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Effectiveness of the MIND Research Institute's Algebra Readiness Curriculum on Student Achievement Implemented in a Tiered Response to Intervention Model

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Effectiveness of the MIND Research Institute’s Algebra Readiness Curriculum on Student Achievement Implemented in a Tiered Response to Intervention Model

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A Dissertation
Presented to
the Faculty of the Morgridge College of Education
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In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

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by
Nancy Quick
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Advisor: Dr. Karen Riley
Abstract

Algebra is typically the gatekeeper for higher-level math coursework. Low math performance on standardized assessments impedes access to these higher-level math classes. Limited math progress in high school affects future career opportunities and quality of life. High school students who have historically struggled with math need interventions that work. Research indicates that the research on reading interventions far outnumbers the research on math interventions. With the passage of new federal legislation requiring schools to document progress for all groups of students and allowing schools to use Response to Intervention models, the need for evidence based math interventions becomes even more pressing. This research examined the effects of the MIND Research Institute’s Algebra Readiness program in order to provide an evidence base for this intervention. In this pre-experimental study, two groups of high school students were exposed to the MIND Research Institute’s Algebra Readiness program and changes in their math performance were analyzed. Students with and without diagnosed disabilities, were assigned to two tiers of intervention. The double dosed students received instruction with the core Algebra I curriculum and the Algebra Readiness program whereas the single dosed students only received the Algebra Readiness intervention. Independent samples t-tests; ANCOVA, percentage change, qualitative teacher responses and correlation procedures were used to analyze the scores of students.
The outcome measures included scores on a post test, grade level curriculum based measures and the state sponsored standardized assessment. Because of limited sample size and possible lack of sensitivity of the outcome measures, overall no significant growth was detected. The students who were double dosed showed significant growth on the end of year, grade level assessment as compared to those students who only participated in the intervention. Although the sample size was small, the effect sizes were large, indicating potential practical significance for this intervention. This research suggests that using visual spatial representations in a mostly nonverbal, computer-based format may show promise as an intervention for students. More research with a larger sample size and a more controlled research design will help definitively answer the questions about the effectiveness of the Algebra Readiness program.
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Chapter I: Introduction

Educational policy and practice in the United States have changed substantially since 2001. With the passage of The No Child Left Behind Act (NCLB) in 2001 mandating 100% proficiency for all students in reading and math by 2014 and the revisions in the 2004 reauthorization of the Individuals with Disabilities Education Act (IDEA) (PL 108-446), schools are working to become more flexible in identifying and responding to student needs. Two of the most significant changes stemming from this legislation, the promotion of Response to Intervention (RTI) models for student-based problem solving and the requirement to monitor the progress of all students using grade level standards, have transformed educational practices. VanderHeyden (2011) calls this no less than system reform. Developing RTI structures to allow struggling students systematic and early access to research-based interventions could help more students regain and maintain progress at grade level standards. Students would no longer need to be identified with a learning disability or fail in order to receive more intensive instruction. To adapt to the requirements in these laws, schools must organize tools, time, and people to more flexibility to meet students’ needs.

Federal Laws

The reauthorization of IDEA (2004) included language requiring Response to Intervention (RTI) models in student based problem-solving and disability diagnosis:
In determining whether a child has a specific learning disability, a local educational agency may use a process that determines if the child responds to scientific, research-based intervention as a part of the evaluation procedures. (IDEIA 2004, Sec. 614.b.6.B)

Fletcher (2006) summarizes research and findings about the development of RTI. Two ideas discussed were that quality instruction must be addressed as a factor in student progress (Bouck & Flanagan, 2010; Maccini & Gagnon, 2000), and early intervention for struggling students could be effective in closing achievement gaps and preventing formal learning disabilities (Bryant, Bryant, Gersten, & Scammacca, 2008; Chaim, Lappan, & Houang, 1988; Clements & Sarama, 2007). RTI methods require schools to systematically screen all students, monitor progress, and use research-based interventions in conjunction with the ensuing data to make informed decisions about students’ academic programming. RTI models provide a tiered approach to interventions that increase or decrease in intensity and frequency based on student progress. School teams monitor how a student responds to core instruction in the general classroom (tier one) before moving into a research-based strategic intervention (tier two) thus saving the most intensive interventions and often an accompanying learning disability diagnosis for tier three. RTI models are embedded with tools to systematically screen and monitor the progress of all students. This allows schools to use data to identify students at high risk for failure and to design appropriate interventions before the student falls significantly behind in grade level standards. RTI structures allow for early identification and help define appropriate research-based intervention materials to remediate deficits and prevent further failure (Clements & Sarama). Fletcher (2006) calls students who continue to fall
farther behind because of inadequate instruction and remediation, “instructional casualties.” Providing targeted instruction quickly to close skill gaps is more effective for more students.

**Response to Intervention.** Although RTI was meant to be preventative in addressing underlying skill gaps with early prescriptive interventions that could prevent a widening achievement gap, it also serves another purpose (Ehren, 2011). RTI offers an alternative to the previously used “discrepancy model” for educational disability diagnoses. The discrepancy model compared an individual’s IQ score to an achievement score in order to determine if a significant gap was present. This gap, typically a grade level difference of 1.5–2.0 years (Baca, Wexler-Love, & Saenz, 2008) is needed to diagnose a learning disability or provide special education services, was present. In this model, many students would have to wait until the gap was severe enough to warrant a systematic intervention. This diagnosis of a severe discrepancy generally did not happen until fifth grade for many students (Fuchs & Fuchs, 2007). By working with the student in a timely manner, not only could it possibly prevent a learning disability diagnosis by addressing the specific skill deficit with more intensive services (Clements & Sarama, 2007), but the body of evidence from the student’s response to the interventions could also be used to identify students with a learning disability, if, when given this increased support, he or she continued to demonstrate inadequate growth on grade level progress monitoring data. Not only would the RTI data and interventions provide a means to possibly prevent a growing achievement gap, it would provide a basis for identification of
a learning disability, and it could identify areas of sustained weaknesses to focus on for remediation (Clements & Sarama; Ehren 2011; Fletcher, 2006).

**No Child Left Behind.** In addition to the structural changes necessary to implement RTI models, NCLB (2001) requires individual student achievement to be measured by a state standards-based test. Ninety-eight percent of students with known disabilities must be included in the state assessment data and on individual school accountability reports in order for states to make adequate yearly progress (AYP). NCLB (2001) requires states to grade schools on student progress and impose school sanctions if students fail to show adequate yearly progress (AYP). Student progress is measured on state assessments and scores are disaggregated to determine growth patterns and adequate yearly progress for each subgroup assessed. The goal is to help all students achieve proficiency on state standard-based tests regardless of disability status (NCLB, 2001).

Adding to this new paradigm, NCLB also requires all teachers (including Special Education teachers) to become highly qualified in the subject areas they teach so all students have access to rigorous foundational core instruction. Teachers must “demonstrate subject-matter competence for each NCLB core academic subject they teach” (NCLB, 2001). In order for students to access highly qualified teachers to receive academic “double doses,” both school structures and instructional delivery must change. A double dose is defined as a scheduled academic core content class coupled with an additional academic core intervention class using a research-based method (Wilson Language Training Corporation, 2008). The scheduling structure in schools must now
include this intervention time. Instructional delivery, how the content is taught, must be flexible to meet the needs of students and include systematic checks for progress. These progress checks are used to determine if teachers need to realign or reteach the content based on student mastery of concepts. Accommodating these changes entails significant restructuring in schools to develop tiered interventions and to develop systems to provide timely and fluid access to them (Fuchs & Fuchs, 2007). Duffy (2008), identifies some of the challenges of implementing RTI in high schools, including access to high school (age) appropriate interventions and making sure they are exposed to grade level content as much as possible since they will be assessed on the grade level standard.

**Research-based Curricula.** The research community has responded by developing specific curricula and standards-based progress monitoring tools. The U.S. Department of Education’s What Works Clearing House, lists 35 interventions with some research base for math. This compares to the more than 70 evidence-based reading and writing interventions. Thus, reading and writing interventions outnumber the math and science interventions more than 2:1 (NCES, 2012). Since so much of this research has focused on reading development, the research in math development and effective interventions has lagged (Geary, Hamson, & Hoard, 2000; Mazzocco, 2001).

This study aimed to explore and to evaluate the effectiveness of a computer-based visual-temporal math intervention, the MIND Research Institute’s Algebra Readiness program. Using RTI methods, the intervention was delivered in two tiers of intensity to high school students who have demonstrated weakness in math achievement. The
Colorado State Assessment Program (CSAP) scores and a systematic assessment tool, McGraw Hill’s Acuity assessments, as well as an Algebra Readiness content post-test were used as measures of individual progress.

**Statement of the Problem**

Over the past two decades, educational research has been focused on analyzing the underlying skills and developing systematic interventions critical for learning to read, however comparatively little research has explored the cognitive skills and interventions needed to learn math (Kulak, 1993, Swanson & Jerman, 2006). As a result, teachers have a multitude of reading interventions linked to specific reading deficits, whereas research in and for math interventions is lagging (Gersten, Chard, Jayanthi, Baker, Morphy, & Flojo, 2009; Gersten, Clarke, & Mazzocco, 2007). Since students identified with reading disabilities often have overlapping problems learning math (Kulak, 1993, Geary & Hoard, 2005), researchers are using this foundation of reading research to identify similarities and differences in diagnosing and intervening with math disabilities (Bryant, Bryant & Hamill, 2000; Geary, Hamson & Hoard, 2000; Kulak, 1993; Swanson & Jerman, 2006). Many students not identified with a formal learning disability continue to demonstrate poor performance in math (NCES, 2011). In addition to these students, an estimated 5-8% of students who exhibit some form of math disability (Geary & Hoard, 2007) also consistently struggle with math (NAEP, 2011).

Since the 2004 reauthorization of IDEA encourages schools to use RTI models to quickly address student learning concerns, and NCLB applies sanctions for schools that
do not show adequate yearly progress for all students, schools need access to research-based interventions to help bridge math achievement gaps. Research supports the idea that visual representations are important in the development of math (Bouck & Flanagan 2010; Chaim, Lappan, & Houang, 1988; Geary, Hamson, & Hoard, 2009; Gersten, Chard, Jayanthi, Baker, Morphy, & Flojo, 2009; National Mathematics Advisory Panel, 2008; National Research Council, 2006; Van Garderen, 1999), and that a graduated sequence of instruction helps students move from concrete to more abstract applications of math concepts (Bouck & Flanagan, 2010; Bryant, 2008; Fuchs, Fuchs, & Hollenback, 2007; Maccini & Hughes, 2000; Witzel, 2005). By providing a visually-based math intervention using a graduated sequence of instruction, in two tiers of intensity, this research explored the effects of such programming on math achievement for students who have demonstrated math skill weaknesses.

**Purpose of the Study**

The purpose of this study was to assess the effectiveness of The MIND Research Institute’s Algebra Readiness intervention, a computer-based visual-temporal math program, for high school students who have struggled to understand pre-algebra concepts essential for successful completion of Algebra I. The study explored the impact of this novel visual and mostly nonverbal intervention on student progress when used as a tier two (double dose) intervention or as a tier three (solo) intervention class. It explored the impact of the curriculum on student progress for students diagnosed with learning disabilities and those without a diagnosis. In addition, this study also explored if any
strands within the curriculum were more effective at increasing the math skills of the students.

**Algebra Readiness Program Design**

The Algebra Readiness curriculum was developed by the MIND Research Institute. This program uses computer games to reinforce the concepts necessary to solve problems algebraically. The Algebra Readiness program uses a spatial temporal approach to teach 2nd-7th grade mathematical concepts in one year. The Mind Research Institute defines spatial temporal reasoning as, “the ability (of the) brain to hold visual, mental representations in short-term memory and to evolve them in both space and time, thinking multiple steps ahead” (Mind Research.net, n.d.). The MIND Research Institute’s Algebra Readiness (AR) math intervention aims to develop underlying spatial cognitive skills by providing engaging visual animations and requiring students to manipulate these representations to solve problems. These problems are designed to bolster understanding of mathematical concepts. In this program, students are required to solve visual problems devised to explain concepts such as fractions, proportional reasoning, equations with rational numbers, coordinate planes, and other advanced topics. When the student correctly manipulates visual representations presented on the computer screen, an animated character moves up levels through a series of more challenging content (MIND Research, 2011).

The National Research Council (2006), as part of the Behavioral and Social Sciences and Education Division of the National Academy of Sciences, defines the
unique qualities of spatial thinking as, “a constructive amalgam of three elements: concepts of space, tools of representation, and processes of reasoning” (p. 12). The highly visual Algebra Readiness program incorporates these qualities in its design. The program utilizes very little language and has two instructional delivery components. The first component is similar to other math curricula in that teachers introduce content through whole group instruction, although the textbook is notable for brevity of verbal descriptions and the extensive use of pictures and diagrams. The unique aspect of the program is the second component during which the concepts are reinforced and generalized with additional practice in a nonverbal computer game.

Spatial Temporal Math (ST Math) provides the basis of the computer games used in the Algebra Readiness curriculum and is language free, (Graziano, Petersen, & Shaw, 1999). This language free approach uses no numbers, symbols or words. The approach is different from traditional methods that teach ratios and proportional reasoning using language-analytic methods. Students learn and practice multistep problem solving with increasing complexity (MIND Research Institute, n.d.) by virtually manipulating visual representations of the problem. When the student correctly manipulates (solves) the visual problem, it creates a pathway for the animated character, Jiji, to cross the screen and move on to more complex problems. The program visually demonstrates why an answer is correct or incorrect to help the student understand the concept. The self-paced computer program allows for differentiation among students and groups of students and
supports teaching to specific skills gaps. It also tracks student progress since students work through the games at their own pace (MIND Research Institute, 2012).

The National Research Council Education Division (2006) identifies spatial representations as powerful learning tools because:

First, creating spatial representations is a powerful way to encode new information that one wishes to recall at a later time. Second, generating images of “old” information that has already been learned and of the situations in which it was learned can powerfully aid in recalling the information at later times. Third, some problems are more readily solved using spatial representations, whereas in other cases, trying to use spatial representations can interfere with problem solving. (p. 281)

By providing multiple visual examples and limiting unnecessary language by using animated representations in a computer game format, the Algebra Readiness program is designed to facilitate students’ understanding of these math concepts more readily. The animated visual representations of the underlying concepts are designed to reinforce student visual spatial skills.

The MIND Research Institute’s Algebra Readiness program also incorporates the National Mathematics Panel’s findings (2008) in the design of the program. These findings recommend using the critical ideas of developing spatial visualization skills, using visual number lines and focusing on fractions to develop the foundations for algebra in curriculum design. In addition, the de-emphasis on language, which may impact student acquisition of content since some reading and math disabilities appear to share core underlying deficits (Geary & Hoard, 2001; Hale & Fiorello, 2004; Mazzocco, 2005; Murphy, 2007) that could involve language processing, simplifies the presentation
of the material. Gunderson, Ramirez, Bailock and Levine (2012) found that developing spatial skills in young students can improve the development of numerical knowledge in later grades. By using a computer-based game format, which relies less on verbal communication and more on visual representations, the students are exposed to a different way to see and learn the concepts.

The Algebra Readiness program incorporates tasks involving visual number lines when demonstrating fraction concepts. The graduated sequence of instruction is structured so that students practice manipulating these visual representations before learning the more abstract mathematical symbols that explain the concept in mathematical language (MIND Research Institute- Math Initiative, nd).

**Spatial Temporal Math Research Base**

**Phase I.** The initial Spatial Temporal Math research was conducted in two phases - a pilot study and the main study. In the pilot study (Graziano, Petersen, & Shaw, 1999), two stages were implemented to assess the effectiveness of the spatial temporal training. Stage one consisted of training 102 second grade students in Orange County, California on spatial temporal tasks. The students used the ST Math computer games to transform mental images in space and time by “folding and unfolding around multiple axes in the x-y plane, rotations around the z axis, 180 degree flips around the x and y axes and translations in the x-y plane” (p.141). In the second stage the students were asked to apply these spatial temporal skills to math and science problems, again solving these problems visually, without using words or using numerical algorithms. Each game is set
up with an introduction that has five constants 1) there is only one correct answer out of three to four choices, 2) no penalties are applied for incorrect answers, 3) visual feedback is provided to validate the correct answer or show how the reasoning was faulty, 4) students can attempt to answer as many times as necessary to obtain the correct one, and 5) the order of the answers does not change for each attempt on the problem. After the introduction phase, students worked on the “real” game, and their scores were registered and analyzed. Points were awarded for correct responses and taken away for incorrect or slow responses. Levels were reset if a student failed to show mastery of the content on the specific tasks. The 102 students in the pilot study were placed in three groups. Two groups, from a school with below average demographics, were given training in either ST Math or English. Group one had training with the ST math computer program. Group two had English language training (reading, writing, and spelling) on the computer. Group three (control) was given no lessons on the computer and these students were selected from a school with above average achievement scores. An ST Math Video Game Evaluation Program assessment was created that consisted of a one hour training on the ST math games for those students who were not exposed to the ST Math intervention. After participating in the one hour training, all the students were tested on the ST Math Video Game Evaluation program that used 44 nonverbal questions to determine their level of understanding of spatial temporal symmetry, rotations, and folding and unfolding figures. The results of the pilot study showed that the students given the ST Math training scored 36% higher than the control English training group. The ST math group also
scored higher 14% higher than the no lesson group which was formed from a school with an above average demographic school.

