On the other hand, the numerical result shows that the vent located in the clamped end can decrease the stiffness of the beam resonator and then lead to the decline of the frequency. However, the vent in the center of the beam showed in present analysis just leads to the loss of weight of the beam resonator, similar to decrease the value of 'm' of a mass-spring system and therefore increases the frequency of the beam.

Another set of analysis is performed to the beam with two vents that are symmetrical distributed. The geometry is shown in Fig. 3.6. The size of the single vent

![Figure 3.6 Schematic view of (a) the beam with two symmetrical distributed vents and (b) the corresponding half-model.](image-url)
is the same as the previous section that is still 3.33\,\mu m wide and 10\,\mu m long. To increase the computational efficiency, the symmetry of the vented beam is used as indicated in Fig. 3.6b. The result when the single beam is moved from the clamped end to the center is shown in Fig. 3.7. It has been found that the maximum increase in the quality factor is as much as 40.6\% when both vents are located in the clamped ends of the CC beam, while the frequency is lowered by only 1.9\%, in comparison with the solid beam. On the other hand, as the vents move to the center region (V_L =95\,\mu m) of the beam, the quality factor increases by 17.5\%, while the frequency increases by 4.1\%. The same as previous study, the much higher quality factor for vents located in the clamped-ends compared with those located in the beam center implies that TED in solid beams is
predominantly contributed by the heat flow generated in the regions of clamped-ends of the beam resonator. What’s more, the result of the change in the quality factor and frequency of the beam with single and symmetrical vents are all listed in Table 3.3 to be conveniently compared with each other.

Table 3.3 Results under different geometries - I.

<table>
<thead>
<tr>
<th>Changes*</th>
<th>Single vent</th>
<th>Symmetrical Dual-vent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Clamped-end</td>
<td>Beam center</td>
</tr>
<tr>
<td>Quality factor</td>
<td>16.9%</td>
<td>8.3%</td>
</tr>
<tr>
<td>Frequency</td>
<td>-0.9%</td>
<td>2.0%</td>
</tr>
</tbody>
</table>

* The increased percentage is with respect to the result of the solid beam without vents.

3.3.3. Geometric Optimization of the Vent Length

The vent size effects on the quality factor and the frequency are investigated for two vent locations: the clamped ends and the beam center, as illustrated in Fig. 3.8, since these locations lead to the maximum quality factors. The analysis of the length effect was performed by altering the beam length from 5μm to 95μm with a constant width of 3.33μm. The result is shown in Fig. 3.9. As expected, the quality factor increases with the vent length for both locations. In more detail, the quality factor increases almost linearly with the vent length when the vent located in the clamped end. Meanwhile, for the vent located at the beam center, the increase in the quality factor is fairly slow at the beginning but gains its speed when the vent length approaches the beam length. The result of this comparison implies that the vents in the clamped ends
can improve the quality factor more effectively than the vents in the beam center. Figure 3.9b shows that the frequency decreases when the vents are moving closer to the clamped end, since the presence of the vent section in the clamped ends causes a major loss of the structural stiffness. By implementing a vent of maximum length 95μm in present study, we have obtained a maximum increase in the quality factor as high as 347.3% while the frequency is lowered by 66.2% when the vents are located in the clamped ends. By comparison, the quality factor is raised by 308.9% and its frequency is reduced by 63.8% for the centered vents with the same length of 95μm.
Figure 3.9 Effects of the length of symmetrical vents located at (a) the clamped end and (b) the beam center.
In addition to the vents located in the clamped ends or the beam center, the effects of vents at other locations are also investigated. Figure 3.10 shows a beam with three vents, each $3.33\mu m$ wide. The maximum full length of the vents is maintained as $95\mu m$ for consistency. The computational result is shown in Fig. 3.11. It can be seen that the peak quality factor occurs when the vent at either location approaches its maximum length. In fact, the maximum increase in the quality factor is 322.1% corresponding to the longest vents located at the clamped ends. This result is higher than the maximum increase in the quality factor of the longest vents at the beam center, which is 263.9%. This demonstrates once again that the vents located at the clamped ends can improve the quality factor more efficiently than the vents located at the beam center. On the other hand, the change in the resonant frequency is opposite to that in the quality factor for the vented beam. Such a trend indicates that the vents in the clamped
ends can reduce the structural rigidity of the system more significantly than other locations. Moreover, all the most important numerical results in this section is provided in Tab. 3.4.

Table 3.4 Results under different geometries - II.

<table>
<thead>
<tr>
<th></th>
<th>Maximized Length</th>
<th>Quality factor*</th>
<th>Frequency*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetrical</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Clamped-ends</td>
<td>347.3%</td>
<td>-66.2%</td>
</tr>
<tr>
<td></td>
<td>Center</td>
<td>308.9%</td>
<td>-63.8%</td>
</tr>
<tr>
<td>Combination</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Clamped-ends</td>
<td>322.1%</td>
<td>-64.8%</td>
</tr>
<tr>
<td></td>
<td>Center</td>
<td>263.9%</td>
<td>-60.1%</td>
</tr>
</tbody>
</table>

* The increased percentage is with respect to the result of the solid beam.
3.3.4. Geometric Optimization of the Vent Width

Based on the previous study, it has been shown that the highest quality factor can be achieved by symmetrically placing long vents in the clamped ends. In this section we are focused on the investigation of vent width effect on the geometry showed in Fig. 3.8a. The width of the vent is altered from 1.2μm to 9.2μm with different lengths of 35μm, 65μm and 95μm. The result in Fig. 3.12a indicates that vents of greater size (longer and wider) exhibit higher quality factors. For vents with constant length the quality factor increases sharply with the width. We also observed that the maximum quality factor of 65μm-long vents are slightly lower than 95μm-long vents but significantly higher than that of 35μm-long vents. It turns out that the highest achievable quality factor based on the present study is 3,801 times that of the solid beam when the vent is 95μm long and 9.2μm wide. The change in the frequency is almost linear, as shown in Fig. 3.12b, meanwhile it converges to a much lower value as the width increases. The change in the quality factor and frequency of the CC beam resonator with the largest width under different length in the numerical analysis are indicated in Tab. 3.5.

Table 3.5 Vent width 9.2μm - CC beam.

