Solar Photovoltaic System Modeling and Control

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ABSTRACT

To realize the benefits of grid-connected photovoltaic system, it is extremely important to reduce energy losses and improve reliability of grid-connected PV systems with high PV penetration. In this thesis, three different Maximum Power Point Tracking (MPPT) strategies named as Perturbation & Observation MPPT, Incremental Conductance MPPT and Fuzzy Logic Control MPPT have been analyzed, simulated and compared with each other to improve the efficiency of power conversion. A general discussion to counteract partial shading effect in several aspects is also provided in this thesis. In order to improve reliability of the system, an optimum current control loop with suitable control parameters are achieved by comparing three different PI controller parameter design methods, including self-tuning method, trial and error method and mathematical analysis method. Considering the complexity brought by modulation and demodulation process between abc stationary frame and dq0 synchronous frame of PI control loop, P+ Resonant controller, with simple control loop structure and zero steady-state error is also designed in this thesis. At last, LCL filter is analyzed and modeled under both steady state and disturbing condition. The effects of LCL filter for improving disturbance rejection capability and dynamic performance of the system is verified.
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CHAPTER 1: INTRODUCTION

1.1 Introduction

Today, PV systems are widely applied to off-grid generation applications [1-1] such as traffic warning lights, telecommunications, security systems and so on. Normally, when the electricity demand exceeds the supply of PV system, wind system or conventional electric generator can be added with PV system to create a hybrid system. In this case, PV system could be developed to provide power for remote area without or with poor supply from power grid. PV system has many benefits including portability, low operating costs, environmental benefits, stand-alone capability, modularity, safety and reliability, etc.

While the basic expected application of PV system in worldwide is to achieve the stand-alone PV system, some highly industrialized countries such as the US, the European countries and Japan have already realized the grid-connected photovoltaic systems [1-2].

However, comparing with other renewable energy resources, photovoltaic generated electricity is still more expensive. In this case, it is extremely important to reduce energy losses and improve reliability of PV systems. This thesis studies and
analyzes the grid-connected PV system and attempts to find more reasonable solutions to solve those problems.

Starting from PV array analysis and modeling, chapter 2 simulates the relationship between PV array’s output characteristics and environmental conditions, which is considered as a fundamental knowledge for the subsequent MPPT algorithm study and partial shading effect study.

When photovoltaic panels work under different temperature and illumination level, each photovoltaic panel will generate unique characteristic curve since the output power of photovoltaic panel varies as a function of the output voltage. Each photovoltaic panel has a unique maximum power point. At the maximum power point, the corresponding voltage changes as environmental temperature or irradiation level changes. Thus, it is necessary to track the maximum power point of the PV array in order to maintain a high output power efficiency of the PV generation system. Considering the high installation cost of PV array, the process of tracking Maximum Power Point maximizes the efficiency of photovoltaic energy system in photovoltaic conversion process. All in all, the MPPT process helps reduce the system cost and in the meantime enhance conversion efficiency.

Three different MPPT algorithms have been carefully studied and analyzed in chapter 3 which includes the Perturbation and Observation (P&O) MPPT strategy, Incremental Conductance (IncCond) MPPT strategy and Fuzzy Logic Control (FLC) MPPT strategy [3-5] [3-6]. these MPPT strategies are analyzed and compared with each other in MATLAB SIMULINK under different environmental conditions, such as
varying temperature, varying irradiation level and varying both temperature and irradiation level. In the same chapter, a comparison on reliability as well as algorithm speed has been carried out between those three methods.

Shading effects is another serious problem for the photovoltaic array distributed generation system, especially for large scale installation of PV array. Generally speaking, the total efficiency of photovoltaic generation conversion will be reduced due to partial shading. This kind of energy waste could bring very serious economic problems considering the high cost of PV investment. When parts of the PV array are shaded which is defined as partial shading, it could lead to hot spot problem which poses a severe damage to the PV array. If excessive heating problem exists for a certain period of time, the PV cell would be burned out and an open circuit in the shaded string would result.

Chapter 4 mainly studies on shading effects and corresponding MPPT strategies. A MATLAB model is presented to illustrate the relationship between the multiple maximum power points and partially shading conditions for an entire PV array system.