**Phase II.** In the main study (Graziano, Petersen, & Shaw, 1999), 170 second grade students participated in a four month study examining math performance of two groups. This study compared the math performance of those students who received piano keyboard training in conjunction with ST computer math games training versus control groups. Five second grade classrooms were assigned one of the following four conditions over four months: 1) students received keyboard training and ST math computer game training, (piano & math group) 2) students received computerized English language training and the ST Math computer game training (English & math group) 3) one half of the students received ST math training or English language training only (English or math group) and 4) two classes received no training in either ST math or English language computer programs (control groups). Again, using the ST Math Video Game Evaluation program, the results of the main study were 15% higher for the group who received the piano training in conjunction with the ST Math computer games when compared to the students who received the ST Math and English training. Even more, the ST Math piano group scored 27% higher than the ST Math and English group on the 16 questions related to fractions and proportional reasoning. Both groups performed 100% better than the control groups from the same school. The results are promising, although they still require confirmation with outside assessments that are not developed by the program originators.
Although the Algebra Readiness program incorporates the ST Math games into the curriculum, currently no piano training is involved in the presentation of the material. For the purposes of this study, piano instruction was also not intended as part of the program. The materials are presented in a specific sequence and the computer games are used to reinforce key concepts important to developing algebra skills.

**Rationale**

In almost any school, it is easy to identify students who have struggled in traditional math courses and select them as the target for math interventions. It is more difficult, however, to find appropriate interventions matched to those individual student’s needs. Murphy (2007), in a search of studies on reading and math disabilities highlights the significance of the gap in the research. Over the same 21 year period, 1077 articles on reading disabilities and 231 articles on math disabilities were found. The end result of this imbalance in research is that not only the 5-8% of students in the United States and in other countries estimated to be affected by a Math Disability (Geary, 1993; Mazzocco, 2005; Murphy, 2007; Swanson, 2006) but those who continue to struggle with math without a formal diagnosis, have less access to evidence-based interventions to help them learn. As a result, many continue to demonstrate poor performance on grade level standards in math (NCES, 2011). By examining the differential impact for groups of students exposed to the MIND Research Institute’s Algebra Readiness intervention, this study will add to the math research so more tailored interventions can be offered to students who struggle making the cognitive shift from concrete representations of
mathematical calculations to understanding abstract algebraic concepts. This is especially important in that many students have received math interventions previously and continue to fail and this has direct implications for high school graduation rates.

**Research Questions**

To evaluate the effectiveness of the MIND Research Institute’s Algebra Readiness intervention for students participating at two tiers of intervention the following questions were explored: 1) What is the impact of the MIND Institute Algebra Readiness intervention on algebra skills for students who have struggled with or failed a pre-algebra or algebra course in their previous year of school? 1a) Does this intervention impact some groups of students more than others? 2) What is impact of the MIND Institute’s Algebra Readiness intervention on math achievement for students who are receiving special education services via an IEP? 2a) Does the RTI tier of intensity have a differential impact on the achievement for these identified students?

Independent samples t-tests were used to compare the scores of students with or without an IEP and those in a double or single dose using Acuity C, and CSAP scores as the test dependent variables. ANCOVA was used with the MDTP pretest as the covariate to test the differences between students in the double dose as compared to the single dose. This ANCOVA procedure was also used to compare the IEP groups—those with IEPs and those without. The MDTP posttest was used as the dependent variable.
Chapter II: Literature Review

Background

Since the implementation of IDEA 2004, which introduced RTI, and the 2001 NCLB Act which required states to show accountability for all student growth, educational practices have changed radically. To understand the impact of incorporating these changing aspects of educational policy into practice, this review outlines the theory base for RTI and the structures associated with schools that implement it. This review also explains ideas about tiers of intervention and curriculum and instructional delivery, as well as the schedules and the sequence of instruction in schools. It reviews the importance of algebra and its role as a gatekeeper in students’ future outcomes. Additionally, it explores the cognitive leap required for students to access algebraic ideas and reasoning and how math interventions can bridge this gap for students with and without disabilities. The review also provides a basis for understanding math programming and interventions for students. Also, math disabilities and their relation to general learning disabilities, including whether or not a diagnosis helps determine programming, are explored. Finally, the review examines cognitive building blocks, especially visual representations that are essential to understanding math concepts.
Response to Intervention (RTI)

History. The 1975 Education for All Handicapped Children Act (Public Law 94-142), the predecessor statue for The Individuals with Disabilities Act (2004), mandated individualized educational programming for students with disabilities. Before Public Law 94-142 was enacted, approximately one in five children with disabilities were educated in public schools (U. S. Office of Special Education Programs, 2007). More egregious, many individual states had enacted laws to specifically exclude children with disabilities in their schools. Many school-aged children lived in institutions or attended special schools with little to no access to appropriate evaluations, services or programs to address their special needs. By guaranteeing a free and appropriate public education this landmark law significantly increased educational opportunities for students with disabilities, although with varying levels of success (National Center for Educational Statistics, 2011). To increase the accountability for the educational progress and eventual outcomes of students with disabilities, Response to Intervention (RTI) was introduced in 2004 when the federal government reauthorized the Individuals with Disabilities Education Act (IDEA), now called the Individuals with Disabilities Education Improvement Act (IDEIA). This iteration of IDEIA included three significant changes embedded in Part B, the Assistance for Education of All Children with Disabilities. 1) It moved away from the discrepancy model as the only means for defining special education eligibility, 2) it mandated progress monitoring for all students and 3) it allowed the use of RTI techniques (tiered interventions) to intervene with and identify students
who require extra resources. Combined with the U.S. Congress’ reauthorization of the 1965 Elementary and Secondary Education Act (ESEA), now known as the No Child Left Behind Act (NCLB), in 2001, these updates stimulated significant educational reforms.

**No Child Left Behind.** No Child Left Behind (NCLB) is a broad law focused on the achievement of all students, not just those with disabilities. It is called the “nation’s general education law”, but it has far-reaching consequences all school programs for students who have historically struggled, including those identified with learning disabilities (National Dissemination Center for Children with Disabilities, 2011). It requires, among other things, an improvement in the academic achievement of the disadvantaged (Title I), a focus on improved teacher preparation, training and recruiting (Title II) and increased accountability for student results, also known as adequate yearly progress (AYP) (Title VI), (NCLB, 2001). Within the NCLB directives, states are required to use research-based programs as the foundation of educational curricula. It also includes the requirement for states to, “set measureable goals for student improvement” (p.268), ensure all students participate in state assessments and demonstrate adequate yearly progress, and hire highly qualified teachers (Rosenberg, Sindelar, & Hardman, 2004).

Specifically, NCLB requires schools to annually assess all students in reading and math using a single statewide accountability system. In Colorado, the statewide assessment is the Colorado Student Assessment Program (CSAP). States are also required
to set targeted achievement goals and are graded on Annual Yearly Progress (AYP) towards those goals. AYP indicators of student progress include student achievement scores as measured by the state assessment, graduation rates, retention rates, and attendance rates among other factors. AYP is used to identify what schools need improvement, corrective action or even restructuring. The statewide assessment scores are then disaggregated for ethnic minorities, students with disabilities and identified students from low-income households. The goal is for all students to achieve grade level proficiency in reading and math by 2014. Students who fail to meet the targeted benchmark level for each grade would then have the option to transfer to another public school and receive outside tutoring. Those schools that continue to show a high number of students who do not meet the required targets face sanctions. These sanctions include reorganizing the faculty and staff at the building level or even closing schools altogether. Using this structure, NCLB emphasizes the accountability for and importance of education for all.

**The Discrepancy Model.** Before 2004, IDEA endorsed a discrepancy model whereby students were required to show a gap between cognitive ability and achievement in order to qualify to receive Special Education services. With the introduction of RTI models in the IDEIA and the added responsibility for continuous, measurable student growth embedded within NCLB, educational focus shifted from ensuring access for students with disabilities a Free and Appropriate Public Education, (FAPE) to ensuring accountability for student progress (Fletcher, 2006; Rosenberg, et al., 2004). Before RTI methods were
widely used in schools, often schools did not build in the allotted time or the structures necessary for systematic access to interventions for students who showed signs of struggling. Students would be referred for a special education evaluation to determine if they would qualify for extra help based on how far behind their peers they scored on an individualized academic skills assessment. This significant achievement/ability gap allowed these students, now identified as learning disabled, to gain access to special education teachers and other related service providers (speech therapists, occupational therapists, psychologists, etc.) as well as a modified curriculum or standard. When Response to Intervention was added as an option in the 2004 reauthorization, students could receive more intensive interventions and data would be collected to determine how they “responded” before they qualified for Special Education services. Since the requirement for a significant discrepancy between an IQ score and a measure of achievement was no longer necessary for special education qualification (and often ensuing resources), it allowed schools to offer interventions to all students who struggled academically and or behaviorally without postponing these services until a significant gap emerged. The intent was to provide early intervention so students no longer needed to fail significantly before receiving systematic assistance. RTI methods assist educators in determining qualification for Special Services based on the student’s response to an intervention before being labeled as learning disabled. Since IDEIA specifically states that lack of appropriate instruction must be ruled out before a student can be identified as learning disabled, this compels schools to examine their instructional methods and curricula and encourages them to provide timely interventions. The data collected during
the intervention period for the student, complements and informs specialized targeted testing to help determine a learning disability. Within this model, students have an opportunity to remediate gaps in learning before, not after, determining the basis of a disability.

**Progress Monitoring.** NCLB’s inclusion of a mandate for progress monitoring was to ensure the on-going documentation of all students’ academic development. Before NCLB, students with documented disabilities were not always included in statewide testing (NCLB, 2001). IDEIA requires schools to write specific goals on an Individual Education Plan (IEP) for the student based on his or her individual progress. Progress towards these individualized goals was the measure of achievement, not progress towards grade level assessments. Since NCLB requires the inclusion of score reports for all students as an indicator of overall school AYP, the gauge for measuring the academic achievement for students with disabilities has changed. The NCLB mandate to focus on closing the achievement gap, highlights the importance of early intervention, which research suggests is effective (Bryant, Bryant, Gersten, & Scammacca, 2008; Chaim, Lappan, & Houang, 1988; Clements & Sarama, 2007) and becomes harder to close as students fall further behind.

**Response To Intervention Models.** Fletcher, (2006) characterizes RTI as “a set of processes for coordinating high quality service delivery in schools”. The National Center on Response to Intervention defines RTI as a problem-solving model with data based decision making at the core. This core is surrounded by screening, progress
monitoring and multi-tiered prevention systems. Universal screening of all students identifies those who need additional supports to access the general curriculum. For those students identified by the universal screening, a multi-level prevention and remediation system must be present in the schools to provide this necessary support. The information gained from frequent progress monitoring data is used to determine the appropriate curriculum and educational environment for each student. Using evidence based interventions and culturally-responsive teaching techniques are also identified as integral to improving student outcomes. The National Center on Response to Intervention website illustrates this model as shown in Figure 1.

Figure 1

RTI Conceptual Model

Figure 1. Explanation of key RTI components. Adapted from Essential Components of RTI by the National Center for Response to Intervention. Retrieved from: http://www.rti4success.org/
VanDerHeyden et al., (2007), writing on the National Center on Response to Intervention website, outlines the effective implementation steps required for schools to make a positive impact on student outcomes in the following order: universally screen and identify at-risk students who need intervention; implement intervention to resolve the learning issue for the majority of students by providing timely access to research-based interventions; progress monitor by collecting and analyzing the data and adjust as needed to ensure curriculum fidelity and student growth; determine need for moving up a tier or back down into the general classroom; use a body of evidence to make educational placement decisions and to determine eligibility for special education; and use the RTI data to evaluate effective programs. Using RTI methods, tiers of interventions can be structured so any student struggling to meet grade standards receives systematic school-based interventions. Implementing an RTI framework means, schools must frequently probe for potential problems, develop structures to support students who need extra practice and attention, and include evidence-based interventions and progress monitoring data to remediate potential areas of weakness. This needs to be completed before moving to a determination of disability and development of an Individual Education Plan (IEP).

Researchers outline two different approaches to implementing RTI in schools, the standard protocol model and the problem-solving model (Fuchs & Fuchs, 2007; Fuchs, Mock, Morgan & Young, 2003). The standard protocol approach, typically used for academic problems, provides targeted, small group instruction with a systematic and evidence-based curriculum for struggling students. For example, students who have
specific skills gaps in math may be scheduled into both the general grade level class to maintain access to core instruction, as well as an additional math class (double dose) to address the underlying skill gap. This second math class would use a consistent and predetermined research based curriculum and standard protocol for all students. It would provide structured support and skill practice for students who demonstrate difficulty learning the grade level concepts. Curriculum-based measures would be used to gauge student responsiveness to the intervention and placement decisions would be reviewed with regularity.

The problem-solving approach, often used to address behavioral problems, provides individually designed interventions for each student (Fuchs & Fuchs, 2007). Functional behavioral analysis provides the data to develop hypotheses and specialized interventions that might include the modifying the environment, modifying adult interactions, or teaching new skills. For example, a team may look for the root cause for a student blurtng out in a structured classroom setting, and frequently change their hypothesis and intervention technique, based on the student response to the intervention. The problem-solving model is a more fluid structure in that it allows teams to quickly review and change the course of intervention as individual skills are developed.

Response To Intervention Blueprint. The Fuchs and Fuchs’ (2007) blueprint for RTI reports most RTI models use a three tier system. Tiers of interventions are often explained using a pyramid graphic as noted in Figure 2 (Deconde, 2007).
The RTI process is systemic, suffuses the culture of the school, and provides a tiered system for intervention. All students are expected to have access to a viable grade-level curriculum delivered with fidelity at the universal level (Fuchs & Fuchs, 2005; Fuchs and Fuchs, 2007; Marston, 2005). At least 80% of students exposed to this core grade level curriculum are expected to attain proficiency. The classroom teacher identifies those students who demonstrate difficulty understanding specific concepts using curriculum based measures at least three times per year and provides supplemental teaching and practice opportunities using a research based intervention or strategy. Those students who respond well to this the supplemental teaching and demonstrate improved performance (usually with a curriculum-based assessment) no longer receive the
intervention and resume the general classroom schedule. For those students who do not demonstrate increased performance may receive a targeted, more intensive tier of intervention (tier two) based on their progress. If students continue to demonstrate inadequate progress in the tier two intervention, they move into the most intensive (tier three) intervention level. The Individuals with Disabilities Improvement Act aligns with the No Child Left Behind Act (NCLB) in that it requires evidence-based interventions (those with some research on potential effectiveness) and progress monitoring with multiple data points in order to inform analysis throughout the decision making process. Using an RTI structure, schools would be able to provide interventions to ensure that all students show growth as required by NCLB.

**Tier One.** Tier one is the universal level with approximately 80% of all students in the general classroom achieving proficiency with the grade level core curriculum. Currently each state develops its own grade level standards, although a national common core curriculum, identifying national grade level standards, is available for states to adopt (National Governors Association Center for Best Practices, 2010). These national common core standards outline the content expectations for students at each grade level. In tier one, teachers adjust curriculum and instruction as needed and cluster students within the classroom to provide small group interventions—changing groupings but continuing to move forward with the core content. Based on the individual response to the general education teacher adjustments, teachers may change the intervention. Within this level, screening students is ongoing and cutoff scores are suggested to identify those
who might need a tier two, targeted intervention. Since approximately 15% to 20% of
the student population may not make adequate progress in reading based on core
instruction alone (Griffiths, Parson, Burns, VanDerHeyden, & Tilly, 2007), tier two
interventions need to be readily available.

**Tier Two.** If a student is unable to make progress with the supports provided
within the universal level, and continues to struggle, then, based on a review of the data,
an intervention could be modified or a more intensive intervention could be offered, a tier
two intervention. Tier two interventions are referred to as targeted group instruction
(Fletcher, 2008). They are intended to be of short duration as an additional support to the
core curriculum. In tier two, students who experience difficulty with specific skill
development and do not respond to the classroom interventions provided by the teacher,
receive systematic, research-based interventions using specialized instruction by qualified
staff to close any present or emerging skills gaps in addition to the core content. Tier two
interventions increase the intensity of time and resources. In addition to maintaining
access to the general curriculum, the additional support presented at this tier is expected
to provide more potential for the student to make progress. Frontloading interventions
this way prevents students from “falling through the cracks” and allows staff to identify
and remediate concerns as they arise. Based on the progress monitoring data in this tier
two intervention, the student may fully reintegrate into the general classroom. If the
student continues to demonstrate a significant lack of progress, then a tier three
intervention would be considered.
Tier two interventions are delivered via small group instruction either in the classroom or outside the classroom with trained adults. This is often referred to as a double dose, since students are expected to receive core academic instruction in addition to the additional support period in their daily schedule (Wilson Language Training Corporation, 2008). Progress is monitored frequently with curriculum based measurements (CBM) to pinpoint areas of needed focus and to measure growth rates. Providing this level of support in addition to core instruction may help some students catch up to their typical peers and avoids special education services along with a disability diagnosis. Supporting these students in small groups may prevent minor skill gaps from evolving into future learning disabilities. Using RTI, students are not required to fail before they receive these supports. The National Center on Response to Intervention website reports, “There may be a difference between low-achieving students (with or without Learning Disabilities) who respond to intervention and those who do not”.