<table>
<thead>
<tr>
<th>Length</th>
<th>35μm</th>
<th>65μm</th>
<th>95μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality factor*</td>
<td>67,999%</td>
<td>365,706%</td>
<td>380,007%</td>
</tr>
<tr>
<td>Frequency*</td>
<td>-95.7%</td>
<td>-97.8%</td>
<td>-97.3%</td>
</tr>
</tbody>
</table>

* The increased percentage is with respect to the result of the solid beam.
Figure 3.12 (a) Quality factor and (b) resonant frequency of the vented beams as functions of the vent width and length, for vents symmetrically located at the clamped ends.
3.3.5. Geometric Optimization of Vented CF Beam (Cantilever)

By changing the boundary condition of the symmetric plane indicated in Fig. 3.6, to a free end, the CC beam became a clamped-free (CF) beam (or cantilever) with a clamped-free (CF) beam (or cantilever) with a

Figure 3.13 (a) 3D view of CF beam resonator and (b) its corresponding temperature contour of its dominant flexural vibration mode.
half length as indicated in Fig. 3.13. Such CF beam, that is 100μm long and 10μm wide, is studied in this section. The contour of the thermoelastic temperature in its fundamental resonance mode is shown in Fig. 3.13b. The greatest temperature variation takes place in its clamped end. Therefore, the optimal location of the vent to disrupt the heat flow is expected be the clamped end. Its corresponding numerical analysis is performed by altering the location of the vents from the clamped-end to the free-end of the CF beam. The vent is still with a width 3.33μm and length 10μm. The result shown in Fig. 3.14 comes to the conclusion that the optimal location of the vent is just the clamped-end, as expected from the thermoelastic contour analysis. The

Figure 3.14 Quality factor and frequency of the CF vented beam as functions of the vent location.
quality factor is up by 49.9% and the frequency is down by 1.1% compared with that of solid CF beam.

After the optimal position of the vent is found, such vent with different length but fixed width, 3.33μm, in the CF beam is analyzed and the results are shown in Fig. 3.15. The quality factor increases quickly at first but slower when the length of the vent approaches to the length of the entire beam. Such trend is almost opposite to the change in the centered vent in clamped-clamped beam as shown in Fig. 3.9b. The quality factor is increased by 586% and the frequency is reduced by 49.8% when the vent reaches its maximum length of 95μm in present work. On the other hand, the width of the vent is

![Figure 3.15](image)

Figure 3.15 The results of the CF beam with a vent at its clamped end as functions of the vent length.
Figure 3.16 (a) The quality factor and (b) resonant frequency of the CF beam as a function of the vent width.
also investigated in the same way as the vented CC beam. The result of the quality and frequency as a function of the width of the vents showed in Fig. 3.16a and 3.16b, respectively, is also similar to the vents located in the clamped ends of the CC beams. To the vented cantilever, the maximum quality factor achieved in this work is as high as 2257 times that of the solid cantilever when the vent is 95μm long and 9.2μm wide. The maximum increased percentage of the quality factor and its corresponding change in the frequency conducted in this section is listed in Tab. 3.6.

Table 3.6 Vent width 9.2μm - CF beam.

<table>
<thead>
<tr>
<th>Length</th>
<th>35μm</th>
<th>65μm</th>
<th>95μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality factor*</td>
<td>14,932%</td>
<td>70,757%</td>
<td>225,599%</td>
</tr>
<tr>
<td>Frequency*</td>
<td>-93.3%</td>
<td>-96.5%</td>
<td>-95.7%</td>
</tr>
</tbody>
</table>

* The increased percentage is with respect to the result of the solid beam.

3.4 Conclusion

A customized finite element method is implemented on the MATLAB platform to investigate the geometric effects of vents on thermoelastic energy loss in clamped-clamped (CC) and clamped-free (CF) beam resonators. The quality factor and resonant frequency are obtained as functions of various geometric parameters including the location, number and size of the vents. For vented CC beams, it has been found that the vent located in the clamped end and the center of the beam can both increase the quality factor more efficiently than other places, though the optimal location is the clamped end that the TED is mainly determined by. The optimum geometry of vented CC beam to improve TED limited quality factor is with two vents symmetrically located in the clamped ends. Although the quality factor can increase with the vent size,
the effect of the vent width is typically more significant than the vent length. In present study, the highest quality factor that is 3801 times that of the solid beam has been achieved. On the other hand, the frequency studied in this work shows that the vents located in the clamped ends can reduce the structural rigidity and hence significantly reduce the resonant frequency. The effect of the vents in cantilever resonators (CF beam resonators) is also investigated by the finite element method. The conclusion is similar to the CC beams.
CHAPTER FOUR: SUPPRESSION OF TED BY PIEZORESISTIVE EFFECT

4.1 Introduction

Although the thermoelastic damping (TED) can be decreased efficiently by adding vents on the MEMS beam resonator, the stiffness of the micro beam will decline and such method is also limited by the fabrication in nanoscale. Therefore, in this chapter, a new method [72] without such problems by utilizing the piezoresistive effect to compensate TED of the micro beam resonator in its flexural mode vibration is proposed and studied. Such new method is mainly inspired by the research of thermally actuated self-sustained oscillator (TPO) indicated in chapter two. The damping loss of the vibration in TPOs is fully compensated by the energy supplied by an electro-thermo-mechanical feedback in the system. Hence, a similar energy supply mode by utilizing the piezoresistivity is applied to compensate TED in the flexural vibration mode of micro beams. The mechanism is firstly explained by a series of differential equations for the couplings among the electrical, thermal and mechanical processes. Thereafter a set of finite element equations are derived and the quality factor is computed from an eigenvalue analysis.