Power converter is an important technology that enables the efficient and flexible interconnection of PV array and power grid. The grid converter design is introduced in chapter 5. The conventional grid connected VSI (Voltage Source Inverter) is a three-phase bridge circuit controlled by IGBTs, which operate according to the control signal generated by control system. Each IGBT works as a controllable switch to be turned on and off and thus controls both magnitude and phase angle of the output voltage. PI controller, which has a wide range of application, is conventionally used to complete the control loop of the grid connected VSI. In this thesis, three design methods for selecting
PI controller parameters are provided. These are self-tuning method, trial and error method and mathematical analysis method.

Additionally, in chapter 5, P+ Resonant controller is compared with PI controller for grid connected VSI. P+ Resonant controller eliminates steady-state error for most stationary reference frame linear current regulation systems. Avoiding the complexity of modulation and demodulation process between abc stationary frame and dq0 synchronous frame, P+ Resonant controller transforms the dc control network into an equivalent ac controller, which could directly achieve zero steady-state error in stationary reference frame. The corresponding test completed in MATLAB is provided in chapter 5 to verify the advantages and performance of P+ Resonant controller.

Low-pass LCL filter analysis is also discussed in chapter 5. On one hand, grid connected PWM converter generates low harmonic current distortion at PWM frequency. On the other hand, low frequency harmonic could be produced due to grid voltage background distortion and grid current harmonic distortion. In this case, the low pass filter is necessary to provide high disturbance rejection capability and dynamic performance as well as high power quality.

1.2 Organization of Thesis:

Chapter 1: Introduction

Chapter 2: PV array analysis and modeling

Chapter 3: MPPT analysis & modeling

Chapter 4: Shading effects and application of power electronics
Chapter 5: Control loop design for grid connected voltage source inverter with LCL filter

Chapter 6: Conclusion and future work
CHAPTER 2: PV ARRAY ANALYSIS AND MODELING

2.1 PV Array Output Characteristics

PV array’s output current-voltage curve reflects PV array’s dependence on environmental conditions such as ambient temperature and illumination level. Typically, the illumination level ranges from 0 to 1100 W/m² and the temperature range is between 233 and 353K. Normally, we select 1100 W/m² and 298 K as the reference values for illumination level and temperature respectively.

The relationship between PV array’s output characteristics and environmental conditions could be illustrated from general simulation results of PV array. PV array’s output power is increased as illumination level increases, while PV array’s output power is improved with the decrease of the ambient temperature.

2.2 PV Array Modeling in PSCAD/EMTDC
Figure 2.1 reflects a simple equivalent circuit of a photovoltaic cell [2-1]. The current source which is driven by sunlight is connected with a real diode in parallel. In this case, PV cell presents a p-n junction characteristic of the real diode. The forward current could flow through the diode from p-side to n-side with little loss. However, if the current flows in reverse direction, only little reverse saturation current could get through. All the equations for modeling the PV array are analyzed based on this equivalent circuit.

\[ I = I_{sc} - I_d = I_{sc} - I_0 \left[ \exp \left( \frac{V + I R_s}{nV_T} \right) - 1 \right] \]  
\[ I_{sc} = I_{sc,ref} \left[ \frac{S}{1000} + \frac{J}{100} (T - T_{ref}) \right] \]  
\[ I_0 = A T^r \exp \left[ \frac{-E_g}{n k T} \right] \]

where \( E_g \) is band-energy gap, whose unit is eV.

\[ E_g = 1.16 - 7.02e - 4 \times \frac{T^2}{T - 1108} \]
\[ E_{g,\text{ref}} = 1.16 - 7.02e - 4 \times \frac{T_{\text{ref}}^2}{T_{\text{ref}} - 1108} \]  \hspace{1cm} (2.5)

\[ I_{0,\text{ref}} = I_{SC,\text{ref}} \times \frac{1}{\exp\left(\frac{V_{O\text{C,ref}}}{nV_T}\right) - 1} \]  \hspace{1cm} (2.6)

Inserting (2.5) and (2.6) into (2.3) to get A:

\[ A = \frac{I_{0,\text{ref}}}{T_{\text{ref}}^r \times \exp\left[-\frac{E_{g,\text{ref}}}{nKT_{\text{ref}}^r}\right]} \]  \hspace{1cm} (2.7)

\[ V_T = m \times \frac{kT}{q} \]  \hspace{1cm} (2.8)

where \( V_T \) is the thermal potential of a module, whose unit is V.