**Tier Three.** Those who continue to demonstrate low achievement while receiving the tier two intervention may move into a more intensive, tier three intervention. A comprehensive evaluation may also be necessary (Griffiths et al., 2012) at this time. Tier three interventions are scheduled for longer periods and with greater intensity (De Conde, 2007), although there is some variation in the implementation across the country (Fuchs, Mock, Morgan & Young, 2003). Sometimes tier three interventions are reserved for students already identified with a learning disability (as in this study), and sometimes
states use it as the most intensive tier wherein the data about responsiveness is used for identification purposes (Fuchs & Fuchs, 2005, Fuchs & Fuchs, 2007; Fuchs et al., 2003). This combines an intervention method and eligibility process and in one step. The comprehensive individualized assessment is used to identify the student’s specific strengths and weaknesses and provides more evidence for possible qualification of a formal learning disability that significantly affects the student’s progress and requires Special Education services. All the previous screening and data from the tier two progress monitoring probes provide a body of evidence to support the disability diagnosis. By analyzing the body of evidence, school teams can use this assessment to target and program for specific cognitive concerns that may be creating a barrier to successful access to the curriculum. This stage would allow the team to develop an individualized education plan (IEP) for the student and will assist the team in better developing a program of study.

Tier three would also be available to students with documented disabilities who need extra support, accommodations, small group instruction and perhaps a modified curriculum in place of or in order to access the general curriculum. Specific supports identified in the body of evidence from the previous interventions would be used to develop an Individual Education Plan. Although some models place students with IEPs outside the RTI process, this study used the RTI model that places students with identified disabilities within a tier three structure.
Research base for Response to Intervention. Fuchs and Fuchs (2007) also stress the importance of implementing and evaluating the effectiveness of tiered interventions to better define the structures that work and so students can be properly placed. If the core curriculum does not address the needs of at least 80% of the students then quality of instruction cannot be ruled out as a cause for academic concerns. IDEIA (2004) paragraph (4)(A) specifically states that a child cannot be determined to have a disability if there is a lack of appropriate instruction in reading or math. Schools now have to consider whether instruction in the general setting was provided with integrity and fidelity before determining a disability. Using research based interventions and collecting individual data on the student’s response to the intervention allows schools to establish a disability diagnosis based on more reliable information. This is how RTI helps in learning disability diagnosis (Griffiths et al., 2012).

VanDerHeyden, Witt, and Gilbertson (2007), in a study aimed to assess the accuracy of RTI decision-making models in determining whether a student is appropriately referred to and qualified for Special Education services, reported increased accuracy for students who were referred and decreased referrals to Special Education following a tier two intervention. The results demonstrated that when school psychologists were trained and the decision-making protocols were followed with fidelity, placement costs and actual placements in to Special Education programs were significantly reduced.
Fuchs, Fuchs, and Hollenback, (2007) also found a decreased number of special education referrals after using a tier two intervention with first grade students. Using an RTI framework, schools have to learn how to screen for potential problems, develop structures to support students who need extra practice and attention, and include evidence based interventions and progress monitoring data in order remediate potential areas of weakness. Vaughn, Linan-Thompson, and Hickman (2003) used the standard protocol method of grouping and regrouping second grade reading students in a supplemental instructional group to determine eligibility for qualification of a learning disability. By providing supplemental instruction in 10 or 20 week installments and analyzing assessments, they determined which students responded to the initial curriculum and which ones needed an increased intensity. Based on the student’s responses, they found that 22 of the 24 students were able to reintegrate into the general classroom and continue to make progress within the core reading instruction. For those students who did not respond to the increasing levels of intervention, these trials contributed concrete data to inform special education eligibility determination.

**Why Algebra Is Important**

Mastering Algebra is important for several reasons. Science, Technology, Engineering and Mathematics (STEM) careers are on the rise and considered key 21st century skills (Terrell, 2007). STEM careers include scientists, computer/information technologists, engineers, and those occupations who use techniques directly related to mathematical theory. The Bureau of Labor Statistics “projects job growth for STEM
occupations as a whole between 2004 and 2014” (p. 32). These projections include a 15% increase in natural science occupations, a 31% increase in technology related fields, a 12% change in engineering fields and a 10% increase in math related fields. The math skill requirements for this work ranges from those acquired by high school graduates, to those skills learned in on the job training programs, to advanced mathematics as required in a Ph.D. degree program. Logical analysis, data analysis and proficiency in advanced math are important for all these future occupations.

**Shift to Earlier Proficiency.** Because successful completion of algebra is so important, there has been a shift to earlier proficiency in grade level curricula. Algebra skills have begun to appear earlier on state standards assessments. In some states educational policy reflects this shift in thinking about the mathematical sequence of coursework in that many students are required to take Algebra I in 8th grade even though they may lack the requisite skills to succeed (NCES, 2011). The National Center for Education Statistics (2011) also reports that 34 percent of 8th graders took Algebra I in 2010. Also, more than 66% of students who completed a National Assessment of Educational Progress evaluated rigorous curriculum, took Algebra I before entering high school (Nord, Roey, Perkins, Lyons, Lemanski, Brown, & Schuknecht, 2011). A rigorous curriculum in math is defined as taking four years of high school math starting with Algebra I, including pre-calculus or higher (NCES, 2011).

**Algebra High School to College.** Algebra has long been recognized as the first course in a sequence of higher level math classes, in essence, becoming a gatekeeper class for
advanced math (Mazzolo, 1995; Stinson, 2004). The task group on conceptual knowledge for National Mathematics Advisory Panel Final Report (2008) “affirms that Algebra is the gateway to more advanced mathematics and to most postsecondary education”. The National Council for Teachers of Mathematics (NCTM) (2011) recommends infusing algebraic concepts throughout elementary curricula to prepare students with the foundational concepts necessary for algebra starting in 8th grade. Many universities require applicants to have three years of math courses starting with Algebra I and encourage a fourth year of advanced math such as pre-calculus or trigonometry. Bozick and Ingels (2007) also reported that 2/3 of those in four year degree programs had taken at least one advanced level math class. Only 1/3 of those enrolled in other postsecondary programs requiring less or no formal training, had taken that sequence (National Council on Disability, 2004). Given these increased expectations and standards for college entry, students need to take a sequence of progressively complex math courses every year in high school to develop these advanced skills.

This also holds true for students who do not plan to attend college. A 2006 American College Testing (ACT) study examining the levels of mathematical readiness high school graduates need to start college or start in the workforce, found that the required math skill base was the same. Both sets of graduates needed a baseline of algebra and algebraic thinking skills to be prepared for Level Three careers and above. Level Three careers are jobs that do not require a bachelor’s degree, but which likely
require some combination of vocational training and/or on-the-job experience, or an associate’s or higher a degree or certification program.

More recently, ACT published scores on the college readiness of the 2012 graduating classes. Only 46% of students of the approximately 1.66 million students who took the 2011 ACT assessment met the mathematics readiness benchmarks (ACT, 2012).

Readiness benchmarks are defined as:

the acquisition of the knowledge and skills a student needs to enroll and succeed in credit-bearing first-year courses at a postsecondary institution (such as a 2- or 4-year college, trade school, or technical school) without the need for remediation. (p. 5)

Students who meet these benchmarks have a 50% chance of obtaining a B or higher and a 75% chance of obtaining a C or higher in a first-year college Algebra class.

Algebra is the pivotal course in the progression of advanced math skills needed for success in college and in the workforce (ACT, 2006; ACT, 2012).

**Bridging Arithmetic to Algebra**

Bridging the cognitive skills from arithmetic to algebra requires a shift in thinking. It not only requires a mastery of the algorithms of arithmetic, but a conceptual shift in abstract reasoning using numbers. The conceptual leap that students take when moving from arithmetic to algebra is harder for some more than others. Algebra introduces the concepts of generality, abstraction and logical reasoning with mathematical models. This instruction provides a basis for logical reasoning and problem-solving in areas other than math.
Researchers have studied various aspects of the cognitive shift from memorizing procedures in algorithms to developing mathematical reasoning skills (Herscovics and Linchevski 1994; Osta and Labban, 2002; Pillay, Wilss, Boulton-Lewis, 1998; Thomas and Tall, 2001). Pillay, Wilss, and Boulton-Lewis (1998) analyzed the progression of student understanding of increasingly complex math concepts. The results indicate that there is a need for instruction that facilitates student progression from operating arithmetically to operating algebraically. The complex concepts of algebra must be developed sequentially from lower to higher abstractions by allowing students time to assimilate quantitative and qualitative changes into their knowledge base (Pillay, Wilss, Boulton-Lewis, 1998).

**Shift in Thinking.** The shift in thinking about numbers as specific entities to a more general understanding and using logical mathematical reasoning to solve problems is essential to learning algebra. Wu (2009) summarizes, “going from the specific to the general is a giant conceptual leap.” Students who begin to show an understanding of the relationship between numbers, not just calculations, are ready to tackle Algebra. Thomas and Tall (2001), note that although algebra is “generalized arithmetic” many students are not ready to take the cognitive leap from the rote understanding of specific arithmetic processes, to the more abstract concept of how procedures can manipulated, in general, as required for solving algebra problems. A process is defined as using step by step algorithm (arithmetic) in different combinations (procedures) to find the same outcome. Thomas and Tall (2001) note that many students have difficulty manipulating procedures
since many remain process-oriented. They have difficulty with, “the shift from arithmetic in everyday situations to the synthetic symbolism of generalized arithmetic” (p. 3). The transition to understanding algebra involving more complex expressions:

is made more difficult by the change in meaning of the symbolism. In arithmetic, the expression \(7 + 4\) is an operational procept in the sense that it has a built-in counting procedure to give the result. In algebra, however, the symbol \(7 + x\) is first an expression for a process of evaluation, which cannot be performed until \(x\) is known. (p. 3)

**Abstract/Mathematical Thinking.** The shift to flexible thinking about math concepts demands more abstract reasoning skills and an increasing use of thinking strategies. Herscovics and Linchevski (1994) studied the problem-solving strategies of elementary students to determine student efficiency with understanding equations. They identified equations with one unknown as an arithmetic task and equations with more than one unknown as algebra. Their findings determined that students who have struggled bridging the arithmetic skills to algebra demonstrated difficulties with “an inability to operate spontaneously with or on the unknown” (p. 61). These students also showed a tendency to, “detach a numeral from the preceding minus signs in the grouping of numerical terms, and had difficulty with the acceptance of the equal symbol to denote a decomposition into a difference” (p. 61).

Osta and Labban (2002) explored the mathematical thinking, using think-aloud strategies and observations, of Lebanese 7th graders before they were formally introduced to algebra. Instead of just reviewing how students understand the unknown and the equality sign (Herscovics and Linchevski, 1994), they focused on the shift from arithmetic procedures to pre-algebraic problem solving. Their findings indicate that
although the students might use their prior knowledge to build algebraic equations, they tended to revert to strategies grounded in arithmetic problem solving methods to solve them:

This anchoring of algebraic thinking in arithmetic, is viewed as an important stepping stone toward algebraic thinking, since algebraic conceptual underpinnings are inherent to them, such as the writing and processing of algebraic and semi-algebraic expressions, the inversion of operations, the meaning of an unknown, a root and an equation, etc. (p. 30)

Such methods of reasoning and gradual symbolizing are a way to facilitate the transition from arithmetic to algebraic modes of problem solving given instruction in formal procedures for solving first degree equations. A solid foundation of basic math skills and concepts, especially understanding fractions, decimals, and percent will help bridge this cognitive gap (National Mathematics Advisory Panel, 2008). In grades five through seven mathematical abstractions are introduced with the concept of fractions, geometry and negative numbers.

The impact of poor performance in early math extends to high school where the sequenced coursework can become even more restrictive. Cavanaugh, (2008) reports that since the 1983 Nation at Risk report was released advocating higher standards (at least three years of high school math) to earn a diploma, minimal graduation requirements often dictate that the lowest math course offered is Algebra I so schools can offer a full progression of higher math coursework (National Mathematics Advisory Panel, 2008). Although the National Council of Teachers of Mathematics recommends more math study for all high school students starting with an Algebra I course, they also recommend that a more flexible curriculum be extended to accommodate diverse student needs.
Before RTI, too often students would find a structure that mandated high expectations, but that offered few classes, outside of Special Education courses available for practice with the prerequisites to Algebra I

**Impact of Poor Performance.** Mastery of the calculation methods necessary for solving basic arithmetic problems facilitates the shift in thinking. According to the Final Report of the National Mathematics Advisory Committee (2008):

> a major goal for K–8 mathematics education should be proficiency with fractions (including decimals, percent, and negative fractions), for such proficiency is foundational for algebra and, at the present time, seems to be severely underdeveloped. (p. xvii)

Many students entering Algebra I classes in middle and high school not only do not understand the basic principles of arithmetic (e.g. mathematical equality), but they often:

> also have difficulty grasping the syntax or structure of algebraic expressions and do not understand procedures for transforming equations or why transformations are done the way they are. These and other difficulties are compounded as equations become more complex and when students attempt to solve word problems. (p.32)

In addition, this report outlines the critical foundations for algebra and benchmarks towards progress of those standards.

**Intervention Technique CSA/CRA**

Scaffolding instruction from concrete to representational to abstract(CRA), sometimes called CSA—concrete to symbolic to abstract, is recommended for students with identified learning disabilities as well as students with low achievement (Maccini, McNaughton, & Ruhl, (1999); Maccini & Hughes, 2000; Witzel, 2005). CRA is a
sequenced instructional technique that moves a student through graduated levels. Instruction begins with concrete manipulation of an object. Concrete manipulatives may include blocks, pie wedges, rods or any other object that a student can manipulate physically to aid in learning the concept. CRA then moves to a representational (semi) manipulation of figures and pictorial representations. Representational images include charts, graphs, drawings and any other pictorial representation that a student can see to more fully visualize the concept. Finally students are exposed to abstract concept mastery using symbols, numbers or letters—CRA. Bouck and Flanagan (2010) recommend “virtual manipulatives” as represented in computer programs as an instructional tool and found them to be effective in facilitating students understanding of mathematical concepts. Since roadblocks to successful implementation of using visual skills in teaching math are that there is not enough time to use manipulatives and it is difficult for teachers to find enough examples (U.S. Department of Education Practice Guide, 2009), these virtual manipulatives can move a student from the concrete representation closer to the abstract representation using CRA (CSA) techniques.

**Response To Intervention Math Research**

Initial research on RTI math interventions (Bryant, 2008), indicated improvement in second grade students’ math achievement using a tier two intervention, but not in the first grade tier two intervention. Both used a CSA sequenced instructional approach (sequenced learning from concrete manipulation/representation to semi concrete representation to abstract conceptualization). Even though this intervention only provided
short-term tutoring over several weeks, the second grade students did show significant improvement.

Fuchs, Fuchs, and Hollenback, (2007) also used a CRA (concrete to representational to abstract) approach in research on tier two interventions with first grade students and found the growth of the students receiving the intervention to be comparable or greater to control peers. This intervention also reduced the number of referrals to Special Education eligibility for those who participated.

**Why Math Interventions Are Important**

NCLB and IDEA mandates, the difficulty bridging the cognitive gap needed to understand algebraic reasoning, the high school math failure rates and subsequent effects, and the persistent achievement gaps, all emphasize the need for math interventions. A report summarizing results of numerous studies and sponsored by the National Research Council, linked a pattern of poor achievement to the likelihood that a student will drop out (Beatty, Neisser, Trent, & Heubert, 2001). Finding interventions to support students is imperative in the face of such critical consequences.

**Math Failure Rates.** Math interventions are so important because a sizeable number of high school students fail at least one academic subject. This failure is one factor among many that places students at risk of not graduating (Battin-Pearson, 2000; Rumberger, 2001; Vallerand, Fortier, & Guay, 1997). Roderick and Camburn (1999) in a study of Chicago public schools, encompassing roughly 28,000 students in the 1992-93 ninth grade cohort, found that 40% of entering ninth graders failed at least one major
academic course. Twenty four percent of these students failed their first semester math course. The progression of this failure increased over time through the second semester of 10th grade when the failure rate increased to about 50%.

In an L.A.Times article (2006), Helfand notes that 48,000 Los Angeles Unified School District ninth grade students took Algebra I with a 44% failure rate and 17% earning Ds (roughly 29,000 students). With 61% of students struggling to pass Algebra I and with requirements of completing Algebra II in order to earn a high school diploma, many of these students may end up dropping out altogether without system wide supports.

Poor academic achievement, including motivational elements related to perceived academic incompetence has consistently been identified as a factor in the rate of high school drop outs (Battin-Pearson, 2000; Rumberger, 2001; Vallerand, Fortier, & Guay, 1997). The effects of low academic achievement have been identified as a mediator in combination with other factors in the “etiology” of a dropout (Battin-Pearson, 2000). In a 2001 presentation, Rumberger notes that in previous studies up to 34% of students reported they dropped out because they were failing. Poor academic achievement is just one of many reasons in a student’s decision to withdraw from school that can be prevented (Rumberger, 2001).

Also, using data from Philadelphia Education Longitudinal Study (PELS), Neild, Stoner-Eby & Furstenberg, (2008), found “ninth-grade course failure and attendance have a substantial impact on the probability of dropping out within six years of starting high school”(p. 546). After controlling for previous experiences, Neild et al. (2008) found,
“the percentage of courses failed in 9th grade is a strongly significant predictor of eventual dropout” (p. 558) and failing one course in a five course load, “increased odds of dropping out by more than one third” (p. 558).