4.2. Methods

4.2.1. Physical Mechanism

The energy loss due to the relaxation of mechanically induced temperature gradients and the resulting irreversible heat flow is show in Fig. 1.7. The compression
in the beam leads to a higher temperature (hot) region while the tensile stress on the opposite side of the beam results in a lower temperature (cold) region. We assume that an electrostatic field is applied on a clamped-clamped silicon beam resonator. When the system starts to vibrate in a fashion shown in Fig. 1.7, the resistance $R$ of the compressed region can be found as

$$R = R_0 (1 + \pi_l \sigma)$$  \hspace{1cm} (4.1)

where $R_0$ is the resistance without the stress load; $\pi_l$ and $\sigma$ are the piezoresistive coefficient and the mechanical stress, respectively. The electrical power $P$ generated is defined as

$$P = \frac{U^2}{R}$$  \hspace{1cm} (4.2)

where $U$ is the voltage across the compressed region. When the resonator has a negative piezoresistive coefficient, such as single crystal silicon after $N$-type doping, $R$ will increase and $P$ will decrease in those regions subjected to compressive stresses. Hence, the piezoresistive effect leads to a reduced electrical power and thus a lower temperature in the compressed region, whereas the temperature in the stretched region increases. Consequently the temperature gradient generated by the thermoelastic effect shown in Fig. 1.7 is attenuated by the reverse temperature gradient induced by the piezoresistivity. The reduced heat low rate results in a less energy loss by TED and a higher quality factor. Briefly, the mechanism is closely related to the coupling among conductive heat transfer, mechanical vibration and electrical piezoresistive effect.

**4.2.2. Heat Transfer**

The heat transfer equation involved in thermoelastic damping is known as

$$\rho c_p \frac{\partial T}{\partial t} - \nabla \cdot (k \nabla \cdot T) = P_m + P_e$$  \hspace{1cm} (4.3)
where $\rho$ is the density of the material, $c_p$ is the specific heat capacity, $T$ is the temperature, $k$ is the thermal conductivity, $P_m$ and $P_e$ are the heat generation rate per unit volume caused by the thermoelastic deformation and the amplitude of the electrical power change, respectively. Without piezoresistive effect, $P_e=0$ and Eq. (4.3) will degrade to the classic thermoelastic problem as discussed in chapter three. The heat generation $P_m$ comes from thermoelastic heating governed by

$$P_m = -\frac{E\alpha T_a}{(1-\nu)} \frac{\partial \varepsilon}{\partial t} \tag{4.4}$$

where $E$ is Young’s modulus, $\alpha$ is the coefficient of thermal expansion, $T_a$ is the ambient temperature, $\nu$ is Poisson's ratio, and the dilatation strain tensor $\varepsilon$ is defined as

$$\varepsilon = \nabla \cdot \mathbf{u} = \nabla \cdot (u_{11} + u_{22} + u_{33}) \tag{4.5}$$

where $\mathbf{u}$ is the displacement tensor; $u_{11}$, $u_{22}$ and $u_{33}$ are the strain components in the $x$-, $y$- and $z$- directions, respectively.

### 4.2.3. Mechanical Vibration

The equation of motion for an elastic solid is obtained from the following force equilibrium:

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \nabla \cdot \mathbf{\sigma} \tag{4.6}$$

where $\mathbf{\sigma}$ is the stress tensor. The stress under mechanical relaxation is defined by Hooke's Law:

$$\mathbf{\sigma} = C \varepsilon = C (\varepsilon_s - \varepsilon_t) = C (\varepsilon_s - \alpha T) \tag{4.7}$$

where $C$ is the stiffness tensor; $\varepsilon_s$ is the strain tensor without thermal effect and $\varepsilon_t$ is the thermal strain tensor.
4.2.4. Electrical Piezoresistive Effect

The continuity equation for the electrical current $J$ under an electrostatic field $E_e$ can be written as

$$\nabla \cdot J = \nabla \cdot \sigma_e E_e = 0 \quad (4.8)$$

where $\sigma_e$ is the electrical conductivity.

The electrical power $P_e$ can be found from

$$P_e = E_e J = \sigma_e E_e^2. \quad (4.9)$$

In the present of piezoresistivity, the electrical conductivity, $\sigma_e$, is composed of two terms: the static component denoted by '$s$' and the harmonic component induced by the mechanical stress denoted by '$h$'. Eq. (4.9) is then rewritten as

$$P_{eh} = (\sigma_{es} + \sigma_{eh}) E_e^2 \quad (4.10)$$

where the harmonic component in the electrical power, $P_{eh}$, is defined as

$$P_{eh} = \sigma_{eh} E_e^2 \quad (4.11)$$

As indicated in Eq. (4.8), the electrostatic field $E_e$ is obtained from,

$$\nabla \cdot \sigma_{es} E_e = 0 \quad (4.12)$$

For single crystal silicon or other semiconductors, in the presence of piezoresistivity resulted from the change in the conduction band by mechanical stress, $\sigma$, leads to a change in the conductivity $\sigma_{eh}$ that is defined by

$$\sigma_{eh} = \pi_i \sigma / \sigma_{es} \quad (4.13)$$

The resistance $R_0$ in Eq. (4.1) is defined as

$$R_0 = \rho_{es} \frac{L}{A} = \frac{L}{\sigma_{es} A} \quad (4.14)$$

where $\rho_{es}$ is the static electrical resistivity; $L$ and $A$ are the length and cross-sectional
area of the conductor, respectively. The harmonic component of the electrical resistivity \( \rho_{eh} \) due to the mechanical stress \( \sigma \) is defined by the following equation [83]:

\[
\rho_{eh} = \rho_{es} \pi_{ij} \sigma = \rho_{es} \begin{bmatrix}
\pi_{11} & \pi_{12} & 0 & 0 & 0 \\
\pi_{12} & \pi_{11} & 0 & 0 & 0 \\
0 & 0 & \pi_{12} & \pi_{11} & 0 \\
0 & 0 & 0 & \pi_{12} & \pi_{11} \\
0 & 0 & 0 & 0 & \pi_{12}
\end{bmatrix} \begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{23} \\
\sigma_{31}
\end{bmatrix}
\] (4.15)

where \( \pi_{ij} \) is the piezoresistive tensor; \( \sigma_{11}, \sigma_{22} \) and \( \sigma_{33} \) are the stress components in the x-, y- and z- directions, respectively. The electrical conductivity, \( \sigma_e \), is defined as,

\[
\sigma_e = \frac{1}{\rho_{es} + \rho_{eh}}
\] (4.16)

which can be expressed in the following form [86],

\[
\sigma_e = \sigma_{es} + \sigma_{eh} = \frac{1}{\rho_{es}} \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} - \frac{1}{\rho_{es}^2} \begin{bmatrix}
\rho_{eh11} & \rho_{eh12} & \rho_{eh13} \\
\rho_{eh21} & \rho_{eh22} & \rho_{eh23} \\
\rho_{eh31} & \rho_{eh32} & \rho_{eh33}
\end{bmatrix}
\] (4.17)