The output Current-Voltage function for a PV array with a string of \( N_S \) modules connected in series and a total of \( N_P \) strings connected in parallel is shown in equation (2.9):

\[ I_O = N_P I_{SC} - N_P I_0 \left(\exp\left[\frac{V_O + I_0 R_S}{nN_S V_T}\right] - 1\right) \]  \hspace{1cm} (2.9)

The data of PV array parameters used in PSCAD/EMTDC model are shown in table 2.1 and table 2.2. The model in PSCAD/EMTDC is presented in appendix 2.1

<table>
<thead>
<tr>
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<td>Parameter</td>
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Table 2.2 Data of PV array structure

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
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<tr>
<td>$N_p$</td>
<td>Number of modulus in parallel</td>
<td>9</td>
</tr>
<tr>
<td>$N_s$</td>
<td>Number of modulus in series</td>
<td>20</td>
</tr>
</tbody>
</table>
2.3 PV Array Output Results

For a single PV cell, the output characteristic of current-voltage curve and power-voltage curve are presented separately as follows. These are the modeling result of single PV cell in MATLAB. The M-file code is illustrated in appendix 2.2.

![I-V curve of a single PV cell](image1.png)

![P-V curve of a single PV cell](image2.png)

Fig. 2.2 I-V curve of a single PV cell  
Fig. 2.3 P-V curve of a single PV cell

For PV array analysis, the PSCAD/EMTDC modeling results are shown as follows:

![Output I-V curve of PV array](image3.png)

![Output P-V curve of PV array](image4.png)

Fig. 2.4 Output I-V curve of PV array at $S=1000$, $T_1=233.0$, $T_2=293.0$, $T_3=353.0$ (Modeled in PSCAD)  
Fig. 2.5 Output P-V curve of PV array at $S=1000$, $T_1=233.0$, $T_2=293.0$, $T_3=353.0$ (Modeled in PSCAD)
As they are shown in figure 2.4 and figure 2.5, the output power of photovoltaic array decreases with the increase of temperature, while the illumination level maintains at 1000 Watt per square meters.

As they are presented in figure 2.6 and figure 2.7, when temperature remains at 293K, the maximum power increases as the illumination level increases.
CHAPTER 3: MPPT ANALYSIS & MODELING

3.1 Introduction of Maximum Power Point Tracking

When photovoltaic panels work under different temperature and illumination level, each photovoltaic panel will generate the unique characteristic curve reflecting the fact that the output power of photovoltaic panel varies as a function of the output voltage. Each photovoltaic panel has a unique maximum power point. The maximum power point and its corresponding voltage change as environmental temperature or irradiation level changes. Thus, it is necessary to track the maximum power point of the PV array in order to maintain a high output power of the PV generation system. Considering the high installation cost of PV array, the process of tracking Maximum Power Point maximizes the efficiency of photovoltaic energy system in photovoltaic conversion process. All in all, the MPPT process helps to reduce the system cost and in the meanwhile enhance conversion efficiency.

Several MPPT algorithms have been carefully studied and well developed nowadays, the most common algorithms are Perturbation and Observation (P&O) MPPT strategy and Incremental Conductance (IncCond) MPPT strategy. The P&O MPPT strategy has comparatively simpler principle and is thus easier to be implemented. But limited by the fixed step size, the tracking speed of P&O MPPT is insensitive to different
condition. Another drawback of P&O MPPT strategy is the large oscillation around the operating point at steady state, which is due to the large step size.

Comparing with P&O MPPT strategy, IncCond MPPT strategy is more complex. IncCond MPPT strategy could make a flexible decision of the next step size based on current judge, a large step size promises fast responding speed while small step size satisfies accurate tracking result. For this reason, it usually leads to a higher cost. The complexity of IncCond MPPT strategy is caused by the design of a reference value $\epsilon$ which determines both the tracking speed and the accuracy of the tracking result. Usually it would take a long time to select a suitable $\epsilon$, for example in my research $\epsilon$ is between $0.005000000000000000000000000000000000000000400750099999999999999$ and $0.0050000000000000000000000000000000000000004007501$. If $\epsilon$ equals the previous value, the step size is not large enough to distinguish the tracking speed of IncCond MPPT strategy from the one of P&O MPPT strategy. However, if $\epsilon$ equals the later value the tracking result turns to unstable and misses the goal in several seconds. The results will be presented in the following section.