**National Assessment of Educational Progress.** NCLB proficiency standards for all students in reading, math and writing by 2014 are a huge undertaking for schools. Highlighting the contrast between expectations and actual performance is the data from the National Assessment of Educational Progress (NAEP) 2011. The National Assessment of Educational Progress (NAEP) is given every two years and measures reading and math achievement for students in grades 4, 8 and 11 across the United States. NAEP is sponsored by the U.S. Department of Education and the National Center for Educational Statistics (NCES), a department within the Institute for Education Sciences. Results are reported using standards of performance. The National Assessment of Educational Progress Standards defines the national core curriculum. NAEP Curriculum Level Standards, also known as achievement levels or performance standards, are categorized as basic, proficient and advanced, They are set by the National Assessment Governing Board based on what students are expected to know at each grade level. NAEP standards of performance do not always align with individual state standards because each state sets their own standards that determine the grade level curriculum. Using the NAEP standards allows for a comparison between state level standards and the national standard. It also allows for analysis of national trends in student performance comparisons over time. It does not allow for state to state comparison of performance. Proficiency on NAEP assessments is defined as “competency over challenging subject
matter” whereas proficiency on state assessments is defined as grade level performance (NCES, 2011). NAEP standards use three categories of performance as shown in Table 1 (NCES, 2011).

Table 1
Achievement Level Descriptions

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>Partial mastery of prerequisite knowledge and fundamental skills for proficient work at each grade level.</td>
</tr>
<tr>
<td>Proficient</td>
<td>Solid academic performance for each grade assessed</td>
</tr>
<tr>
<td>Advanced</td>
<td>Superior performance for each grade assessed</td>
</tr>
</tbody>
</table>

Classification for performance levels on NAEP assessments.

The most recent NAEP 8th grade math assessment, shows that 73% of students had a basic level of knowledge and only 36% of students diagnosed with disabilities were at or above the basic level (but not proficient). Only 9% of students with disabilities were at or above proficient on the grade level pre-algebra standards on this eighth grade assessment (NCES, 2011). This illustrates how many students struggle with grade level math concepts and the significant impact on the population of students identified with learning disabilities.

Colorado Student Assessment Scores. Colorado uses the Colorado Student Assessment Program (CSAP) to measure student progress towards achieving the state standards and the NCLB mandates. Students take this test yearly and scores are disaggregated to determine how groups of students are achieving towards predetermined
goals. Based on the data from these tests, many secondary students across the state demonstrate a lack of proficiency in math (CSAP, 2011). The CSAP assessment of ninth grade math performance on the Colorado Student Assessment Program (CSAP) for the year 2011 shows a large percentage (61%) of students scored in the partially proficient and unsatisfactory range. Overall of the 9th graders who took the CSAP in 2011, 31% were unsatisfactory, 30% were partially proficient and 38% were proficient or advanced as noted in Table 2 (Colorado Department of Education, 2012).

Table 2
CSAP Scores for 9th Grade Students in Colorado

<table>
<thead>
<tr>
<th>Grade</th>
<th>Category</th>
<th>Total</th>
<th>% Unsat</th>
<th>% Partial Proficient</th>
<th>% Proficient</th>
<th>% Advanced</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>State: Colorado</td>
<td>59,859</td>
<td>31</td>
<td>30</td>
<td>24</td>
<td>14</td>
</tr>
</tbody>
</table>

Number and percentage of total state student population at different performance levels on the CSAP.

The results for students identified with a learning disability are even more concerning. Only 7% of ninth grade students with IEPS scored at the proficient or advanced levels. 76% of students with IEPs tested at an unsatisfactory level on CSAP and 14% tested at the partially proficient level as noted in Table 3 (Colorado Department of Education, CSAP 2011 Results).

Table 3
CSAP Scores for 9th Grade Students with IEPs

<table>
<thead>
<tr>
<th>Grade</th>
<th>Category</th>
<th>Total</th>
<th>% Unsat.</th>
<th>% Part. Proficient</th>
<th>% Proficient</th>
<th>% Adv.</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>IEP</td>
<td>5,007</td>
<td>76</td>
<td>14</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

Students need access to interventions if they are going to make sufficient progress toward grade level standards. The general curriculum is not designed to fill in missing
skill gaps. This disparity between actual math achievement and the expectation for student growth underscores the importance of closing the achievement gap for all students and even more for students with disabilities. Students will need exposure to algebraic concepts in order to receive grade level content, but they also need more practice in pre-algebra skills in order to use the algebra concepts. By offering tiers of interventions, students with different levels of need can benefit.

Among 8th graders who scored below the 25th percentile on the 2011 NAEP math assessment, 25 percent were identified as students with disabilities. Using tiered interventions for support, almost all students are expected to access the general education curriculum since this is where they will be exposed to grade level concepts. Without tiered interventions to support the basic skills, many students, not just those with IEPs, will continue to fail.

The National Center for Education Statistics (NCES 2011) data represents a stark picture of the achievement gap for students with disabilities. In the NAEP High School Transcript Study (2011), a report on coursework taken in 2009 by American high school graduates, “63 percent of English language learners (ELL) and 45 percent of students with disabilities (SD) completed a below standard curriculum compared with approximately 25 percent of non-ELL and non-SD graduates”. A below standard high school curriculum in math is described as completion of two or less credits of high school math coursework. With almost half of students with identified disabilities taking coursework considered to be below standard, it makes sense that students would need not only
remediation in basic skills, but they also need to stay on track with the general education curriculum in order to show growth on the state standards set to grade level.

**Math Disability—What Is It and Does It Matter?**

Many students without diagnosed disabilities struggle with math (Geary & Hoard, 2005; Kulak, 1993; Mazzocco, Hanich, & Early, 2007; Proctor, Floyd & Shaver, 2005). Some students, oftentimes those with a significant math disability, demonstrate qualitative differences in their processing of information while others simply show a delay in the rate of skill acquisition (Kulak, 1993). A key qualitative difference is automatization or automaticity. Automaticity, allows students to retrieve information quickly using minimal cognitive resources (e.g. recognize a sight word or recognize a math fact). It also “involves assembling small units of information into larger units, which can be retrieved with relatively greater speed and with high levels of accuracy” (Kulak, 1993, p. 667). Students develop automaticity as they are exposed to frequent opportunities to experience and practice tasks. “Children with math disabilities may have difficulty with the automatization of addition facts” (p. 667). Those students with qualitative learning differences who struggle with math also may show cognitive deficits in several different areas including memory (Bull, Johnston, & Roy, 1999; Bull & Scerif, 2001; Geary, Hamson, & Hoard, 2000; Meyer, Salimpoor, Wu, Geary, & Menon, 2011; Murphy, Mazzocco, Hanich, & Early, 2007), or procedural deficits (Geary & Hoard, 2005) or visual spatial deficits (Bull, 1999; Geary, 1993; Geary and Hoard, 2005; Murphy, 2007; and Royera, 2009).
In contrast, students with a delay, progress through learning stages as their typical peers, but at a slower rate, which represents a quantitative difference. Kulak, (1993) theorizes, “the developmental lag hypothesis may best explain the performance of the majority of children who experience problems in reading and math” (p. 670). Tier one and tier two interventions would provide extra time and intensity for these students to help address the lagging skills. Geary & Hoard, (2005) also make note that those students who achieve inconsistent scores on assessments may not have a math disability. They do not appear to have the underlying memory and (or) cognitive deficits of those students who score consistently low and likely do have a math disability. When exploring the broad cognitive abilities and math achievement, Proctor, Floyd, and Shaver (2005) found that 50% of the “low achievement” group of the students in their study again, did not demonstrate a cognitive weakness. They speculated reasons for this may be that instruction of math in the United States varies greatly from state to state, anxiety may impact their scores, students may have had varying levels of practice or poor instruction or lack motivation. Any one of these non-cognitive reasons could impact a student trying to learn such a sequential subject as mathematics. Since math disabilities are not consistently identified, this research will examine both those students with diagnosed disabilities and those students without diagnosed disabilities but demonstrate low achievement.
Math Disability Identification

There are challenges involved in how to screen or even label a student with a math disability. Key reasons for this are that there is no agreement on a standardized diagnostic protocol to use when diagnosing a math disability. There is also no consensus definition for Math Learning Disability (MLD) (Mazzocco, 2005; Murphy et al, 2007).

Determining clear guidelines for diagnosing math learning disabilities is an underserved area of research (Geary, 2007; Mazzocco, 2005; Murphy et al., 2007). Two tools that are often used are IQ scores and performance levels on standardized tests with an arbitrary cutoff. In a review of research, (Mazzocco, 2005 and Murphy et al., 2007) noted that students who score at or below the 35th percentile can be diagnosed with a math disability. Comparably, other professionals use more stringent cutoff scores when diagnosing a math disability for students, and recommend using scores at or below the 25th, 15th, and 9th percentiles (Mazzocco, 2005). Some professionals even advocate using qualifying cutoff scores at each different level to more accurately diagnose the type of math disability in students (Murphy, 2007). How to identify a student with a math disability remains an area of research where there is little consensus.

Types of Math Disabilities

Further complicating the issue, it appears there are different subtypes of math disabilities. Some math disabilities that are part of a larger reading disability that share core underlying deficits while other math disabilities are more impacted by a specific deficit (Bull,1999; Geary,1993; Geary & Hoard, 2001; Geary & Hoard, 2005; Hale &
The core deficits for students who struggle with math are an area of consensus for neuropsychologists, cognitive psychologists and even genetic researchers (Geary, 2007; Geary & Hoard, 2005; Mazocco, 2005; Swanson, 2006). Hale and Fiorello (2004), advocate for diagnosing the cognitive characteristic strengths and deficits for struggling students using neuropsychological assessments. This would allow more targeted interventions for students to practice and build specific skills. The way information is represented, either linguistically or visually, impacts conceptual learning. Many research findings can be consolidated into three distinct subtypes of math disabilities 1) procedural deficits, 2) semantic memory deficits, and 3) visual-spatial deficits (Bull, 1999; Geary, 1993; Geary & Hoard, 2005; Murphy, 2007; Royera, 2009).

Geary and Hoard (2005) use a conceptual and procedural deficit model to explain math disabilities that include, “underlying deficits in (1) the central executive and (or) (2) in the information representation or manipulation (i.e., changing the way the information is represented) systems of the language or visuo-spatial domains” (p. 261). The central executive influences all types of learning through attention and memory input whereas the visual spatial deficits impact math more specifically and may best be described as impairment between information representation and the ability to change the form of this representation, “as they might affect magnitude representation” (Geary & Hoard, 2005, p. 264). This model reinforces a spatially-based form of math disability. Adding to this work, in a study of kindergarten and first grade students, Geary (2007) identified three
generally agreed upon specific underlying cognitive deficits students with math
disabilities typically exhibit are, 1) delayed conceptual development, 2) impaired working
memory (central executive) and retrieval and 3) a slower speed of processing.

Executive functioning (comprising working memory), along with other cognitive
building blocks, is important to the development of math skills, (Geary, Hoard, Byrd-
Craven, Nugent, & Numtee, 2007). Learning disabilities and executive functioning are
closely connected, especially when considering how students maintain and manipulate
information in working memory as opposed to working memory span alone (Bull,
Johnston, & Roy, 1999; Bull & Scerif, 2001; Geary, Hamson, & Hoard, 2000; Meyer,
Salimpoor, Wu, Geary, & Menon, 2011; Murphy, Mazzocco, Hanich, & Early, 2007).
The central executive provides resources to memory systems including the phonological
loop and visuo-spatial sketchpad. The phonological loop receives auditory-verbal
information and allows for rehearsal to help one remember it. The visuo-spatial
sketchpad helps one remember visual imagery, allowing for mental rotation and
facilitates mental math skills (Feifer & DeFina, 2005). The central executive, as
explained by Baddeley and Hitch’s (1974) and Baddeley’s (2000) multi-component
working memory models, is key in directing shifting and maintaining attention. This
central control system directs attention to visual or auditory stimuli to the visual spatial
sketchpad or the phonological loop, which is then combined with sensory input and
manipulated. They both use temporary stores although the phonological loop uses an
auditory rehearsal system while the visuospatial sketchpad, “holds information into
separate visual, spatial and possibly kinaesthetic components” (Baddeley, 2000, p. 418) of the brain. They work in combination with an episodic buffer to form long term memory (learning). As students grow older, there is a shift from reliance on phonological cognitive skills, to visuo-spatial representations in mathematics problem-solving (Meyer et al., 2011).

Visual spatial deficits often manifest as difficulty representing spatial relationships, difficulty estimating the magnitude of large numbers, frequent misinterpretation of spatially represented information and other mathematical concepts (Geary, 2005; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). Each of these areas was noted to need more research to fully understand and program interventions.

**Visual Spatial Skills and Math**

Research supports the idea that visual spatial skills and visual representations are related to developing mathematical skills (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; National Research Council, 2006). Visual spatial abilities are those skills that allow students to see and mentally manipulate objects in their mind’s eye. These skills allow students to form representations of concepts and to process information without words or labels (National Research Council, 2006). Gunderson, Ramirez, Beilock, & Levine, (2012), using a longitudinal study, found that developing spatial skills can improve children’s development of numerical knowledge by helping them understand a linear spatial representation of numbers. In addition, one mechanism linking spatial skill with math achievement is acquiring a representation of a spatial, linear number line.
The Institute of Educational Statistics Practice Guide also recommends using a visual number line to accustom students to new concepts. After reviewing several meta analyses (only about 50 studies total), Ketterlin-Geller, Chard, and Fein (2008) found consistency with the hypothesis that spatial skills can improve children’s development of numerical knowledge by helping them to acquire a linear spatial representation of number. They also concluded that:

explicit, systematic instruction that involves extensive use of visual representations appeared to be a moderate to strong approach for supporting mathematical learning for both low-achieving students as well as those with learning disabilities. (p. 35)

The Chaim, Lappan, and Houang (1988) results indicate that visual spatial skills improve with practice. “Both sexes demonstrated equal potential for acquiring significant gain from the training. “When these spatial visualization skills have been attained, they last and even continue to develop over time” (p. 68). Also, Van Garderen (1999) found “significant and positive correlations” between spatial visualization measures and applied math word problem solving measures (p. 503).

Nonetheless, the impact of other factors cannot be overlooked when evaluating the link between visual representations and math achievement. In a review of the literature, Gersten, Beckmann, Clarke, Foegen, Marsh, Star, & Witzel, (2009) determined, after analyzing the evidence of the studies proposing that visual representations be used, that there was only moderate evidence for this recommendation since many of the studies were multifaceted and examined many “complex multi component interventions” (p. 30). Since it is difficult to parse out the effects of the visual
representations, more research is needed to clarify the important aspects of visual learning and the sequence of cognitive steps from concrete to representational to abstract.

**Math Interventions**

Understanding math disabilities is integral to developing interventions. Research in the development and effectiveness of interventions specifically designed to tap and encourage growth of deficient underlying cognitive skills (visual-spatial, sequencing, memory skills, concept formation, etc.) may directly benefit students identified with math disabilities. The Institute of Education Sciences in the What Works Clearinghouse lists recommendations for using RTI with math including providing sufficient visual representations of mathematical ideas with explicit and systematic instruction (2009 U. S. Department of Education Practice Guide). Although there are various subtypes of math disorders, this research will focus on the visual spatial component as identified in previous research (Bull et al., 1999; Geary, 1993; Geary et al., 2007; Proctor, Floyd, & Shaver, 2005). Bryant, Bryant & Hammill (2000) concludes the ability to discriminate among the various subtypes of math disabilities will, “have implications for those who design and implement technology adaptations for students with academic problems” (p.174). Since students who struggle with math may or may not have been identified with a math disability, interventions to meet the needs of both learners need to be developed.
Summary

The movement in school policy to align the precepts of NCLB with the most recent version of IDEIA has implications for programming for students who struggle with math. Additional research is necessary to provide appropriate intervention research to help schools close the persistent and widespread achievement gap in math (NCES, 2011). Even those students without a diagnosed math disability are at risk since there is no consistency about how to diagnose and intervene. RTI has begun to change the educational landscape by building structures to allow students to some flexibility to access special services. With this policy shift, school staff are responsible for collecting data on the impact of interventions and using a problem-solving model providing tiered levels before moving to develop an assessment for special services. No longer will students need to be identified in order to participate in systematic remedial instruction.

The shift in mathematical reasoning necessary for successful completion of algebra helps students learn problem-solving and patterns of thinking that will benefit them in later coursework and future careers. Improving math achievement is critical for students entering an increasingly technical world requiring problem-solving skills learned in algebra. The Algebra Readiness curriculum focuses on skill development in pre algebra mathematical concepts in an effort to help students become more facile at bridging this leap in mathematical reasoning necessary for successful completion of algebra.
Understanding how visual spatial cognitive skills impact student learning is important in developing interventions for students entering algebra. The ability to visualize representations of key mathematical concepts is difficult for many students, which makes learning the abstract concepts of algebra even harder. Providing evidence-based interventions to address this deficit has the potential to help many students bridge the gap from concrete to abstract reasoning required in higher level math courses.

**Research Questions**

The research questions aim to explore the effect of the MIND Research Institute’s Algebra Readiness program by measuring impacts on student learning using performance on CSAP, ACUITY C, and or MIND post test scores as outcome measures. More specifically,

1) What is the impact of the MIND Institute Algebra Readiness intervention on algebra skills for students who have struggled with or failed a pre-algebra or algebra course in their previous year of school?