For two-dimensional problems, Eqs. (4.15) and (4.17) is degraded to:

\[
\rho_{eh} = \begin{bmatrix}
\rho_{eh11} \\
\rho_{eh22} \\
\rho_{eh12}
\end{bmatrix} = \rho_{es} \begin{bmatrix}
\pi_{11} & \pi_{12} & 0 \\
\pi_{12} & \pi_{11} & 0 \\
0 & 0 & \pi_{44}
\end{bmatrix} \begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{12}
\end{bmatrix}
\] (4.18)

and

\[
\sigma_e = \sigma_{es} + \sigma_{eh} = \frac{1}{\rho_{es}} \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} - \frac{1}{\rho_{es}^2} \begin{bmatrix}
\rho_{eh11} & \rho_{eh12} \\
\rho_{eh12} & \rho_{eh22}
\end{bmatrix}
\] (4.19)

Therefore,

\[
\sigma_{eh} = -\frac{1}{\rho_{es}^2} \begin{bmatrix}
\rho_{eh11} & \rho_{eh12} \\
\rho_{eh12} & \rho_{eh22}
\end{bmatrix}
\] (4.20)

Substituting Eq. (4.18) to Eq. (4.20), resulting in:

\[
\sigma_{eh} = -\frac{1}{\rho_{es}} \begin{bmatrix}
\pi_{11} \sigma_{11} + \pi_{12} \sigma_{22} \\
\pi_{44} \sigma_{12}
\end{bmatrix} \begin{bmatrix}
\pi_{11} \sigma_{11} + \pi_{12} \sigma_{22} \\
\pi_{44} \sigma_{12}
\end{bmatrix}
\] (4.21)

4.2.5. Finite Element Formulations

In harmonic vibration, the temperature and displacement have the following forms,

\[
T = T_0 e^{i\omega t}
\] (4.22)
\[ U = U_0 e^{i\omega t} \]  \hspace{1cm} (4.23)

\[ V = V_0 e^{i\omega t} = \frac{\partial u}{\partial t} e^{i\omega t} = i\omega V_0 e^{i\omega t} \]  \hspace{1cm} (4.24)

where \( \omega \) is the angular frequency and generally complex when damping is present; \( V \) is the velocity; \( T_0, U_0, \) and \( V_0 \) are the amplitudes of temperature, displacement and velocity, respectively.

By applying the interpolating function \( N \) (i.e. the shape function) to Eqs. (4.3) \sim (4.5), we obtain the following finite element equations:

\[(K + \omega H)T + \omega FU + P_{eh} = 0 \]  \hspace{1cm} (4.25)

where \( K, H \) and \( F \) are the coefficient matrices, \( T \) is the nodal temperature, \( U \) is the nodal displacement and \( P_{eh} \) is the harmonic change in the electrical power caused by piezoresistive effect. The nodal strain is given by

\[ \varepsilon = B U = \nabla \cdot (NU) \]  \hspace{1cm} (4.26)

where \( B \) is the strain-displacement function in the finite element method.

Similarly, Eqs. (4.6) and (4.7) are converted into the following form using the same interpolating function,

\[ LU - GT + \omega MV = 0 \]  \hspace{1cm} (4.27)

where \( L, G, \) and \( M \) are the coefficient matrices, and \( V \) is the nodal velocity. In addition to Eqs. (4.25) and (4.27), one more equation is needed to solve for the three unknown variables \( U, T \) and \( V \). The third equation has already been shown in Eq. (4.24). That is,

\[ V = i\omega U \]  \hspace{1cm} (4.28)

where the method to compute the coefficient matrices \( K, H, F, L, G \) and \( M \) has already been indicated in chapter three.
The vector form of Eq. (4.11) can be expressed as

\[ P_{eh} = (\sigma_{eh} E_e)' \cdot E_e \]  \hspace{1cm} (4.29)

where \( E_e \) is obtained from Eq. (4.12) and

\[ E_e = \begin{bmatrix} E_{e11} \\ E_{e22} \end{bmatrix} \]  \hspace{1cm} (4.30)

Substituting Eqs. (4.21) and (4.30) into Eq. (4.29) yields

\[ P_{eh} = Y\sigma = -1/\rho_{dc} \cdot \begin{bmatrix} \pi_{11} E_{es1}^2 + \pi_{12} E_{es2}^2 \\ \pi_{12} E_{es1}^2 + \pi_{11} E_{es2}^2 \\ 2\pi_{44} E_{es1} E_{es2} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} \]  \hspace{1cm} (4.31)

Substituting Eqs. (4.7) and (4.26) into Eq. (4.31) leads to

\[ P_{eh} = Y\sigma = YCE = YCBU \]  \hspace{1cm} (4.32)

In an elemental domain,

\[ P_{eh} = SU \]  \hspace{1cm} (4.33)

where

\[ S = \iint N^T CYB dx dy \]  \hspace{1cm} (4.34)

4.2.6. Eigenvalue Equations

Combining Eqs. (4.25), (4.27), (4.28) and (4.33) yields,

\[ \begin{bmatrix} -K & S & 0 \\ G & -L & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} T \\ U \\ V \end{bmatrix} = i\omega \begin{bmatrix} H & F & 0 \\ 0 & 0 & M \\ 0 & I & 0 \end{bmatrix} \begin{bmatrix} T \\ U \\ V \end{bmatrix} \]  \hspace{1cm} (4.35)

where \( I \) is an identity matrix. This is a standard eigenvalue equation. The eigenvalue of the equation is the angular frequency \( \omega \) (i.e. the eigenfrequency) whose corresponding eigenmode is denoted by the eigenvector \( U, V, \) and \( T \). When the electrical field is absent, \( S \) equals zero and Eq. (4.35) degenerates to the situation involving TED alone as Eq.(3.43) which is discussed in chapter three. By formulating the velocity field to be
independent of the displacement, the originally quadratic Eq. (4.6) is reduced to two first-order equations, i.e. Eqs. (4.27) and (4.28)

In the above eigenvalue formulation, the same as Eq. (3.6), the quality factor is defined as,

$$ Q = \frac{1}{2} \frac{|R e(\omega)|}{|l m(\omega)|} $$

where Re(ω) is the real part of ω giving the angular frequency in the presence of both thermoelasticity and piezoresistivity; Im(ω) is the imaginary part of ω representing the attenuation of vibration. The finite element algorithm is developed in the form of a customized MATLAB code.