Another MPPT is Fuzzy Logic Control (FLC) MPPT strategy [3-5] [3-6], which uses linguistic rules to describe next step’s direction and size, and in this case makes the tracking process flexible. Instead of finding out a tedious long reference value such as IncCond MPPT strategy does, FLC MPPT strategy expresses all the possible conditions and judgmental rules in Fuzzy Control rule table which is modeled in FIS block of MATLAB SIMULINK. By modifying the definition of all the conditions the tracking speed could be changed, the detailed analysis of FLC MPPT strategy and MATLAB SIMULINK results will be illustrated in section 3.4.
In addition to previously introduced MPPT strategies, some other strategies have also been studied such as Maximum Power Voltage (MPV) based method [3-7], which build a direct connection between the duty cycle of DC-DC converter and the output power of PV array. The advantage of this method is to avoid PI controller design and for this reason the PV generation system got simplified and the cost got reduced. Nonlinear MPPT control strategy [3-8] [3-9] has also been studied in some papers, this strategy could be applied to more complex conditions and still achieve accurate results. In this thesis, the last two MPPT strategies (MPV and nonlinear MPPT) are not studied in details.

3.2 Perturbation and Observation MPPT strategy

![Fig.3.1 the principle of P&O MPPT strategy](image)

The principle of P&O MPPT strategy is to periodically vary next step direction by a fixed factor $\pm \Delta P_{PV}/\Delta V_{PV}$, which is considered as the perturbation cycle. As shown in figure 3.1, regardless of where the tracking point firstly starts, the final goal is to arrive at the steady state operation region around the maximum power point. By comparing the
current PV array output power with that of the previous perturbation cycle, the decision
of the subsequent perturbation direction can be made as follows:

If the PV array output power increases, the subsequent voltage perturbation
should continuously increase in the same direction, otherwise the voltage perturbation
direction should be reversed in the next perturbation cycle. In this case, the operating
point of the system gradually moves towards the maximum power point and finally
oscillates around it in steady state region.

Two parameters need to be designed carefully to achieve faster tracking of
maximum power point with smaller P&O MPPT strategy voltage step size. One of them
is the time interval between iterations while another one is the step size of each voltage
perturbation. Large step size $\Delta V_{PV}$ leads to fast tracking of the maximum power point
under varying atmospheric conditions yet results in reduced overall average power
conversion in steady state due to large oscillations around the maximum power point.
Likewise, the design of time interval between iterations should leave enough operating
time for computer calculation, but if the time interval is designed too long, the MPPT
algorithm will lose the fast response capability to a varying environmental condition.
The flowchart of P&O algorithm is shown as below:

![Flowchart](image.png)

Figure 3.2 flowchart of P&O MPPT strategy

Based on the flowchart of P&O MPPT strategy, the algorithm model has been completed in MATLAB SIMULINK, which is shown in figure 3.3. The detail model will be expressed in appendix 3.1
The P&O MPPT strategy results in constant environmental condition are shown as figure 3.4(a) and figure 3.4(b). The test temperature maintains at 290K and the irradiation level maintains at 1100W/m².

Fig. 3.4(a) output I-V curve of P&O MPPT strategy in constant condition T=290K, G=1100W/m²

Fig. 3.4(b) output P-V curve of P&O MPPT strategy in constant condition T=290K, G=1100W/m²
From figure 3.4(a) and 3.4(b), the operating point starts from 0V and tracks the maximum power point along PV module characteristic curve, it stops around 0.8V which based on the figure indicates the maximum power point.

Based on previous results, P&O MPPT strategy achieves the goal of tracking maximum power point in a constant condition. In real life, where both temperature and irradiation level changes unpredictably, the previous test could not demonstrate the adaptability of P&O MPPT strategy. In this case, varying conditions test need to be provided as well.

The first test is under varying temperature and constant irradiation level, the test parameters has been listed in table 3.1. The additional part model is given in appendix 3.2.