1a) Does this intervention impact some groups of students more than others?

2) What is impact of the MIND Institute’s Algebra Readiness intervention on math achievement for students who are receiving special education services via an IEP?

2a) Does the RTI tier of intensity have a differential impact on the achievement for these identified students?
Independent samples t-tests were used to compare the scores of students with or without an IEP and those in a double or single dose using Acuity C, and CSAP scores as the test dependent variables. ANCOVA was used with the MDTP pretest as the covariate to test the differences between students in the double dose as compared to the single dose. This ANCOVA procedure was also used to compare the IEP groups—those with IEPs and those without. The MDTP posttest was used as the dependent variable.
Chapter III: Method

Introduction

This study explored the effect of a pre algebra intervention on student’s math performance given two levels of intensity. The level one group of students, the tier two group, was composed of students with identified disabilities and students with no formal diagnosis, but who scored below proficient on their state grade level standards assessment. They participated in a double dose intervention attending both the Algebra Readiness intervention class and the district-adopted general education Algebra I class daily. The level two group, a tier three group, was composed of only students with identified learning disabilities. They participated in the intervention as their only math class daily. This chapter details the participants, curriculum, assessment instruments, and data analysis used in this study.

Participants

Twenty-seven ninth and tenth grade students and two eleventh grade students in a suburban high school near Denver, Colorado participated in this study. Students were selected in August 2010 based on their previous year’s math grade. The students had all scored at or below 65%. Also the previous year’s teachers recommended the selected
students based on their attendance and effort. Those students selected had attended class consistently and turned in their homework but still performed poorly. Students who demonstrated poor attendance or those who did not complete at least 50% of their homework from the previous year were excluded from participation in the study. All the students in the tier three intervention group had a developed IEPs and received Special Education services. The students in the tier two intervention were mixed, some had diagnosed learning disabilities with IEPs and others did not. Colorado State Standards recognize two levels of competence—PA for proficient or advanced and BP for below proficient on grade level work. All of the students who participated in this study scored Below Proficient in math standards on their previous year’s CSAP.

The previous year’s teachers also recommended assignment to the intervention level or tier. Students assigned to the tier two intervention were estimated to be ready for Algebra I by the previous years’ teacher and or counselor, so they were enrolled in both an Algebra I class and the Algebra Readiness intervention class, resulting in a double dose. Seven students without IEPs were included in the tier two intervention. For the students in the tier two level, the Algebra Readiness class was scheduled as an elective course. Since students voluntarily participated in the Algebra Readiness class, some elected to drop the course at the end of the first semester and chose a different elective course. Students who did not agree to share their data were also allowed to take this class.

Students assigned to the tier three intervention were scheduled in the Algebra Readiness intervention as their only math class. All were receiving Special Education...
services and had Individual Education Plans (IEPs) which allowed them to have this course replace the required grade level math class. They were estimated to need additional work with fractions, decimals and percents by their previous year’s teachers.

Tables 4- 6 identify students with and without IEPs by grade and by dose. Table 7 identifies students by grade and gender.

Table 4

<table>
<thead>
<tr>
<th>Grade</th>
<th>With IEP</th>
<th>Without IEP</th>
<th>Total</th>
</tr>
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<tr>
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<td>13</td>
<td>6</td>
<td>19</td>
</tr>
<tr>
<td>10th</td>
<td>8</td>
<td>0</td>
<td>8</td>
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<tr>
<td>11th</td>
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<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
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<td>29</td>
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Table 5

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<th>Without IEP</th>
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<td>6</td>
<td>12</td>
</tr>
<tr>
<td>10th</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
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<tr>
<td>Totals</td>
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Table 6

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<th>Grade</th>
<th>With IEP</th>
<th>Without IEP</th>
<th>Total</th>
</tr>
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<tbody>
<tr>
<td>9th</td>
<td>6</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>10th</td>
<td>8</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>11th</td>
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<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
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<td>0</td>
<td>16</td>
</tr>
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</table>

Table 7

<table>
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<tr>
<th>Grade</th>
<th>Male</th>
<th>Female</th>
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<tbody>
<tr>
<td>9th</td>
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<td>6</td>
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<td>10th</td>
<td>5</td>
<td>3</td>
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<td>11th</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>19</td>
<td>10</td>
</tr>
</tbody>
</table>
Curriculum

There were two curricula used in this study. All the students used the MIND Research Institute’s Algebra Readiness program. The students in the tier three group used only this curriculum. No other math instruction was presented. Students in the tier two level participated in the Algebra Readiness program and the district Algebra I curriculum, College Preparatory Math.

**Algebra Readiness.** The Algebra Readiness math curriculum was developed by the MIND Research Institute, a neuroscience and education research based non-profit corporation. It incorporates recommendations from the National Council for Teachers of Math and the National Mathematics Advisory Panel (2008) in its design. Fuchs (2012) recommends six key points for implementing effective math interventions: instructional explicitness, instructional design that eases the learning challenge, teaching a strong conceptual base for procedures, an emphasis on drill and practice, cumulative review as part of drill and practice, and motivators to help student regulate their attention and behavior and to work hard. The Algebra Readiness math intervention incorporates all these recommendations to varying degrees into the program. There are two components of the Algebra Readiness curriculum, a classroom text that the teacher uses during whole group instructional time and a computer games practice component.

The Algebra Readiness computer games provide a nonverbal approach. The student is required to virtually manipulate visual stimuli on the screen to move an animated character through increasing more complex levels of mathematical concepts. It
is essentially a computerized version of the use of math manipulatives. The task to be performed is visually demonstrated with little to no reading required. The visual representations of the concepts, paired with immediate feedback, allow the student to monitor his/her own progress. The visual, spatial representations of mathematical concepts using the animated character to achieve increasing levels of proficiency, reinforce the “gamelike” qualities of this program. The students interact with the character by manipulating a visual representation of the concept until completing (mastering) a level. This moves the character forward to more challenging tasks. These interactive exercises allow students to see and manipulate the visual examples and help them learn math content without using extraneous language. Graziano, Peterson and Shaw (1998) demonstrated that engaging the cognitive spatial temporal reasoning skills using the math games in the Algebra Readiness program can improve student understanding of general math and science concepts. Students learn to move the character through different levels by solving a series of puzzles using number lines and blocks on the screen, thereby practicing underlying mathematical concepts. The software gives immediate feedback by demonstrating the consequence of the attempted solutions, reinforcing the concept with a visual representation of both the correct and incorrect solution. When students demonstrate proficiency in this animated version, exercises using language-based representations, including the appropriate math symbols, follow. Teachers can assign practice modules individually for each student, effectively differentiating the curriculum each student pursues on the computer. When students attempt a problem three times and fail to achieve mastery, the game locks them out of
that module and sends a message to the teacher that the student is “stuck”. This allows
the teacher to reset the game and follow up with additional instruction. Unit tests allow
teachers to check progress more frequently. Based on each student’s individual progress
in the Algebra Readiness computer games, teachers can assign and reteach concepts to
each student based on any gaps in their preparation (MIND Research Institute, (nd).
Using the text in the classroom three days a week, the teacher facilitates expression of
these visual representations using math language and symbols. This whole group
instructional time, in addition to helping students articulate mathematical reasoning,
allows teachers the time to reinforce and further develop the concepts by making
connections and introduce concepts using the accompanying text.

**College Preparatory Mathematics.** The Algebra I curriculum used at the high
school is *College Preparatory Mathematics—Algebra Connections Volumes 1 and 2
version 3.1 (CPM).* This program is designed to develop mathematical problem-solving
and critical thinking skills using an investigative approach. The teacher directs the lesson
to the large group for the first half of the class period. Afterwards, students work together
and use think alouds as a basis to refine and generalize the new information. The course
is designed “to develop multiple strategies to solve problems and to recognize multiple
ways of understanding concepts” (p. 1). The course is structured around problems and
investigations that build the conceptual understanding of these algebraic topics and an
awareness of connections between the different ideas.” The Algebra I content is
organized around seven core ideas with student groups exploring these ideas through
discussion and examples. CPM gives students pieces of math information, demonstrates how to put it together and then allows practice so students can eventually discover the gestalt on their own (Dietiker, Baldinger, Cabana, Gulick, Shreve, & Lomac 2006).

Recruitment

As this program was adopted for trial by the high school, recruitment started with the registration process during conversations between students and teachers and counselors. Parents were informed of the school's adoption of this program via phone calls and signed consent forms informing them about the research aspects of the program. Students who enrolled in the intervention classes and chose not to allow their data to be used still received the math intervention, however, their data were not included in the research. Also the data from students who did not complete the program were not used.

Since this research was performed in an applied setting, and many variables could not be controlled, it provided some evidence about the effectiveness of the curriculum as delivered in a natural setting. Some of the variables which may have affected the outcomes include typical student maturation, effort of students, previous exposure to content, length of exposure to content, types of previous interventions, etc. These variables were uncontrolled in this design.

Design

This study utilized a pre-experimental design with students enrolled in two high school classrooms without randomization and a control group. The purpose of this
research was to analyze any change in math performance for students who have traditionally failed math or showed low response to previous math interventions. This research sought not to explore the replacement of the traditional Algebra I math course, based on standards and reviewed by state boards, rather, explored whether this non-traditional intervention had an impact on a select group of students. Since failure of Algebra I in the schools has been and continues to be an ongoing and entrenched problem for many students (CSAP 2011; NAEP, 2011), more effective research based interventions are needed. This research explored the effects of the MIND Research Institute’s novel spatial reasoning approach, measuring progress with the same standardized tests states use to assess overall student achievement (CSAP and Acuity). In addition, for the two groups, tier one - those who participated in this intervention program as an add on to the traditional Algebra I curriculum and tier two - those who participated in the intervention class alone, pre and post tests were used to explore more closely any change in individual performance.

**Fidelity**

Accuracy and fidelity to treatment protocols are paramount in delivering instruction and assessing intervention outcomes and provide a basis for future studies to replicate the treatment (Dumas, Lynch, Laughlin, Smith, & Prinz, 2001; Gersten, Fuchs, Compton, Coyne, Greenwood, & Innocenti, 2007; Moats & Lyon, 1996). Both affect both internal and external validity as well as the statistical power of the outcome analysis (Moncher & Prinz, 1991; Dumas, Lynch, Laughlin, Smith, & Prinz, 2001). Monitoring
fidelity involves measuring the precision and consistency with which the interventionist presents the treatment to all the participants. It includes teacher activities such as participating in content training, using teacher manuals, adhering to instructional protocols, collecting data about pacing, and monitoring student results (Bellg, Borrelli, Resnick, Hecht, Minicucci, Ory, Ogedegbe, Orwig, Ernst, Czajkowski, 2004).

In this study, fidelity and quality of instruction were addressed by implementing a shorter, pilot version of the curriculum for the two tiers of intervention six months previous to the start of the study. A Special Education teacher was paired with an NCLB highly qualified Algebra I general education teacher to teach two classes, an Algebra I class and the Algebra Readiness intervention in a semester long pilot study (18 weeks). The Special Education teacher co-taught the Algebra I class with the general education math teacher in the tier two (double dose) group. For the tier three (solo) intervention group, the Special Education teacher alone implemented the Algebra Readiness program in a small group setting for students with identified learning disabilities. Both teachers participated in one full day of training conducted by the MIND Research Institute and two follow up sessions via phone for technical and curricular supports, including teaching techniques presented in the initial session and the teacher manual. This was the standard training for teachers administering ST math, aimed to familiarize them with the program. The sequence of the pilot intervention curriculum was tailored by the Mind Institute’s consultant so that the students would gain exposure to critical aspects of the Algebra Readiness program since they were presented with an abbreviated content. This prepared
the Special Education teacher for the full year project, as she was familiar with the
content and pacing of the curricula in both the Algebra I and the Algebra Readiness
courses. The two teachers scheduled one hour weekly for content planning. The Special
Education teacher continued to email the MIND Research Institute team with questions
as needed throughout the intervention.

Procedure

Approximately thirty, ninth, tenth and eleventh grade students completed a 36-
week (one school year) math intervention class taught by a Special Education teacher at
one of two levels (tiers) of intervention. The students who were selected by their teachers
and their parents signed consent and assent forms agreeing to allow their data to be
analyzed after the study. The students and parents were informed about the study before
they signed the consent/assent forms either in a face-to-face meeting or via phone during
the registration process. All the students had scores below proficient on their previous
year’s CSAP.

The Special Education teacher was trained in the Algebra Readiness program by
participating in the pilot study during the previous year with a highly qualified Algebra
teacher. She taught both the double dose class using the Algebra Readiness intervention
and the Algebra I class with the general education teacher in this pilot. She also taught
the single dose class for students identified with IEPs without the general education
teacher using only the Algebra Readiness curriculum. After the pilot project training
period, the teacher had ongoing contact with the student support team via emails to
discuss pacing, content, alternative methods of teaching and any other questions that might arise. In the tier two intervention, one group of students (approximately 50% with IEPs and 50% without IEPs) took both a general education Algebra I class and the Algebra Readiness Math intervention class, a double dose. In the tier three intervention group, all the students had been identified with a diagnosis of a disability that affects learning and took the Algebra Readiness intervention class only. For each group, the teacher presented whole class instruction in a fifty minute period using the text three days a week. For at least 90 minutes a week, students worked individually in the computer lab with the computer games.

The sequence of testing started with a MIND Research Algebra Readiness pretest during the first two weeks of school. Within the first six weeks of school the students took the Acuity A assessment, screening students’ readiness for the anticipated grade level content as defined by Colorado state content standards. In December and January, students took the Acuity B assessment. In February and March students took the CSAP—the state standards test. Within the last six weeks of school, the students took the Acuity C assessment, aligned with the summative assessment for the state grade level content standards (CSAP). Finally within the last two weeks of school, students took the Mathematics Diagnostic Testing Program (MDTP) post-test assessing the curriculum content knowledge. The teacher used weekly progress reports, embedded in the Algebra Readiness program, to inform instruction. In addition to the Algebra Readiness weekly progress data, the teacher used the Acuity assessments three times during the year to
monitor individual progress toward state standards. The aim of these probes and monitoring tools was to allow students enough practice time to master the skill. The students completed the state assessment (CSAP) in February/March and the Acuity C assessment in April before the end of the instruction. The Algebra Readiness curriculum-based post test (MDTP) was administered in May, at the end of the program. Student progress was evaluated using pre and post assessments and Acuity tests as well as CSAP scores. Times at which the data were captured are listed in Figure 3.

Figure 3

Assessment Process

Assessments/ Instruments

The assessments used to measure student growth were Acuity, the Mind Research Institute’s pre and post assessments and the Colorado State Assessment Program (CSAP). Acuity and CSAP assessments are based on Colorado grade level standards. Acuity
assessments are used as formative and predictive assessments, whereas the CSAP is used as a summative assessment of student progress towards standards. The Mind Research Institute’s pre and post assessments are based on specific Algebra skills as identified in The Mathematics Diagnostic Testing Project (MDTP).

**Colorado Student Assessment Program.** The Colorado Department of Education (2011), describes the Colorado Student Assessment Program (CSAP) as a measure of academic achievement for students across the state based on grade level standards. Model Content Standards are used as benchmarks for reporting performance levels. Model Content Standards were adopted in 1995 by the State Board of Education as expectations of what students are should know and be able to do at specific grade levels. Performance level descriptors are based on scaled scores. For ninth grade math the designations are as follows: Advanced Proficient (628 - 890), Proficient (577 - 627), Partially Proficient (521 - 576), and Unsatisfactory (310 - 520). CSAP provides overall information about student performance. It also breaks down standards and provides scores on specific content and sub content strands. According to the Colorado Department of Education (2011) technical reports, there are four content standards and three sub content standards for ninth and tenth grade math: 1) Number Sense and Computational Techniques, 2) Algebra Patterns and Functions, 3) Statistics and Probability and 4) Geometry and Measurement. The three sub content areas include: 1) Linear Pattern Representation, 2) Proportional Thinking, and 3) Geometry. The format of the math CSAP for ninth and tenth grade includes calculation and applied reasoning.
problems, as well as constructed response questions. Each constructed response is expected to take about 15 minutes, as the student may need to draw graphs, extend patterns or use spatial reasoning to solve and explain their reasoning (Colorado Department of Education, 2011). Each are scored using a 0-4 rubric. The ninth grade math test comprises 60 items with 45 multiple choice questions (four choices) and 15 constructed responses. Of the constructed responses 6 are worth 2 points, 6 are worth 3 points and 3 are worth 4 points. Students are allotted 65 minutes to complete each of the three sections of CSAP math questions. The tenth grade math assessment comprises 58 questions of which 43 are multiple choice and 15 are constructed response. Scale Score Correlations by Content Standard and Sub content Area on the 10th grade test range from .70 to .91 and .80 to .94 for the ninth grade assessment (CDE, 2011, Tables 212 & 213). All content standards and sub content areas are moderately to highly correlated, as would be expected. Using Cronbach’s alpha reliability coefficients for content standards, the internal consistency reliability estimate for the total score for the 9th grade test was .93 and for the 10th grade test it was .92. The sub content reliability estimates ranged from .39 to .78. A validity correlation coefficient of .96 from grade 9 score to grade 10 scores shows strong predictability between the Acuity assessment scores and the CSAP assessment scores (CSAP, Table 252). Divergent validity shows intercorrelations among math content areas by grade level (Tables 249, 250) from .75 in Reading and to .85 in Science. An IRT model was used to calibrate test items.
**Acuity.** The McGraw Hill Acuity tests are administered to all students as formative assessments three times a year. These assessments use a mix of computer-based and pen and paper assessments and take approximately 45 minutes to complete. Acuity assessments are used as universal screeners in the RTI process to help identify those at risk students who may extra support. The content of Acuity assessments are specifically aligned with Colorado State standards for each grade level and are predictive of student’s CSAP score (CTB/McGraw-Hill, 2008). The items in each of the three assessments are “specific to the time of assessment” and based on predicted progress toward the items on the CSAP summative assessment. The items are also in the same proportion as the CSAP content (CTB/McGraw-Hill, 2008).