4.3. Results and Discussions

4.3.1. Power Change by Piezoresistive Effect

As discussed in the former section, the TED can be compensated by the reduced temperature difference due to the change in the electrical power. Such power change can be computed by Eq. (4.33) as a function of the displacement. A schematic of the electrical connection for providing the electrical field is shown in Fig. 4.1. To illustrate the temperature distribution by the piezoresistive effect, the power change in the silicon beam is computed when the driving force 'F' is assumed constant. The result is shown in Fig. 4.2. The micro beam in the numerical analysis is assumed as single crystal silicon with length 200µm, width 10µm and thickness 20µm. The voltage $V_e$ applied across the silicon beam is 2V. The center of the beam is exerted by a concentrated force $F=2\times10^{-10}N$, and both ends of the beam are clamped. The properties of the single crystal silicon after N-type doping (with a negative piezoresistive coefficient) are listed in Table 4.1. The analysis is performed in the two-dimensional...
plane strain condition. To minimize the computational effort, the model is simplified into a half beam by taking advantage of its geometric symmetry, as shown in Fig. 4.2b.

![Figure 4.1 Schematic of the beam resonator under a central driving force with a static electrical field.](image)

Table 4.1 Material properties of single crystal silicon after N-type doping.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus ($Pa$)</td>
<td>$1.57 \times 10^{11}$</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.22</td>
</tr>
<tr>
<td>Thermal expansion coefficient ($1/K$)</td>
<td>$2.6 \times 10^{-6}$</td>
</tr>
<tr>
<td>Thermal conductivity ($W/m K$)</td>
<td>90</td>
</tr>
<tr>
<td>Specific heat ($J/Kg K$)</td>
<td>700</td>
</tr>
<tr>
<td>Density ($kg/m^3$)</td>
<td>2,330</td>
</tr>
<tr>
<td>Temperature ($K$)</td>
<td>300</td>
</tr>
<tr>
<td>Piezoresistivity, $\pi_{11}/\pi_{12}/\pi_{44}$ ($1/\text{Pa}$)</td>
<td>$-102.2/53.4/-13.6 \times 10^{11}$</td>
</tr>
<tr>
<td>Static electrical resistivity ($\Omega m$)</td>
<td>$1 \times 10^{-4}$</td>
</tr>
</tbody>
</table>
The nodal strength of the electrical field was computed from either the finite element analysis for a complex geometry, or just the equations $E_{e11}=V_e/L$ and $E_{e22}=0$ for a straight beam with uniform cross-section. It was followed by a stress-strain analysis for the nodal values of the displacements, stresses and strains. The result was used to find the electrical power change from Eq. (4.33) directly or a combination of Eqs. (4.18), (4.20) and (4.29) sequentially. It turns out that both the direct and the sequential methods lead to the same result, which can be considered as a validation of Eq. (4.33) despite the fact that the piezoresistive effect is anisotropic. The change in the

Figure 4.2 The distribution of the nodal power change (W/m$^2$) under a constant central driving force of (a) clamped-clamped beam resonator and (b) its simplified half model.
Figure 4.3 Results of (a) the transverse displacement and (b) power change of the upright corner node (shown in Fig. 4.2b) in the convergence test.
nodal power is shown in Fig. 4.2 based on the 9-node quadratic quadrilateral elements with 51 nodes along the length and 13 nodes along the width. It has been found that the distribution of power change leads to a reversed temperature gradient in comparison with that shown in Fig. 1.7.

Additionally the relationship between the results and the mesh density was studied by employing the 4-node linear quadrilateral (Q4) elements and 9-node quadratic quadrilateral (Q9) elements with various node numbers. The results of the transversal displacement and the power change at the node where the force is exerted are shown in Fig. 4.3. The horizontal axis of Fig. 4.3 represents the number of nodes along the length of the half beam, while the number of nodes along the width is 5, 9 and 13. The result shows that the Q9 elements converge faster than the Q4 elements even when a coarse mesh is used with only 5 nodes (i.e. two Q9 elements) along the width and 21 nodes (i.e. ten Q9 elements) along the length. In addition, Fig. 4.3 shows that the results using 9 or 13 nodes along the width are quite close to each other, for both Q4 and Q9 element types.

4.3.2. Convergence Test

The convergence studies on the quality factor along with Re(ω) and Im(ω) were also performed by using both Q4 and Q9 elements. When the electrical field is absent (i.e. \( V_e = 0 \)), the system degenerates to a standard TED problem and a number of analytical methods exist in the literature. In order to compare the numerical results of TED with the analytical results reported in Ref. [55], we set Poisson’s ratio to zero. The geometry and material properties of the silicon beam remain the same as those used in the previous section. The analysis was performed under the following two conditions:
(a) Quality factor $10^4$

(b) Re($\omega$) $10^7$
Figure 4.4 Convergence test of (a) the quality factor, (b) the Re(ω) and (c) the Im(ω) using Q4 and Q9 elements with different mesh densities when $V_0=0V$. 
Figure 4.5 Convergence test of (a) the quality factor, (b) the Re$(\omega)$ and (c) the Im$(\omega)$ using Q4 and Q9 elements with different mesh densities when $V_0=2V$. 
the applied voltage, \( V_e \), was set to (i) 0V (standard TED) and (ii) 2V through the beam (i.e. 1V through the half beam).