<table>
<thead>
<tr>
<th>Table 3.1 test parameters of varying temperature condition</th>
<th>First group: temperature increases</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jump time (s)</td>
<td>0</td>
</tr>
<tr>
<td>Temperature (K)</td>
<td>T1=240</td>
</tr>
<tr>
<td>Irradiation level (W/m²)</td>
<td>G1=1100</td>
</tr>
<tr>
<td>Second group: temperature decreases</td>
<td></td>
</tr>
<tr>
<td>Jump time (s)</td>
<td>0.6</td>
</tr>
<tr>
<td>Temperature (K)</td>
<td>T1=340</td>
</tr>
<tr>
<td>Irradiation level (W/m²)</td>
<td>G1=1100</td>
</tr>
</tbody>
</table>
First group
Fig.3.5(a) output I-V curve of P&O MPPT strategy in varying temperature condition

Second group
Fig.3.5(b) output P-V curve of P&O MPPT strategy in temperature varying condition
Fig. 3.5(c) zoomed in figure of output P-V curve of P&O MPPT strategy in temperature varying condition (first group)

As shown in figure 3.5(a) and 3.5(b), from time 0~0.3s, the first group operating point starts from 0V and tracks the maximum power point along the first PV module characteristic curve under T1=240K. At 0.3s when temperature increases to T2=290K, the operating point jumps to the corresponding second PV characteristic curve which clearly shown in the zoomed in figure 3.5(c). Again, at 0.6s the operating point jumps to the third curve and in the last oscillates around the maximum power point. The second group operating point starts from 0V and tracks along the PV module characteristic curve under initial temperature of T2=290K until the first jump time comes, then it jumps to the next curve corresponding to T1=340K. After this, the operating point jumps towards lower temperature curve two times. For each of them, the operating point could achieve the maximum power point as shown in above figures.
The second test is under varying irradiation level and a constant temperature, the test parameters as listed as table 3.2. The additional part model will be shown in appendix 3.3.

Table 3.2 test parameters of varying irradiation condition

| First group: irradiation increases |  |  |
| Simulation time (s) | 0 | 0.3 | 0.6 |
| Temperature (K) | T1=290 | T2=290 | T3=290 |
| Irradiation level \((\text{W/m}^2)\) | G1=100 | G2=600(initial) | G3=1100 |

| Second group: irradiation decreases |  |  |
| Simulation time (s) | 0 | 0.6 | 0.9 |
| Temperature (K) | T1=290 | T2=290 | T3=290 |
| Irradiation level \((\text{W/m}^2)\) | G1=1100 | G2=600(initial) | G3=100 |

Fig. 3.6(a) output I-V curve of P&O MPPT strategy in varying irradiation condition
As shown in figure 3.6(a) and 3.6(b), from time 0~0.3s, the operating point of first group starts from 0V and tracks the maximum power point along the first PV module characteristic curve under $G_1=100W/m^2$. At 0.3s when temperature changes to $G_2=600W/m^2$, the operating point jumps to the corresponding second PV characteristic curve. At 0.6s the operating point jumps to the third curve and in the last oscillates around the maximum power point.

Based on previous analysis, P&O MPPT strategy could achieve the goal of tracking maximum power point under both constant and varying conditions.

3.3 Incremental Conductance MPPT Strategy

Incremental Conductance MPPT method is one of the most widely used MPPT strategies which has the advantage of fast Maximum Power Point Tracking. Compared with Perturb & Observe (P&O) MPPT strategy, Incremental Conductance MPPT method combines and utilizes the unique characteristics of both the output P-V curve and I-V
curve of the PV array, and thus tracks the maximum power point faster and more accurately.

The characteristics of PV array’s output curve for MPPT study is shown in figure 3.7(a) and 3.7(b), which uses the result of PV array model operating under the reference environmental conditions. When step size is larger than a certain value (\(\text{abs}(\text{d}V_{\text{PV}}) > \varepsilon\)), it has distinct difference between the red region and the blue one as shown in figure 3.7(a), both of which are not close enough to the maximum power point region.

The obviously opposite characteristics are present between red and blue region:

In the red region: when the operation point is moving towards to the Maximum Power Point, it has \(d_i_{\text{PV}} < \varepsilon\), the next step should on the same direction. When the operation point is moving opposite to the Maximum Power Point, it has \(d_i_{\text{PV}} > \varepsilon\) the next step should on the opposite direction.