Acuity questions are “developed according to state standards, pacing guides, scope and sequence, and other common state-related curriculum” (Acuity, 2009, p. 9). “Approximately 12–15 items are carried forward from assessment to assessment (form A to form B, form B to form C, and form C to form A of the next grade)” (CTB/McGraw-Hill, 2008, p. 3) with each assessment becoming more challenging as students progress through the course content (CTB/McGraw-Hill, 2010). Questions on Acuity A (taken in August/September) have more review content (i.e. there are common items for the 8th grade Acuity C and 9th grade Acuity A), whereas questions on Acuity B (taken in December/January) are aligned with the middle of the grade level content and Acuity C (taken in April/May) reflects year end grade level content and standards.
The math constructed response items have obtainable scores of two and three points in addition to the same format for the multiple choice questions making it comparable to the CSAP. The design closely reflects the structure and format of the CSAP assessments and is predictive of CSAP scores, the state level grade level standard assessment. The item format, content and length are all modeled on the CSAP with senior editors reviewing content for item difficulty. The reported Feldt-Raju form reliability estimates for the math assessments range from .84 to .87 (Colorado Acuity Technical Report, 2009, Appendix B, Table 7). An IRT model fit using theory calibration, scaling, and equating was also used.

**Mathematics Diagnostic Testing Project.** The MIND Research Institute’s Algebra Readiness pre and post tests were developed jointly by California State University and the University of California under the umbrella of the Mathematics Diagnostic Testing Project (MDTP). The MDTP was developed as a California statewide project to, among other things, measure student readiness for math coursework, in this case, Algebra I (MDTP, nd). The DASKALA Algebra Readiness pretest (MDTP), assesses a student’s preparation for Algebra I. It is typically given to students, “the year before or near the beginning of a first year algebra class,” (MDTP, 2012). Test scores are then compared to both teacher ratings of the students’ readiness for the next mathematics course and student performance on the diagnostic posttest given at the end of the elementary algebra course. An R-Biserial measure is used to measure the consistency of an item with the rest of the test, although this correlation was not reported. Content
validity of each item is also reviewed using the correlation of item student performance on the item with performance measures at the end of the course according to the manual. Item discrimination is evaluated by comparing the overall test performance of students who choose each available response and by comparing the performance of each of the five quintiles of students based on total test score with the performance of the other quintiles.

**Assessment Limitations**

Traditionally, students with identified learning disabilities show a lower level of response to intervention than those students who have no diagnosed disabilities (Fogen & Deno, 2001; Jordan, Kaplan, & Hanich, 2002; Tran, Sanchez, Arellano, & Swanson, 2011; Al Otaiba, & Fuchs, 2002). Based on studies of children in early grades who have participated in reading interventions, Fuchs, Marston, and Shin (2001) recommend using curriculum-based measures to establish growth for students with learning disabilities, recognizing the high stakes for schools who fail to show adequate growth for students. CSAP and Acuity are designed to assess grade level, Algebra I, standards as determined by the State of Colorado. Students in the tier three intervention will not have been exposed to the general Algebra I curriculum when taking these assessments. With this in mind, it is unlikely the CSAP and Acuity measures precisely captured the growth of students with disabilities participating in this intervention. The students in the double dose intervention had been exposed to Algebra I content and had an opportunity to reinforce their skills using the AR intervention, therefore, the CSAP and Acuity measures
more likely captured the effect of the intervention. The posttest as compared the pretest on the AR program most likely captured the effect of the intervention since these tests were designed to assess the growth based on the curriculum. Albeit, in a meta analysis of responsiveness to reading interventions for students with learning disabilities, Al Otaiba and Fuchs (2002) assert that even:

> defining unresponsiveness exclusively in terms of growth may also be problematic: Higher performing students may show little growth but demonstrate acceptable performance levels; lower performing students, including many with disabilities, may make relatively impressive growth but have an unacceptably low performance level. (p. 312)

They summarize that there is no typical non-responder, and expect, “that unresponsive students will be characterized by considerable intra- and inter individual variation, which would seem to defy precise applications of treatments to learner strengths and weaknesses” (p. 313).

The limitations in the measures of the growth of students with IEPs, based on the impact of RTI policies, is a relatively new area of research. Since this study used a state standards assessment as part of the analysis, it would be expected that these assessments may not accurately represent achievement of students with a demonstrated slow learning curve who have struggled for years without consistent, evidence-based interventions.

**Data Analysis Plan**

Five analyses were performed on the data. 1) Independent samples t-tests were used to compare the scores of students with or without an IEP and those in a double or single dose using MDTP posttest, Acuity C, and CSAP scores as the test dependent
variables. Grade and gender were also explored as factors. 2) An ANCOVA was used with the MR pretest as the covariate to test the differences between students in the double dose as compared to the single dose. This ANCOVA procedure was also used to compare the IEP groups—those with IEPs and those without. CSAP, Acuity C, and the MDTP posttest were used as the dependent variables. 3) Correlation coefficients were computed to identify any relationships among the independent and dependent variables. 4) The percentage change from pre to post test in individual topic areas was also examined. 5) The teacher’s input about the curriculum based on a brief interview was analyzed thematically.

**Research Questions:**

The research questions aim to explore the impact of the MIND Research Institute’s Algebra Readiness program by measuring impacts student learning using performance on CSAP, ACUITY C, and or MIND post test scores as outcome measures. More specifically,

1) What is the impact of the MIND Institute Algebra Readiness intervention on algebra skills for students who have struggled with or failed a pre-algebra or algebra course in their previous year of school?

1a) Does this intervention impact some groups of students more than others?

2) What is the impact of the MIND Institute’s Algebra Readiness intervention on math achievement for students who are receiving special education services via an IEP?
2a) Does the RTI tier of intensity have a differential impact on the achievement for these identified students? Correlation coefficients were calculated to explore the relationships among IEP status, dose, grade, and gender for each of the outcome measures.

Hypothesis 1: There is a statistically significant difference over time (pre to post, or pre to midterm to post) on i.CSAP, ii.ACUITY C, and iii.MIND posttest.

The mean scores of students in the different doses, with and without IEPs; were compared using pre and post test scores on the MDTP. T tests were used to calculate the difference between the pre and post test scores on the MDTP posttest, the CSAP, and the Acuity C test. Then an ANCOVA was used to compare groups (dose, IEP status) reflecting the effect of the intervention on performance. Independent variables include exposure to Algebra Readiness as double dose, solo dose, and IEP status. ANCOVA assesses significance of group differences while controlling for the effect of the MDTP pretest. The significance and effects of IEP status and tier of intervention (dose) on dependent variables was explored. Independent variables included exposure to Algebra Readiness as double dose, solo dose, IEP Status, Gender, and Grade (9th or 10th). Dependent variables included CSAP performance, ACUITY C, and MIND Research Algebra Readiness assessments. Groups examined are shown in Table 8.
<table>
<thead>
<tr>
<th>Variables</th>
<th>Male</th>
<th>Female</th>
<th>Single Dose</th>
<th>Double Dose</th>
<th>IEP</th>
<th>No IEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>9th</td>
<td>13</td>
<td>6</td>
<td>6</td>
<td>13</td>
<td>12</td>
<td>7</td>
</tr>
<tr>
<td>10th</td>
<td>5</td>
<td>3</td>
<td>8</td>
<td>0</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>Totals</td>
<td>18</td>
<td>9</td>
<td>14</td>
<td>13</td>
<td>20</td>
<td>7</td>
</tr>
</tbody>
</table>

Students with IEP = 20; Students without IEP = 7; Double Dosed students = 13; Students with IEPs and DD = 6; Students without IEPs and DD = 7; Students Solo Dose = 14; Grade 9 = 19; Grade 10 = 8; Male = 18, Female = 9
Chapter IV: Results

Introduction

This chapter presents the findings of the study. It outlines the descriptive statistics, explains the tests of assumptions, and presents the results of the data analysis using t tests, ANCOVA, and correlations, among other statistical methods. Mastery scores for the strand areas of the curriculum for each class are also detailed.

Both t-tests and analysis of covariance (ANCOVA) were the main statistical analyses used to address the research questions. First a diagnostic analysis was used to test underlying assumptions including the following: normality of distributions, homogeneity of variance, homogeneity of regression, and linearity of dependent variables and covariates with the following results.

Descriptive Statistics and Test of Assumptions

Table 9 summarizes the basic descriptive statistics for each of the main variables of interest in this study. It was noted that generally the skewness and kurtosis coefficients were negative. The normality of the data distribution was further assessed using the Kolmogorov-Smirnov test (see Table 10). When broken down into the IEP and non-IEP groupings, results indicated that the Math CSAP scores and the MDTP post-test scores
were approximately normally distributed. Acuity and MDTP pre-test scores had indications of non-normal distributions. However, even though both t-tests and ANCOVA are considered robust to violations of normality with a sufficient number of cases, in this study only five students were without an IEP which affected the distribution. As long the assumption of normality is not severely violated, the actual Type I error rates approximate minimal rates for t tests (Stevens, 2002). However when data are severely platykurtic, power is reduced in t tests and ANOVA-type tests (Stevens, 2002). Based on the distributions in the current study, results are interpreted with caution.

Homogeneity of variance was assessed using Levene’s test. The variance of the IEP and non IEP scores across the dependent variables was tested and the results are summarized in Table 11. Results indicated that Acuity and MDTP pre and post-tests scores met the assumption of homogeneity of variance. The Math CSAP had a slight violation. This violation to homogeneity of variance (as indicated by the Levene’s test), was addressed by using an adjusted t value. This adjusted t provides a correction factor in the computation to account for the potential bias of unequal variances between the two groups being compared.
Table 9

Descriptive Statistics - Assessments

<table>
<thead>
<tr>
<th>Variables</th>
<th>n</th>
<th>M</th>
<th>SD</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math CSAP</td>
<td>19</td>
<td>524.84</td>
<td>47.96</td>
<td>.37</td>
<td>.52</td>
</tr>
<tr>
<td>Acuity</td>
<td>17</td>
<td>489.82</td>
<td>61.42</td>
<td>-1.07</td>
<td>.132</td>
</tr>
<tr>
<td>MR Pre-test</td>
<td>22</td>
<td>15.59</td>
<td>4.77</td>
<td>.04</td>
<td>9</td>
</tr>
<tr>
<td>MR Post-test</td>
<td>23</td>
<td>21.17</td>
<td>7.06</td>
<td>.09</td>
<td>.48</td>
</tr>
</tbody>
</table>

Table 10

Summary of the Kolmogorov-Smirnov Test Results

<table>
<thead>
<tr>
<th>Variables</th>
<th>Groupings (based on IEP Status)</th>
<th>Statistic</th>
<th>df</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math CSAP</td>
<td>With IEP</td>
<td>.146</td>
<td>11</td>
<td>.200</td>
</tr>
<tr>
<td></td>
<td>Without IEP</td>
<td>.198</td>
<td>5</td>
<td>.200</td>
</tr>
<tr>
<td>Acuity C</td>
<td>With IEP</td>
<td>.158</td>
<td>11</td>
<td>.200</td>
</tr>
<tr>
<td></td>
<td>Without IEP</td>
<td>.424</td>
<td>5</td>
<td>.004</td>
</tr>
<tr>
<td>MDTP Pre-test</td>
<td>With IEP</td>
<td>.187</td>
<td>11</td>
<td>.200</td>
</tr>
<tr>
<td></td>
<td>Without IEP</td>
<td>.473</td>
<td>5</td>
<td>.001</td>
</tr>
<tr>
<td>MDTP Post-test</td>
<td>With IEP</td>
<td>.196</td>
<td>11</td>
<td>.200</td>
</tr>
<tr>
<td></td>
<td>Without IEP</td>
<td>.211</td>
<td>5</td>
<td>.200</td>
</tr>
</tbody>
</table>
Table 11
Summary of Levene’s Test Results

<table>
<thead>
<tr>
<th>Variables</th>
<th>Levene Statistic (df = 1, 14)</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math CSAP</td>
<td>4.730</td>
<td>.047</td>
</tr>
<tr>
<td>Acuity</td>
<td>.022</td>
<td>.884</td>
</tr>
<tr>
<td>MR Pre-test</td>
<td>1.871</td>
<td>.193</td>
</tr>
<tr>
<td>MR Post-test</td>
<td>2.480</td>
<td>.138</td>
</tr>
</tbody>
</table>

Since analysis of covariance (ANCOVA) was used to test some of the hypotheses, the assumption of homogeneity of regression and linearity between the covariate and dependent variables was also tested.

**Homogeneity of Regression**

To test the assumption of homogeneity of regression, a univariate ANOVA was conducted with the MR post-test as the dependent variable, and IEP status and the interaction between IEP status and MDTP pre-test as independent variables. The interaction between IEP status and MDTP pre-test was not significant ($F(1, 18) = .009, p = .93$). This indicated that the assumption of homogeneity of regression was not violated.

**Linear Relationship Between Covariate and Dependent Variable**

To check for a linear relationship between the covariate and the dependent variable, a scatterplot was generated. The grouped scatterplot between the MDTP post-test and MDTP pre-test indicated that the slopes of the regression lines were roughly parallel (see Figure 4). This is an indication that the assumption was met.
Correlations

The correlation between Dose and Acuity C was statistically significant, $r = .52, p < .05$ (see Table 12).
Table 12
Correlations between Variables of Interest

<table>
<thead>
<tr>
<th></th>
<th>Math CSAP 10-11</th>
<th>Acuity C 10/11</th>
<th>MR Pretest /45</th>
<th>MR Posttest</th>
<th>Grade</th>
<th>Gender 1F</th>
<th>IEP Status</th>
<th>Dose</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math CSAP 10-11</td>
<td>1.00</td>
<td>.37</td>
<td>.70**</td>
<td>.83**</td>
<td>-.17</td>
<td>.41</td>
<td>.22</td>
<td>.42</td>
</tr>
<tr>
<td>*Acuity C 10/11</td>
<td>.37</td>
<td>1</td>
<td>.23</td>
<td>.38</td>
<td>-.55*</td>
<td>.17</td>
<td>.09</td>
<td>.51*</td>
</tr>
<tr>
<td>MDTP Pretest /45</td>
<td>.70**</td>
<td>.23</td>
<td>1</td>
<td>.86**</td>
<td>-.07</td>
<td>.25</td>
<td>-.10</td>
<td>.31</td>
</tr>
<tr>
<td>MDTP Posttest</td>
<td>.83**</td>
<td>.38</td>
<td>.86**</td>
<td>1.00</td>
<td>-.20</td>
<td>.15</td>
<td>.07</td>
<td>.40</td>
</tr>
<tr>
<td>Grade</td>
<td>-.17</td>
<td>-.55*</td>
<td>-.07</td>
<td>-.20</td>
<td>1.00</td>
<td>-.10</td>
<td>-.43*</td>
<td>.57**</td>
</tr>
<tr>
<td>Gender 1F</td>
<td>.40</td>
<td>.17</td>
<td>.25</td>
<td>.15</td>
<td>-.10</td>
<td>1.00</td>
<td>-.25</td>
<td>.18</td>
</tr>
<tr>
<td>IEP Status</td>
<td>.22</td>
<td>.09</td>
<td>-.10</td>
<td>.07</td>
<td>-.44*</td>
<td>-.25</td>
<td>1.00</td>
<td>.52*</td>
</tr>
<tr>
<td>Dose</td>
<td>.42</td>
<td>.52*</td>
<td>.31</td>
<td>.40</td>
<td>-.57**</td>
<td>.18</td>
<td>.52*</td>
<td>1.00</td>
</tr>
</tbody>
</table>

**p < 0.01 level (2-tailed).
*p < 0.05 level (2-tailed).

Math CSAP 10/11 and the MDTP post test were highly correlated, $r = .83$, $p < .01$. The MDTP pretest and the CSAP 10/11 were moderately correlated, $r = .70$, $p < .01$.

The MDTP pretest and the MDTP posttest were highly correlated, $r = .86$, $p < .01$. Grade and Acuity C were moderately negatively correlated ($r = -.55$, $p < .05$). As students reached tenth grade their Acuity C score tended to be lower. There were no 10th grade students in the double dose indicating that by the time the students reached 10th grade they were more impacted and received Special Education services. They were not exposed to the Algebra curriculum tested by Acuity C.
Dose and Acuity C were moderately related \((r = .52, p < .05)\). Students in the double dose condition were exposed to an Algebra I curriculum in addition to the Algebra Readiness program and tended to score higher.

Dose and grade were moderately negatively correlated \((r = -.57, p < .05)\) and significant at the .01 level. No 10th grade students were in the double dose condition.

Dose and IEP status were moderately correlated \((r = .52)\) and significant at the .05 level. Only students with IEPS were in the single dose condition.