The analytical solution [55] of the quality factor for the standard TED problem is 14,646.7 while the numerical analysis yields 14721.1 by the Q9 elements with a 13×101 finite element mesh (13 nodes along the width and 101 nodes along the length). The error in the numerical result is approximately 0.508% compared to the analytical solution. The results of the quality factor along with Re(\( \omega \)) and Im(\( \omega \)) are also shown in Fig. 4.4 and Fig. 4.5 when the applied voltage is set to 0V and 2V, respectively. The convergence trends are similar to each other not matter with (2V) or without piezoresistive effect (0V) included in the numerical analysis. In addition, similar to the convergence test of the power change discussed previously, the Q9 elements show a faster speed of convergence than the Q4 elements. The results using 9 nodes and 13 nodes along the width are noticeably close to each other when Q9 elements are used. To balance the effort of computation and accuracy, the results based on Q9 elements with

<table>
<thead>
<tr>
<th>( V_0 )</th>
<th>Result</th>
<th>Q9 Mesh (W-nodes×L-nodes)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q )</td>
<td>14,723.65</td>
<td>14,721.06</td>
<td>0.0176</td>
</tr>
<tr>
<td>Im(( \omega ))</td>
<td>444.156</td>
<td>444.215</td>
<td>-0.0133</td>
</tr>
<tr>
<td>Re(( \omega ))</td>
<td>13,079,200</td>
<td>13,078,640</td>
<td>0.00428</td>
</tr>
</tbody>
</table>

Table 4.2 Results from the convergence studies
two different mesh densities have been compared in Table 4.2. The numerical error shows that the accuracy based on the 9×41 mesh is more favorable in the current study. The analyses involved in the following sections are therefore based on this mesh density alone.

4.3.3. Parametric Studies
4.3.3.1 Strength of electrostatic field

The quality factors of the beam resonator under different strengths of the electrical field, \( E_{es11} \) (\( E_{es22} \) is always zero in present study), are obtained and shown in Fig. 4.6a. The geometric parameters and material properties remain the same. A sharp peak is seen in Fig. 4.6a when the electrical field has a strength of \( 2.3 \times 10^4 \) V/m. To explain the reason for this peak, the angular frequency of vibration, \( \text{Re}(\omega) \), accompanied by the attenuation, \( \text{Im}(\omega) \), are also presented in Fig. 4.6b. It shows that the peak of the quality factor in Fig. 4.6a is caused by the value of \( \text{Im}(\omega) \) which is approaching zero. In addition, \( \text{Im}(\omega) \) decreases with the strength of the electrical field applied on the system. This trend confirms that the electrical field can indeed reduce the energy loss caused by TED.

The regression analysis shows that there exists a strong correlation between \( \text{Im}(\omega) \) and the electrical field since the coefficient of determination is equal to 1. Hence, an interpolating function can be obtained from curve fitting based on a limited number of data points. Furthermore, the fitted function shows that \( \text{Im}(\omega) \) equals zero when the electrical field has a strength of \( 2.2972 \times 10^4 \) V/m. According to Eq. (4.36), in that situation the quality factor approaches infinity and TED is completely compensated by the piezoresistive effect. In the current work we define the strength of the electrical
Figure 4.6 (a) Quality factor and (b) the Re(\(\omega\)) and Im(\(\omega\)) as functions of the strength of the electrical field.
field at zero $\text{Im}(\omega)$ to be the critical value of the electrical field (CEF). The applied voltage is denoted as $V_{CEF}$. The values of CEF and $V_{CEF}$ are dependent on the beam geometry and material properties.

When the strength of the electrical filed is greater than CEF, $\text{Im}(\omega)$ will have a negative value. It implies that the change in the electrical power acts as a driving force for the kinetic energy in vibration. By introducing piezoresistivity, the energy loss due to TED can be compensated, leading to a higher quality factor in resonance. Theoretically, the amplitude of vibration would increase indefinitely, leading to material failure if the system had a sufficiently high input energy. However, in reality the increased vibration amplitude with sufficiently high quality factor will trigger the nonlinear dynamic status of the system [87,88]. The change in the angular frequency $\text{Re}(\omega)$ as a function of the strength of the electrical field is also shown in Fig. 4.6b. Contrary to the result of $\text{Im}(\omega)$, $\text{Re}(\omega)$ rises as the electrical field increases its strength. However, the change in $\text{Re}(\omega)$ is merely 0.001% given the present range of the electrical field, which is negligible compared to the change in $\text{Im}(\omega)$.

4.3.3.2 Material Properties

The parametric study in the following sections is focused on the value of CEF when TED is entirely compensated by the piezoresistive effect. The CEF is estimated from the curving fitting on the results of $\text{Im}(\omega)$ under different electrical field strengths. Figure 4.7a indicates the change in the quality factor as a function of the electrical field strength through the beam for different values of Poisson’s ratio. The estimated CEF from curve fitting is then shown in Fig. 4.7b as a function of Poisson’s ratio. The change is almost linear and the rate is approximately $1.14 \times 10^3 \text{V/m}$. On the other hand,
Figure 4.7 (a) Im(ω) as a function of the strength of the electrical field and (b) the value of CEF as a function of Poisson's ratio.
Figure 4.8 (a) $\text{Im}(\omega)$ as a function of the strength of the electrical field and (b) the value of CEF as a function of Young's modulus.
Fig. 4.8 shows the effect of Young’s modulus when Poisson's ratio is set to 0.22 as denoted in Tab. 4.1. Quite different from the effect of Poisson’s ratio, when Young’s modulus varies, the values of CEF do not differ significantly as seen in Fig. 4.8a. However, a more extensive investigation (shown in Fig. 4.8b) has revealed that the change rate of the CEF remains as a linear function of Young’s modulus. Nevertheless, such a change rate is only 0.5 V/m per GPa and apparently negligible.

**4.3.3.3 Beam Aspect Ratio**

The studies on the geometric effects are shown in Figs. 4.9, 4.10 and 4.11. Figure 4.9a shows the quality factor as a function of the electrical field with different beam lengths while the width, W, is maintained as 10μm. Figure 4.9b shows the CEF decreases nonlinearly with the beam length. However, as the length increases by 167% from 120μm to 320μm, the decrease in the CEF is merely 4.08%. The applied voltage of the corresponding CEF can also be found in Fig. 4.9b. Interestingly $V_{CEF}$ increases linearly with the beam length and the rate of the change is approximately 0.022V/μm.