In the blue region: when the operation point is moving toward to the Maximum Power Point, since \(d_i_{\text{PV}} > \varepsilon\) the next step should move in the same direction. When the operation point is moving opposite to the Maximum Power Point, since \(d_i_{\text{PV}} < \varepsilon\), the next step should move in the opposite direction.

Comparing with P&O method which demands fine judgment and thus more iteration steps for every operating point, the first advantage of Incremental Conductance method is that iteration time at above regions can be reduced.
As the operating point moves close enough to the maximum power point, the previous detection method would be problematic. This is because of the lack of accuracy of the detected Maximum Power Point caused by large step size. In this case, the step size will be regulated smaller where \( \text{Abs}(dV_{pv}) \leq \varepsilon \) as it was shown in figure 3.7(b). There is distinct difference of slope polarity between the right side and the left side of the Maximum Power Point. In this case, it is necessary to take advantage of P&O strategy at the close region of maximum power point. The judgmental methods of selecting suitable size and direction of next step could be concluded as follows:

On the left hand side: When the operating point is moving toward the Maximum Power Point, we have \( \frac{dP_{pv}}{dV_{pv}} > \varepsilon \), and thus the next step should move along the same direction. When the operating point is moving opposite to the Maximum Power Point, we have \( \frac{dP_{pv}}{dV_{pv}} < \varepsilon \), and thus the next step should move along the opposite direction.
On the right hand side: When the operating point is moving opposite to the Maximum Power Point, we have $\frac{dP_{PV}}{dV_{PV}} < \varepsilon$, and thus the next step should move along the opposite direction. When the operating point is moving toward the Maximum Power Point, we have $\frac{dP_{PV}}{dV_{PV}} > \varepsilon$, and thus the next step should move along the same direction.

In order to reflect the P&O strategy in the Incremental Conductance method, the equation of $\frac{dP_{PV}}{dV_{PV}}$ need to be transformed to another form as $\frac{dP_{PV}}{dV_{PV}} \times \frac{1}{V_{PV}}$. Since $\frac{1}{V_{PV}}$ is always positive the polarity of $\frac{dP_{PV}}{dV_{PV}}$ would not change.

$$\frac{di_{PV}}{dV_{PV}} = \frac{1}{V_{PV}} \frac{dP_{PV}}{dV_{PV}} = \frac{d(V_{PV} \cdot i_{PV})}{V_{PV} \cdot dV_{PV}} = \frac{i_{PV}}{V_{PV}} - \frac{di_{PV}}{dV_{PV}} \tag{3.1}$$

In this case, the flow chart of IncCond MPPT strategy is shown as figure 3.8.
Fig. 3.8 flow chart of IncCond MPPT strategy

Based on figure 3.8 the IncCond MPPT strategy control model is completed in MATLAB SIMULINK which is presented in figure 3.9. The IncCond MPPT strategy results under both constant condition and varying environmental conditions are included in the following part.
First of all, the test begins under the constant condition where temperature maintains at 290K and irradiation level maintains at 1100W/m². The MPPT tracking results are shown in figure 3.10(a) and 3.10(b).
From figure 3.10(a) and 3.10(b), the operating point starts from 0V and the maximum power point is tracked along PV module characteristic curve. The algorithm stops around 0.8V. The algorithm performs just like the previous P&O MPPT strategy. For IncCond MPPT strategy design, the reference parameter $\epsilon$ is very sensitive. If $\epsilon$ is too small the tracking speed is reduced. However if $\epsilon$ is set too large, the tracking goal may be missed and the algorithm may become unstable. The example is shown in figure 3.11(a) and 3.11(b) in which $\epsilon$ is changed from previous value of 0.001 to 0.01.
The test parameters for varying temperature condition are the same as those in Table 3.1. The test results are shown in figure 3.12(a) and 3.12 (b)
The parameters for testing varying irradiation condition are the same as those in Table 3.2. The test results are shown in figure 3.13(a) and 3.13(b).