**DASKALA Data**

DASKALA is the computer program that presents the MDTP tests online and organizes data details of students’ individual and group performances. Using the MDTP pre and post test data, the percentage change in the number of students who reached critical levels of items correct was calculated. The results are noted in Table 13. The critical level for each topic is, “the minimum number of correct responses to show adequate preparation in that area” (MDTP, 2012).
Table 13  
*Topic Strands in the MDTP Test*

<table>
<thead>
<tr>
<th>Topic</th>
<th>Pre—number of students at critical level</th>
<th>Post—number of students at critical level</th>
<th>Percent Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>DAPR</td>
<td>9</td>
<td>13</td>
<td>44% increase</td>
</tr>
<tr>
<td>DECM</td>
<td>1</td>
<td>7</td>
<td>600% increase</td>
</tr>
<tr>
<td>EQTN</td>
<td>9</td>
<td>11</td>
<td>22% increase</td>
</tr>
<tr>
<td>EXPS</td>
<td>1</td>
<td>9</td>
<td>800% increase</td>
</tr>
<tr>
<td>FRAC</td>
<td>2</td>
<td>2</td>
<td>No change</td>
</tr>
<tr>
<td>GMMS</td>
<td>8</td>
<td>10</td>
<td>25% increase</td>
</tr>
<tr>
<td>GRPH</td>
<td>3</td>
<td>7</td>
<td>133% increase</td>
</tr>
<tr>
<td>INTG</td>
<td>5</td>
<td>4</td>
<td>20% decrease</td>
</tr>
</tbody>
</table>

Note: DASKALA Data Abbreviations: Data, Analysis, Probability & Statistics: DAPR; Decimals their Operations and Applications: Percent: DECM; Simple Equations and Operations with Literal Symbols: EQTN; Exponents and Square Roots; Scientific Notation: EXPS; Fractions and their Applications: FRAC; Measurement of Geometric Objects: GMMS Graphical Representation: GRPH; Integers, their Operations, and Applications: INTG.

The three topics that showed the greatest improvement in number of students reaching critical levels of items correct were in the Decimals: Their Operations and Applications; Percent, Exponents and Square Roots; and Scientific Notation Scores and Graphical Representations cluster areas. Other topics showed more moderate increases and Fractions and their Applications showed no change.

**Results of Analyses**

In order to evaluate the effectiveness of the Algebra Readiness curriculum, t-tests and ANCOVA were used to compare the scores of the students in the different groupings - those with and without IEPs and those exposed to the double dose versus the single dose. These tests answered the research questions: What is impact of the MIND Institute’s Algebra Readiness intervention on math achievement for students who are receiving special education services via an IEP? Does the RTI tier of intensity have a differential impact on the achievement for these identified students?
The Math CSAP mean score of the IEP group was lower (M = 518.86, SD = 54.23) compared with the non-IEP group (M = 541.60, SD = 17.73) but the difference was not statistically significant, \( t(17) = -1.377, \ p = .19 \). Cohen’s \( d \) was 0.48 indicating a moderate effect size. Note that the sample sizes were uneven: IEP group (n = 14) and non-IEP group (n = 5) and equal variance was not assumed based on the Levene’s test result \( F = 5.105, \ p = .04 \). An adjusted \( t \) value was accordingly used considering that the homogeneity of variance assumption was not met.

The comparison of the Math CSAP scores between the single dose (M = 501.75, SD = 63.98) and the double dose group (M = 547.64, SD = 23.18) indicated that the difference was not statistically significant, \( t(17) = -1.685, \ p = .13 \) (See Table 14). Cohen’s \( d \) was 0.95 indicating a large effect.

Table 14

<table>
<thead>
<tr>
<th>Variable</th>
<th>IEP</th>
<th>Non-IEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math CSAP</td>
<td>M 518.86</td>
<td>M 541.60</td>
</tr>
<tr>
<td>Acuity C</td>
<td>M 486.33</td>
<td>M 498.20</td>
</tr>
</tbody>
</table>

The Acuity C scores of the IEP group were lower (M = 486.33, SD = 59.52) compared with the non-IEP group (M = 498.20, SD = 72.30). The difference was not statistically significant, \( t(15) = -.353, \ p = .73 \). Furthermore, Cohen’s \( d \) value (\( d = .18 \)) suggests a small effect size. Note that the sample sizes were uneven: IEP group (n = 12) and non-IEP group (n = 5).
The comparison of the Acuity C scores, however, indicated that the double dose group (M = 512.55, SD = 52.94) had a higher mean score compared to the single dose (M = 448.17, SD = 57.02). The difference was statistically significant, \( t(15) = -2.335, \ p = .034 \) (see Table 15). Furthermore, Cohen’s effect size value, \( d = 1.17 \) suggests potential practical significance (Cohen, 1988).

Table 15

<table>
<thead>
<tr>
<th>Grouping</th>
<th>Single dose</th>
<th>Double dose</th>
<th>t</th>
<th>df</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math CSAP</td>
<td>501.75</td>
<td>541.64</td>
<td>-1.68</td>
<td>8</td>
<td>0.13</td>
</tr>
<tr>
<td>Acuity</td>
<td>448.17</td>
<td>512.55</td>
<td>-2.33</td>
<td>15</td>
<td>0.05</td>
</tr>
</tbody>
</table>

**ANCOVA**

Results indicated that the adjusted mean scores after statistically removing the effect of the covariate indicated that the non-IEP group had a higher mean MDTP Post-test score (M = 22.94, SE = 1.37) compared to the mean score of the IEP group (M = 19.77, SE = .84). However, after adjusting for pre-test scores, the effect of the IEP was not statistically significant, \( F(1,19) = 3.88, \ p = .064 \), partial \( \eta^2 = .17 \) (See Table 16).
Table 16
*Summary of ANCOVA (IEP Grouping) on MDTP Posttest*

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>p</th>
<th>partial $\eta^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MR Pre-test</td>
<td>1</td>
<td>722.30</td>
<td>64.90</td>
<td>&lt;.001</td>
<td>.774</td>
</tr>
<tr>
<td>IEP</td>
<td>1</td>
<td>43.22</td>
<td>3.88</td>
<td>.064</td>
<td>.170</td>
</tr>
<tr>
<td>Error</td>
<td>19</td>
<td>11.13</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Dependent Variable: MDTP Post-test

Similarly, analysis indicated that the double dose group had higher MDTP post-test scores ($M = 21.30, SE = 1.06$) compared to the scores of the single dose group ($M = 19.84, SE = 1.17$). However, after adjusting for pre-test scores, the effect of the dose was not statistically significant, $F(1, 19) = .813, \ p = .378, \ partial \eta^2 = .04$ (See Table 17).

Table 17
*Summary of ANCOVA (Dose Grouping)*

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>p</th>
<th>partial $\eta^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MR Pre-test</td>
<td>1</td>
<td>577.454</td>
<td>44.927</td>
<td>.000</td>
<td>.703</td>
</tr>
<tr>
<td>Dose</td>
<td>1</td>
<td>10.453</td>
<td>.813</td>
<td>.378</td>
<td>.041</td>
</tr>
<tr>
<td>Error</td>
<td>19</td>
<td>12.853</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Dependent Variable: MDTP Post-test

The results of these tests suggest that there was no significant difference in the scores between the students who were double dosed and the students who had the single dose controlling for pretest scores. Also, the students with IEPs did not show a significant difference when compared to students without IEPs. It is noted that the comparison between students with IEPs and those without IEPS was unbalanced and this small sample size may not capture true differences.
Summary of Study/Results

In summary, the following research questions were addressed and results reported:

Research Questions:

Did the MIND Institute’s Algebra Readiness have an impact on high school students’ scores on standardized grade level tests? Did the intensity of exposure to math curriculum and intervention have a differential impact on student achievement? Was there a differential impact for students identified with learning disabilities versus those without an identified learning disability?

In short, no significant growth was detected on CSAP, Acuity C, and the MDTP posttest. None of the measures showed a significant difference between students with IEPs and those without IEPs. Students with the double dose of math showed a significant difference on the Acuity C measure as compared to students who only participated in the single dose condition. Those students who participated in the double dose condition had significantly higher scores on Acuity C and the results indicate a large effect size. Grade level was significantly correlated with the Acuity C assessment. Except for the fraction strand, students showed increases, as measured by the number of students who reached critical levels for correct answers, in all concept strands. The teacher reported that students reacted positively to the intervention and noted concerns about students would generalize the material to other contexts.
The results of this study were inconclusive. Although the analysis indicated that generally the curriculum was not effective, the DASKALA data critical levels indicated student growth. Moderate to large effect sizes for the students in the double dose groups on both the CSAP and the Acuity C measures indicate that the Algebra Readiness intervention program may have some potential practical significance. The power analysis suggests that the small data set likely influenced the non-significant outcome. The small data set was not powerful enough to find a significant effect. The students who participated in this study, those with and without an IEP, did not demonstrate measurable differences in their math achievement. Those students who were offered the Algebra I curriculum in addition to the AR intervention performed significantly better on the Acuity C assessment which measured end of year grade level standards. This was the only significant difference found. The double dosed students performed better than the single dosed students on the Acuity C measure that predicts their eventual CSAP score. Perhaps if CSAP had been given at the end of year, an effect might have been found.
Chapter V: Discussion

Introduction

This chapter presents an overview of the problem. It also provides a summary of the study, analyzes and discusses the results, and draws conclusions about the implications of RTI, math assessments and interventions. It concludes with recommendations for future studies.

Overview of the Problem

Learning math is a problem for many students, not just those diagnosed with learning disabilities. According to national and state assessments, student with and without disabilities continue to show persistent deficits in math skill development (NCES, 2011; CSAP, 2011; National Council on Disability, 2004). Many of these students, who have trouble negotiating the generalization of arithmetic skills into more abstract thinking about numbers, are not identified with disabilities (NCES, 2011). Part of this is because researchers disagree about precise standards for identifying a student with a math disability (Geary, 2007; Mazzocco, 2005; Murphy et al., 2007), although most agree about core cognitive deficits that impact math learning (Bull, 1999; Geary, 1993; Geary & Hoard, 2001; Geary & Hoard, 2005; Hale & Fiorello, 2004; Mazzocco, 2005; Murphy et al., 2007; Swanson, 2006). Learning math is enhanced by strong visual spatial
skills (Gunderson, Ramirez, Beilock, & Levine, 2012; Geary & Hoard, 2005; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; National Research Council, 2006; Van Garderen, 1999) as well as adequate development of executive functioning skills (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Geary, 2007). Research indicates best practices for designing math interventions include a visual design (Bouck & Flanagan, 2010; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; National Research Council, 2006) and a graduated sequence of instruction (Maccini & Hughes, 2000; Witzel, 2005; Fuchs, Fuchs & Hollenback, 2007) as integral parts of the math curriculum. Learning algebra also requires a shift in thinking (Wu, 2009; Osta & Labban, 2002; Thomas & Tall, 2001; Pillay, Wilss, Boulton-Lewis, 1998; Herscovics & Linchevski, 1994). Difficulty mastering this shift combined with the impact of poor preparation in early math skills (NAEP, 2011; Helfand, 2006; Rumberger, 2001; Battin-Pearson, 2000; Roderick & Camburn, 1999; Vallerand, Fortier, & Guay, 1997) are two factors that often inhibit student math development.

Algebra, in particular, is a gatekeeper for the higher level math courses (Mazzolo, 1995; Stinson, 2004) and is difficult for many students to master. Failure rates for first year algebra students are high (Roderick & Camburn, 1999; Helfand, 2006; NCES, 2011) and contribute to student dropout rates (Battin-Pearson, 2000; Rumberger, 2001; Vallerand, Fortier, & Guay, 1997). Students who do not complete four years of math in high school are at a disadvantage in their college studies and in their professional lives. Higher level math skills are important for success in school and careers (ACT, 2006;
Abstract, mathematical problem-solving, as taught in Algebra I and higher level math classes, is important for success in school and in the workplace (ACT, 2006; Nord, Roey, Perkins, Lyons, Lemanski, Brown, & Schuknecht, 2011; Terrell, 2007).

NCLB (2002) holds schools accountable for all student progress using state mandated tests as the measure of performance. The revised IDEA (2004), validated RTI as an accepted method of identifying and providing services for students with and without disabilities. RTI is a productive way to support students quickly without waiting for significant skill gaps to arise (Bryant, Bryant, Gersten, & Scammacca, 2008; Chaim, Lappan, & Houang, 1988; Clements & Sarama, 2007). It changes how, when and which students access school-based interventions. Inherent in the RTI framework, is access to effective research-based interventions which increases the need for more study of potential math programs. Students who fail at math have fewer systematic, structured interventions as compared to reading interventions (NCES, 2012; Geary, Hamson & Hoard, 2000; Mazzocco, 2001). Developing research-based interventions will help students master math skills associated with increased higher level math performance.

Using evidence-based interventions and methods are best practice in providing students with math programs that work (NCLB, 2001). This research aimed to study the effectiveness of the MIND Research Algebra Readiness program with high school students. The Algebra Readiness program aligns with many of the best practices as noted in the National Research Council guidelines. It incorporates visual number lines in the
lessons and presents abstract concepts using a visual representations and manipulations into the design of the intervention thereby reinforcing spatial skills to help with mathematical problem solving. High school students with and without IEPs, participated in this program in two tiers of intensity and the results of their performance on standardized tests were measured and compared. The students who participated in this study all scored below proficient on their previous year’s math CSAP. They were scheduled in this class either as an alternative to their required grade level math class or in addition to their grade level math class. This natural setting using the standardized tests was designed to reflect the true learning environments of student in any high school math class.

**Summary of the Study and Analyses**

The aim of this study was to research the effectiveness of the MIND Research Institute’s Algebra Readiness program using two groups of ninth and tenth grade students in two tiers of intensity. The hypothesis was that exposure to this intervention would show significant results on the tests scores. The students who participated in either level of intensity, did not significantly improve their scores on standardized measures after using the curriculum for a school year. The students in the double dose group did perform significantly better than those in the single dose on the Acuity C measure. In addition, moderate to large effect sizes in the double dose groups indicate potential practical usefulness.
Previous research (Tran, Sanchez, Arellano, & Swanson, 2011; Al Otaiba, & Fuchs, 2002; Jordan, Kaplan, & Hanich, 2002; Fogen & Deno, 2001) indicates that students with math disabilities show slower growth when exposed to interventions. Also RTI research (Fuchs, Fuchs and Hollenback, 2007; Bryant, 2008) indicates that early interventions are more beneficial in ameliorating math difficulties. The students who participated in this study were older (high school age) and had experienced math failure in their previous coursework. They did not significantly improve their scores on any of the standardized measures, although the students in the double dose group significantly improved over the students in the single dose group on one measure. Possibly the double dosing helped impact the scores of the level one students. The AR program provided reteaching and reinforcement of basic math skills and the Algebra I curriculum kept students up to date with grade level concepts. The small sample size likely affected the outcome since the power to detect a true difference was lacking.

Conducting a post hoc power analysis using G*Power, (Faul, Erdfelder, Lang, & Buchner, 2007) revealed that when using the means, between-groups comparison effect size observed in the Acuity C assessment, which was the only test to show significance, with a partial $\eta^2$ of .17 for IEP status and .04 for dose, indicates that approximately 84 participants would be necessary to reach Cohen’s (1988) recommended .80 level.

The assessments used may not have been sensitive enough to capture the growth of the math skills. It is hard to delineate if the lack of significant effect was because of the
small sample size, the lack of exposure to critical concepts for such a long time or if the visual program was just not effective.

**Summary of the Study**

Two groups of ninth and tenth grade high school students, who historically have struggled in math, participated in a math intervention as part of their daily schedule for one school year. The students were exposed to the intervention using two tiers of intensity. One group of students (the tier two group) took a double dose of math daily, the Algebra Readiness intervention class and a general curriculum Algebra I class. The other group of students took only the Algebra Readiness class daily. The performance of the students, as measured by pre and post test scores, progress monitoring probes, and state assessments was analyzed. The results showed that the students in the double dose group scored significantly higher on the end of year probe, Acuity C, than the students in the single dose class. The other measures did not detect any significant growth.

**Results Summary**

1. **Did any group or condition show significant growth as measured by Acuity C, CSAP, or the MDTP post test scores as compared to the pretest scores?**

   No significant result was found. Typically, special education students show significantly slow growth (Al Otaiba & Fuchs, 2002; Fogen & Deno, 2001; Jordan, Kaplan, & Hanich, 2002; Tran, Sanchez, Arellano, & Swanson, 2011). In addition, for students who initially scored far below a cutoff point, grade level tests may not be sensitive enough to
capture their growth. Without exposure to key concepts of the Algebra curriculum, as tested on all three assessments, the students in the single dose group were at a significant disadvantage.

2. Did Math CSAP, Acuity C or the MDTP posttest scores significantly differ between the groups with IEP and without IEP? Using t tests to compare the IEP group and the non IEP group using the Math CSAP and Acuity C scores as the dependent variables, no significant results were found. Using ANCOVA to control for the effect of the MDTP pretest with the MDTP posttest, no significant differences were found between the groups with IEP and without IEP. This is verified in the research about how difficult it is to identify students with a math disability based on the disagreement about what constitutes a math disability (Mazzocco, 2005; Murphy et al., 2007). Only five students without a diagnosed disability were used as a comparison group and this was likely too small to detect a significant effect. The educational casualties, those students who had been removed from the general curriculum years before with little to no exposure to grade level content, were now to be assessed on a grade level assessment. In addition, their scores would be used in a formula to calculate the schools progress towards standards as written in the NCLB Act. It suggests that students exposed to a double dose of math intervention and grade level curriculum (not instead of) may increase performance given grade level expectations. The effect size suggests this may have practicality as an intervention. Implementing another intervention year with a
greater number of students would more definitively answer the research questions left unanswered by the small data set.