On the other hand, Fig. 4.10a shows the quality factor as a function of the electrical field strength with different widths while the length is maintained as 200μm. In Fig. 4.10b, the CEF is shown as a nonlinear function of the beam width. However, the effect of the beam width on CEF is more significant than the beam length. The CEF decreases by 60.5% when the width increases by 166.7% from 6μm to 16μm. In addition, the applied voltage changes in the same trend since the length is maintained constant.
Figure 4.9 (a) Im(ω) as a function of the strength of the electrical field and (b) the value of CEF as a function of the beam length.
Figure 4.10 (a) \(\text{Im}(\omega)\) as a function of the strength of the electrical field and (b) the value of CEF as a function of the beam width.
To further study the change in CEF as a function of the beam aspect ratio, $A_r = L/W$, the results in Figs. 4.9b and 4.10b are reordered and presented in Fig. 4.11a. Apparently the CEF changes in opposite directions as the aspect ratio is altered by the length and the width, respectively: The CEF declines nonlinearly with the aspect ratio when the length is changed, meanwhile it increases linearly with the beam aspect ratio when the width is changed and the corresponding rate is approximately 1,100V/m.

Since TED is compensated by the kinetic energy transferred from the input electrical power, there must exist a correlation between the input electrical power, $P$, and the aspect ratio, $A_r$. We define the cross-sectional area of the beam as $A$, and $A = W\delta$ where $\delta$ is the beam thickness. Substituting Eq. (4.14) into Eq. (4.2), we obtain the input electrical power $P_{CEF}$ as:
\[ P_{CEF} = \frac{V_{CEF}^2}{R_0} = \frac{V_{CEF}^2 A}{\rho_{es} L} = \frac{V_{CEF}^2 \delta}{\rho_{es} L} = \frac{V_{CEF}^2 \delta}{\rho_{es} A_T} \]  

(4.37)

The result is presented in Fig. 4.11b and apparently \( P_{CEF} \) is a linear function of the beam aspect ratio alone. It indicates that more electrical input power is required to compensate TED when the beam resonator has a greater aspect ratio and the corresponding rate is approximately 0.009W per unit aspect ratio of the beam when the thickness is set to \( \delta = 20 \mu m \). In addition, for a beam resonator with the same material property and thickness, \( V_{CEF} \) is a also linear function of the beam aspect ratio alone, as seen in both Eq. (4.37) and Fig. 4.11b. In fact, the slope of the linear regression of the data in Fig. 4.11c indicates that the corresponding rate is approximately 0.22V per unit beam aspect ratio.

4.3.3.4 Scaling effect

We also investigated the CEF on different size scales toward miniaturization of the beam resonators. Figure 4.12a shows the result of CEF when the scaling factor of the resonator beam 200 μm×10 μm changes progressively from \( \times10,000 \) (length 2 m, width 0.1 m) to \( \times1 \times 10^{-5} \) (length 2 nm, width 0.1 nm) in a log-log diagram. However, the applied voltage for the corresponding CEF has a different trend as shown in Fig. 4.12b since the length here changes differently from the scaling factor. Two distinct plateaus can be seen in Fig. 4.12b. The first plateau shows the voltage when the scaling factor is higher than \( \times100 \) (length 20mm, width 1mm) while the second plateau emerges when the scaling factor is lower than \( \times1 \) (length 200μm, width 10μm). Moreover, the change in the corresponding voltage to the CEF from the higher plateau to the lower plateau is only about 10% from 5.12V to 4.60V, meanwhile the size changes dramatically from the meter scale to the nanometer scale.
Figure 4.12 (a) CEF as a function of the scaling factor of the beam geometry in a log-log plot and (b) the applied voltage, $V_{CEF}$ to the corresponding CEF as a function of the scaling factor of the beam geometry.
4.4 Conclusion

The current work presents a method to enhance the quality factor of MEMS beam resonators vibrating in their flexural modes by applying an electrostatic field on the system with a negative piezoresistive coefficient. During the vibration, the temperature distribution caused by the electrical power change can attenuate the thermoelastic temperature difference that leads to the conductive heat flow for TED. To predict the thermomechanical responses of the system in the presence of piezoresistivity, a set of coupled differential equations for the relevant thermal, mechanical and electrical processes are constructed. These equations are then solved by the Galerkin finite element method. The quality factor is thus evaluated by an eigenvalue analysis.

A series of convergence studies have been performed using both linear and nonlinear interpolating functions in the finite element method for achieving better accuracy with less computational effort. The quality factor is computed as a function of the strength of the electrical field. The result demonstrates that the quality factor increases with the electrical field strength. Further analysis reveals that an infinite quality factor is possible when the attenuation of vibration, Im(ω), equals zero, i.e. TED is completely suppressed. The study also shows that the relationship between the attenuation in the energy loss and the electrical field strength can be presented by a mathematical function from curve fitting. Such a function is also implemented to estimate the value of CEF.

In addition, other parametric studies have indicated the impacts of a variety of material properties and geometric parameters on the CEF and $V_{CEF}$. For example, the
CEF increases linearly with Poisson’s ratio whereas Young’s modulus has a negligible effect. It has also been found that $P_{CEF}$ and $V_{CEF}$ are functions of the beam aspect ratio alone. On the other hand, the CEF changes linearly with the scaling factor of the miniature model in a log-log diagram; meanwhile there exist two distinct plateaus of CEF in the plot as a function of $V_{CEF}$. 
CHAPTER FIVE: CONCLUSIONS AND FUTURE WORK

5.1 Conclusions

In this dissertation, the gas recognition and pressure sensing capability of MEMS resonant structures operating as both TPRs/TPOs were investigated by theoretical analysis and experimental measurements. It showed that the TPRs are sensitive to the thermal conductivity of surrounding gas while the frequency shift of the TPOs is mainly affected by the damping altered by the change in the density of ambient gas. The sensitivity trends of the TPO and TPR derived from mathematic models were also confirmed by the measurements for designing more sensitive TPR/TPO gas and pressure sensors. The much higher frequency shift was also observed for the TPOs compared to the TPRs resulting from the same change in the helium concentration and the gas pressure. New TPOs suitable for operating under high pressure range were also successfully designed and tested. Except the higher sensitivity of the TPOs than the TPRs showed by the measurements, the MEMS devices operating in their TPOs mode also have a simpler working mechanism as their vibration is only driven by a DC current. The TPRs/TPOs with robust monolithic crystalline silicon structures can be exposed to the environment directly. Batch-fabrication of such devices using simple and well-established MEMS fabrication techniques can decrease the cost significantly.
In real application, the TPRs and TPOs have the potential as low cost gas thermal conductivity and density/pressure sensors, respectively.