3. **Did Math CSAP, Acuity C or the MDTP posttest scores significantly differ between the single dose and the double dose group?** Using t tests to compare the CSAP scores between the scores of students in the double dose and the single dose groups indicated no significant differences. Using t tests to examine the Acuity C scores of students in the single dose and the double dose group, \( p = .034 \) indicated that the double dose group scores were significantly higher compared to the single dose group. In addition the effect size \( (d = .52) \) indicated potential practical significance for the intervention. Using ANCOVA, the MDTP pre to post test differences were not significant. CSAP scores measure grade level math skills, students who score far below the cutoff point for proficiency may not show enough growth on such a measure. However, Acuity C scores were significantly higher for the double dose group as compared to the single dose group. Students in the double dose group were exposed to the general Algebra I curriculum which likely improved their scores on the Acuity C assessment because it measures grade level content—Algebra I, not pre-algebra skills. So the level of intensity did make an impact on scores.

4. **Did any of the correlation coefficients show significant relationships among IEP status, dose, grade, and gender for each of the outcome measures.** Acuity C appears to adequately measure grade level standards (Table 12). This is important to understand because students performed significantly better when
participating in the double dose condition as opposed to those in the single intervention dose. Acuity C measures grade level content at the end of the term and it predicts future CSAP scores so perhaps with increased exposure to the math curricula, students will show growth on next year’s CSAP. Student growth was detectable on the Acuity C when given at the end of the semester as opposed to the MDTP. Since the MDTP posttest was aligned with a first year algebra I curriculum, it suggests that the CSAP measures more than just an algebra I curriculum.

The findings indicate that although students did not show significant differences on their grade level assessments, the students who participated in the double dose program did outperform their peers who only took the single dose of Algebra Readiness. The low sample size prevented more analyses of progress and likely impacted the significance testing. Moderate to large effect sizes on the CSAP and the Acuity C assessment suggest that given 57 more participants in a larger study, significance would likely be detected.

Limitations

Limitations of the study need to be acknowledged. Foremost, the design of the study did not include a control group. Second, this study had a very small number of participants, which created insufficient power to reject the null. The timing of the assessments was problematic with the summative CSAP assessment administered before the program was completed. Also, the assessment measures may not have been sensitive enough to capture the slow learning curves of the participating students. Another
limitation was that this intervention relied heavily on visual tools to explain the content. This may not have been an appropriate intervention for some students. Finally, students profiles of student strengths and weaknesses were not identified in this study, nor were histories of past interventions collected.

**Design.** The design of this study was a limitation in that there was no control group to compare to the participants in the study. It also did not have a true experimental design with random sampling and selection, the “gold standard” of effective research. The students who elected to participate also represented a restricted population and were more impacted by poor math performance than the typical ninth or tenth grade student. Also the groups were very unbalanced for dosing, and IEP status. There were not enough students without an IEP who completed the entire program. These students were underrepresented in the data, possibly because they did not have the built in support of and consultation services of a special education case manager. Some opted to take a different elective, or did not take the assessments. Possibly two math classes a day was too burdensome and the payoff for the student was too far in the future. Since math credit was not given for this intervention students may have given up an opted to take a summer “crash” course in Algebra1 to earn their math credit more easily. This limited the strength of the study and makes the results hard to generalize. This is a typical challenge in field-based research.

**Power.** The small sample size and unbalanced groups likely had an effect on the outcomes. The small data set confounded some of the results and several analyses
originally planned could not be conducted due to the low number of participants. Some of the original students did not complete the course while others were added to the class and did not experience the full curriculum. This significantly reduced the data set and possible analyses. This also decreased the possibility that the scores would reach a significant level. Two statistical fundamentals make this so, power and statistical significance. Finding significant effects with such a small sample size is difficult. Statistical power is the probability of rejecting the null hypothesis when the alternative hypothesis is true. This is the probability that a researcher will find a legitimate relationship that is statistically significant. “The power of a statistical test is reduced by (among other things) small sample size, a weak underlying relationship, or measures that are clouded by error (low reliability),” (Goodhue, Lewis, & Thompson, 2006, p. 3). With the unbalanced design and low sample size, not enough power was present to detect significant effects. If for instance the sample size n was increased by approximately 57 more students, a significant effect may have been detected. Notwithstanding, the power analysis indicated that the effect size for those in the double dose on the Acuity C variable was large and the effect size on the CSAP assessment was moderate.

One would expect that the students in the double dose group who were exposed to both the intervention and the typical algebra class (two math classes a day) would demonstrate more knowledge of the algebra concepts, and this did appear on the Acuity C measure with a significant difference detected between the students in the double dose group as compared to the students in the single dose group. Still the students in the
double dose group did not show a significant increase in their scores. The data were very limited in that only 16 scores were available for Acuity C 10/11. Obtaining a change score by using the Acuity C 9/10 scores would reduce the data points to only 12 and only five would belong to the no IEP group. Since this would limit the sample for all other comparisons this change could not be assessed. The same issue arose with the CSAP scores. So not only would the sample size be significantly reduced, but the sample size distribution between those students with IEPs versus those students without IEPs would be significantly uneven. This limited sample size and unbalanced groups made the overall power weak and therefore the results are interpreted with caution.

**Timing.** The study showed that students who had the general Algebra curriculum along with the intervention had more growth than those with only AR, on the Acuity C assessment. The Acuity C test was administered in April, after the CSAP, which was given in February/March. The results of the analysis show no significant differences between groups who participated in the two levels of this intervention on the MDTP posttest or the CSAP. The difference between the Acuity C measure and the CSAP was one of timing. Since the Acuity C test was given in April and the CSAP was administered in February/March, the students had time to complete more of the AR program and the Algebra curriculum. The Acuity C test, based on grade level standards, not only measures the end of year growth for a typical 9th and 10th grade student, but it predicts the CSAP score (Acuity, 2009), although the predicted test had already been taken. The students who were exposed to both the AR program and the general Algebra I
curriculum did show significantly more growth than the students who participated in the AR program alone. Timing does not explain the lack of progress on the MDTP posttest, however. The single dose students did not show significant growth on any measure, even on the year end algebra posttest. Perhaps the students who participated in this study demonstrated a “floor effect”. Since nine to ten school years is a long time to wait for a systematic, well-planned, consistent intervention, perhaps students given this intervention earlier may benefit and reach grade level standards earlier.

**Sensitivity.** For those who participated in the single dose condition of this study, these tests may be measuring a standard far above the typical student with learning disabilities, a “floor effect”. The content of the CSAP was just too challenging for the students therefore, it was unable to meaningfully discriminate patterns of growth. Also, for special education students whose school careers have been focused on remediation, not grade level materials, and who may have had limited access to appropriate interventions, grade level standards may not accurately reflect their growth (Allbritten, Mainzer & Ziegler, 2004). The standards tests may did not detect significant differences, which may indicate a lack of sensitivity in the measure. This is concerning because with the implementation of NCLB, all students need to be at grade level by 2014 and students with identified disabilities are included in the analysis for each school and district. The scores of the students with disabilities are also analyzed to determine if the school and school district make Annual Yearly Progress (AYP). If students perform so far below the benchmark on each assessment it is difficult to discern even modest growth using such
broad, normative instruments. This creates an impossible situation for students and schools (Allbritten, Mainzer, & Ziegler, 2004). For diverse learners, the use of more tailored assessments, rather than the traditional state, grade level assessments, needs to be explored. Using more specific measures of pre and post test growth will more likely capture the true effects of this curriculum.

**Slow Learning Curve:** The assessment data may reflect the acknowledged slow learning curve of students with disabilities. Since the students all started below grade level, they did not make enough progress to show measureable growth on the grade level standardized assessments (CSAP and Acuity). Slower acquisition and integration of mathematical concepts because of processing deficits typically found in students with learning disabilities, could possibly explain the results of the MDTP posttest. Other studies have concluded that the MIND Research ST Math intervention, which uses the same games as the Algebra Readiness intervention, shows promise as an effective program for elementary and middle school students (Graziano, Peterson & Shaw, 1999; Rutherford, Kibrick, Burchinal, Richland, Conley, Osborne, & Martinez, 2010). In the preliminary phase of the Rutherford et al. (2010) ST Math study in Orange County, California, researchers reported initial regression results show an “effect for ST Math that is significant at the .05 level with an average effect size of .37” (p. 17). In a more powerful and controlled study, using more sensitive measures, a similar effect with the Algebra Readiness program may more easily be found.
**Content.** Based on the research about the types of math learning disabilities, perhaps the students in this sample did not have significant visual spatial interference. Perhaps this sample of students had processing deficits that were more impacted by verbal/oral language processing or memory or something other and a visual spatial deficit. As with any intervention, it works best when matched to the deficit to address.

**Past Interventions.** It is not documented what interventions these students had been exposed to in the past, if any nor at what grade the students in the tier three class were removed from the general education math curriculum. Lack of exposure to the general math curriculum for many years could confound this study. The teacher reported that students were engaged with the curriculum in a way that was notably different from past students. This is an effect that could be controlled in a more tightly designed study.

Results found that those students who were double dosed did show significant improvement when compared to those students who only participated in the single dose, even though they started out with no significant differences on the pretest. That combined with the large effect sizes suggest that this intervention has practical potential. When a student factor is controlled, as these students were with the MDTP pretest, and one group performs significantly better, this suggests an effect from the intervention. It reinforces the idea that when students will be tested on grade level materials, they should at least be exposed to grade level content. When students have access to grade level curriculum and supports for skills gaps, this program may help. Also, it gives a snapshot of how difficult classroom research can be because it is in a natural and dynamic setting where many
variables cannot be controlled. It does bring to light the lack of sensitivity the high stakes tests have in capturing growth of students with disabilities. The CSAP and the Acuity measures were too broad to detect significant growth, even if some students did improve as shown in the threshold for students accurately answering more critical items on the MDTP posttest. This highlights a flaw in using standards-based tests for students who do not learn at the same pace as peers, are not exposed to grade level content and consistently perform at much lower levels than the standard measures (Allbritten, Mainzer, & Ziegler, 2004). It emphasizes the disconnect between the policy and the actual content that students have available.

**Recommendations/Implications for Future Study**

Several areas are implicated for further follow up. Research design, including more sensitive assessments and timing of intervention would be important. Matching students with a control or comparing the progress of students exposed to a different pre algebra intervention would also be helpful. Finally, increasing the sample size are the three most important recommendations.

**Research Design.** Recommendations for future studies would include a more powerful research design. Matching students with a control group or comparing the progress of students exposed to a different pre algebra intervention would improve generalizability. Increasing the sample size would strengthen the statistical power. Implementing this intervention method before students start their ninth grade curriculum would also be recommended so they would not be so pressured to earn the required
number of math credits in just four years. Possibly a summer intervention class using the AR curriculum would be best since time and other classes would not impinge the student participation.

**Assessments.** Also recommended is using more finely tuned assessments at the end of the project. In this study the growth of the students who were in the single dose group alone was not detectable on any of the standardized measures. Since the state sponsored assessment (CSAP) is the primary method the state and federal education entities measure success for students and schools, the lack of instrument sensitivity to the starting point and growth of students with disabilities needs to be explored further. Using this intervention at a tier two level when working with high school students may be most beneficial since they will be exposed to grade level content as well as practicing underlying concepts in the procedures. This group did show significantly better performance on the Acuity C. Using this as a single dose in the high school is not advised since the content in the general curriculum is much broader and without this exposure, student math development will continue to lag. Using standardized global assessments for students with identified disabilities may not detect actual growth and some measure of this growth needs to be explored so both students and schools are not penalized for persistent and expected slow growth.

**Qualitative Teacher Responses**

The teacher who presented the material over an eighteen month period responded with personal comments about her experience with the program. Overall, she identified
the power point presentations, the range of instructional methods used in the program and the individualized computer practice as the most powerful tools. She attributes this variety in the increased student engagement. Students were eager to complete written exercises in order to earn time on computers to practice skills at their own individual level.

**Best features.** For this teacher, one of the best features of this program was the AR instructional format. The simply worded scripts, examples and preplanned exercises made the program easy to follow and increased fidelity. She also remarked that the power point presentations that required students to respond using multi method techniques, including whiteboards, created more interaction and participation by the students. The spiraling back of content to previously taught concepts was effective at building student mastery. She noted that the nonverbal aspect appealed to the diverse learners. The Algebra Readiness approach appeared to particularly engage students with math and written language learning difficulties or disabilities. By minimizing the written aspect students were able to demonstrate their understanding on the computer screen. These same students showed eagerness to interact and participate during class power point presentations. Students also liked that they were able to progress at their own pace on the individualized computer exercises.

**Behavioral and academic changes.** This teacher was also pleasantly surprised by some behavioral changes in the students while using this approach. She noted increased student interaction and participation during the Algebra Readiness interactive power
point presentations. She attributes this to the creative visual format of the games. The visuals seemed to capture the student attention and they demonstrated more on-task conversations and behaviors. She noted fewer behavioral disruptions and more on-task behaviors. Students even volunteered to read aloud and take on teacher role during pre-planned power points.

**Pride.** She also recognized increased reports of student pride in their math success. Students would tell her when they passed levels making sure she noticed their efforts. She attributes this to the “corny” visual display of recognition the computer rewarded after the student mastered each level. The students, although high school aged, seemed to enjoy this. She reflected that some of the students with learning disabilities and have not experienced much success in school math classes. The program really helped their self-esteem, and parents sent messages that their children came home and expressed pride in their math achievements with the Algebra Readiness program.

**Comprehensiveness.** Finally, this teacher reported the Algebra Readiness program seemed to be more comprehensive than the other pre-algebra math intervention programs she has used. She reports that in her experience, most of the other interventions are drill and practice on basic math skills, but the AR program seemed to cover more pre-algebra topics. She stated, “Algebra Readiness program is a great combination of textbook work, hands on application, individual student computer mastery practice as well as whole class power point instruction. It is the perfect blend.”
Summary and Conclusions

The novel Algebra Readiness program, delivered with unique and visually engaging explanations of deeper math concepts is an intervention focused on improving student’s pre-algebra skills and is based on research into what works. Students need effective interventions that work long before high school when the time pressure to complete so much curriculum becomes overwhelming and the skill gaps become more significant. More math interventions that work are needed for all students who struggle with math, not just those diagnosed with learning disabilities. Evolving RTI techniques and models can help schools design more effective learning structures. These structures will support interventions that need to happen early and focus on important skills needed for abstract mathematical skills.

Math and algebraic problem solving are critical skills in the workplace. Students need interventions to close skill gaps before they lose all exposure to grade level content. They need appropriate measurements to detect growth. All students, not just those with a diagnosed learning disability, who struggle with math need interventions that work,

This study examined the effects of the Algebra Readiness curriculum on a group of students divided into two tiers of intervention. The results indicate that after exposure to the Algebra Readiness program for one academic year, students failed to transfer their knowledge of these concepts as measured by standardized grade level assessments. This is concerning since so much of school time is used to measure student growth and sanctions are attached to schools who demonstrate low performance. Finding a way to
measure the growth of students who have consistently struggled with learning and assessments is so important in this era of high stakes testing. Statistical tests did not detect significant student growth overall, although the large effect sizes indicate that this intervention has practical potential. The small and unbalanced sample sizes affected the likelihood of finding a significant effect.
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Appendix A

Definitions

ACT—American College Testing—a standards-based educational testing program

Acuity—McGraw-Hill—a formative assessment program to assess progress towards CSAP standards

AR—Algebra Readiness—The MIND Research Institute’s pre-algebra curriculum

AYP—Adequate Yearly Progress—NCLB requirement, formula used to determine the achievement of each school district and school.

CBM—Curriculum-Based Measures—method of monitoring student growth using probes from the curriculum

CSAP—Colorado State Assessment Program—NCLB required state assessment of student progress based on grade level content standards.

FAPE—Free and Appropriate Public Education—schools must provide students with disabilities specialized instruction including related services and aids needed to benefit from the instruction.

LD—Learning Disability—Difficulty learning reading writing and math based on impact of one or more psychological processing deficits
MDTP—Mathematics Diagnostic Testing Project—the pre and post test used to assess Algebra I skills

MLD—Math Learning Disability—difficulty learning math based on the impact of one or more psychological processing deficits

MRI—MIND Research Institute—A non profit organization in Irvine, CA, developed the ST Math program

NAEP—National Assessment of Educational Progress—assessment given every two years to measure reading and math achievement for students in grades 4, 8 and 11 across the United States

NCES—National Center for Education Statistics—collects and analyzes educational data, part of the United States Department of Education's Institute of Education Sciences

NCLB—No Child Left Behind—2001 reauthorization of the 1965 Elementary and Secondary Education Act (ESEA) focused on improving educational outcomes for all students

NCTM—National Council of Teachers of Mathematics—organization dedicated to improving the teaching and learning of mathematics for k-12 students.

RTI—Response to Intervention—a multi level prevention system used in schools to provide interventions for students who demonstrate learning difficulties