The rest of the dissertation is focused on two different methods to increase the quality factor of MEMS beam resonator by decreasing the thermoelastic damping (TED). The first method is adding vents to disrupt the heat current caused by the thermoelastic temperature in the case of vibration. Such method for both clamped-clamped (CC) and clamped-free (CF) beam resonators is investigated by a customized finite element method that is implemented on the MATLAB platform. The TED limited quality factor and resonant frequency are obtained as functions of various geometric parameters including the location, number and size of the vents. For vented CC beams, it has been found that the clamped end is the best place for vents to decrease the TED than other places. It also been found that quality factor can be increased by larger vent but the effect of vents' width is more significant than their length. In present numerical analysis, the highest quality factor that is 3801 times with respect to that of solid beam is realized. On the other hand, the vents located in the clamped end decrease the dynamic stiffness of the structure and then lower its frequency significantly. In addition, the effect of the vents in CF beam resonator (cantilever) studied by the finite element analysis shows a similar conclusion to the clamped-clamped beam but with little difference. Since both the change in the quality factor and the frequency are all important performance factor for beam resonator, the numerical result of the vented beam in present analysis provides a useful reference for engineering design.
The second method to enhance the quality factor of MEMS beam resonators vibrating in their flexural modes is applying a static electrical field on it and using the piezoresistive effect. A set of coupled differential equations for the relevant thermal, mechanical and electrical processes are constructed and the corresponding TED limited quality factor are solved by an eigenvalue finite element method. The convergence test is performed to choose a proper mesh for achieving better accuracy and less computational effort. The method to decrease the TED is validated by the numerical result that the quality factor is enhanced when the strength of electrical field through the beam is increased. In addition, the TED can be completely compensated by the piezoresistive effect when the strength of electrical field reaches the value of CEF. The corresponding CEF is obtained when Im(ω) equals zeros calculated from curve fitting function. In addition, other parametric studies have indicated the impacts of material properties and geometric parameters on the CEF. It shows that the value of CEF has opposite trend, though the corresponding voltage and electrical power are solo correlated to the beam aspect ratio no matter altered by the length or width. Moreover, it is observed that the CEF changes linearly with the scaling factor of the miniature model in a log-log diagram.

5.2 Publications
5.2.1 Journal Publications


5.2.2 Conference Publications


5.3 Future Work

5.3.1. The Sensing Application of TPR/TPO

As indicated in chapter two, the design principle for more sensitive TPR/TPO sensor derived from the theory analysis has been proved by the measurement. In future
work, more sensitive TPR/TPO sensors based on these design rules should be designed, fabricated and tested. On the other hand, it is also important to find the best workable range of the TPO because its nonlinear frequency shifts observed in the experiments. Another issue limited the real application of the TPO is the fabrication technique. Successive oxidation and HF release leads to the difference between different TPO batches. Therefore, some other fabrication techniques will be explored, such as electron beam/track lithography.

5.3.2. Geometry Effect on TED

Since using vents to decrease the TED has been successfully studied by the numerical method, but its corresponding experimental work is still needed to perform in the future. However, it should be kept in mind that some optimal geometry could be limited by the fabrication technique. What's more, the vents in the beam will also change the support loss what is another important factor to the total quality factor, thought the gas damping can be eliminated by testing the device in vacuum. Therefore, the change in the support loss of the beam resonator with vents will be investigated by numerical method at first in the future and then the numerical result can be used to guide the corresponding experiments.

5.3.3. Suppress TED by Piezoresistive Effect

The same as the problem discussed above, the corresponding experimental work should be implemented in the future. However, the support loss will not be a problem since the geometry of the beam resonator is fixed. Furthermore, the method to increase the quality factor by using piezoresistive effect could be used for other silicon resonators such as the bulk acoustic resonator, ring resonator and disk resonator. The
numerical study for these resonators with different geometry will be studied by numerical method at first. The numerical result should then be used to design the experiments.

5.3.4. Nanoscale Effect on TED

We propose to extend the current thermoelastic damping theories to the nanoscale regime. The nanoscale effects will be incorporated into the computation of thermal conductivity required in the damping modeling. For silicon, the mean free path of phonons at room temperature is about 43 nm. The thermal conductivities in both thickness and lateral directions can be estimated by an approximation formula provided by Flik et al. [89] when the beam thickness is below this threshold, but no general guidelines exist for predicting values of thermal conductivity and therefore a molecular dynamics simulation will probably be needed for prediction.

On the other hand, Khisaeva et al. [90] studied such effect by applying one relaxation time (ORT) theory [91] which is defined as

\[ q + t_0 \dot{q} = -k \nabla T \]  \hspace{1cm} (5.1)

where \( q \) is heat flux; \( t_0 \) is the relaxation time; \( k \) is thermal conductivity and \( T \) is temperature. For classical method, \( t_0 \) equals zero. Their results showed multiple thermoelastic damping peaks as indicated by Fig. 5.1. However, a simple Bernoulli beam was employed in their model and it is unclear if this effect has appreciable effect on the damping capacity of plates and rings, and for longitudinal and torsional motions as well. Therefore, we propose to make a more extensive investigation using both analytical and numerical approaches, and answer the following three questions: (i) What is the effect of dimensionality on the finite wave propagation and further
thermoelastic energy dissipation? (ii) What is the role of finite wave speed of thermal disturbance in the longitudinal and torsional oscillations of nanoresonators? (iii) How will the finite wave speed interact with fluid damping? To answer these questions, a perturbation method as opposed to transient scheme will be used to derive the exponential rate of the perturbed field.

Fig. 5.1 A comparison of a classical solution for thermoelastic damping with one relaxation time solution for different $\gamma$ [90], where $\gamma$, $\Omega$ and $\psi/\psi_0$ are the normalized time relaxation time, frequency and thermoelastic damping, respectively.
References