Based on previous results, IncCond MPPT strategy could achieve the goal of tracking maximum power point under both constant and varying conditions. However,
since the reference value $\epsilon = 0.001$ is too small, the advantage of fast response of IncCond MPPT strategy could not be well reflected based on previous tests. In this case, the IncCond MPPT strategy results looks similar to the P&O MPPT strategy results. The detail analysis of reference value $\epsilon$ selection will be given in section 3.5

3.4 Fuzzy Logic Control MPPT Strategy

Fuzzy logic is a form of many-valued logic which deals with reasoning that is approximate rather than fixed and exact. In contrast with traditional logic which usually sets two-value logic as true or false, fuzzy logic can have varying values. Fuzzy logic variables may have a truth or false value that ranges in different degrees and be expressed by linguistic variables. In these cases, fuzzy logic control could provide both fast process speed and the needed accuracy to some extent.

Fig.3.14 principle of fuzzy logic control (FLC) MPPT strategy
Based on figure 3.14, the concept of applying fuzzy logic control to maximum power point tracking strategy is to measure PV array characteristics including the voltage variation $\Delta V_{PV}$ and power variation $\Delta P_{PV}$ to get an optimal voltage increase $V_{PV,ref}$. According to different degree of power variation in the positive direction or in the negative one, the reference photovoltaic voltage variation $\Delta V_{PV,ref}$ is increased or decreased respectively in a direction which makes it possible to increase the power $P_{PV}$.

In figure 3.14, when operating voltage changes from A to B, the voltage variation $\Delta V_{PV}$ is small and in positive direction, while the power variation $\Delta P_{PV}$ is big and in positive direction, then the reference photovoltaic voltage variation $\Delta V_{PV,ref}$ should continue on the positive direction and the step size is medium. By considering and comparing a total of 49 possible conditions, the fuzzy control rule could be set in table 3.3 and the rule should not change if a different environmental condition appears at time $t_{chang}$ as shown in figure 3.14.

The optimal voltage increase $V_{PV,ref}$ is obtained from equation (3.2) to (3.4)

$$\Delta P_{PV} = P_{PV}(k) - P_{PV}(k - 1) \quad (3.2)$$

$$\Delta V_{PV} = V_{PV}(k) - V_{PV}(k - 1) \quad (3.3)$$

$$V_{PV,ref}(k) = V_{PV}(k - 1) + \Delta V_{PV,ref} \quad (3.4)$$

where $P_{PV}(k)$ and $V_{PV}(k)$ are the power and voltage of the photovoltaic generator at sampled times (k) and, $V_{PV,ref}(k)$ is the instant of reference voltage.

The control rules are illustrated in table 3.3 with voltage variation $\Delta V_{PV}$ and power variation $\Delta P_{PV}$ as inputs and reference photovoltaic voltage variation $\Delta V_{PV,ref}$ as
the output. The degrees to separate variables are expressed in terms of linguistic variables such as BN for representing “big negative”, MN “medium negative”, SN “small negative”, Z “zero”, SP “small positive”, MP “medium positive”, and BP “big positive”.

<table>
<thead>
<tr>
<th>$\Delta V_{pv}/\Delta P_{pv}$</th>
<th>BN</th>
<th>MN</th>
<th>SN</th>
<th>Z</th>
<th>SP</th>
<th>MP</th>
<th>BP</th>
</tr>
</thead>
<tbody>
<tr>
<td>BN</td>
<td>BP</td>
<td>BP</td>
<td>MP</td>
<td>Z</td>
<td>MN</td>
<td>BN</td>
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<tr>
<td>MN</td>
<td>BP</td>
<td>MP</td>
<td>SP</td>
<td>Z</td>
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<td>MN</td>
<td>BN</td>
</tr>
<tr>
<td>SN</td>
<td>MP</td>
<td>SP</td>
<td>SP</td>
<td>Z</td>
<td>SN</td>
<td>SN</td>
<td>MN</td>
</tr>
<tr>
<td>Z</td>
<td>BN</td>
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<td>SN</td>
<td>Z</td>
<td>SP</td>
<td>MP</td>
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</tr>
<tr>
<td>SP</td>
<td>MN</td>
<td>SN</td>
<td>SN</td>
<td>Z</td>
<td>SP</td>
<td>SP</td>
<td>MP</td>
</tr>
<tr>
<td>MP</td>
<td>BN</td>
<td>MN</td>
<td>SN</td>
<td>Z</td>
<td>SP</td>
<td>MP</td>
<td>BP</td>
</tr>
<tr>
<td>BP</td>
<td>BN</td>
<td>BN</td>
<td>MN</td>
<td>Z</td>
<td>MP</td>
<td>BP</td>
<td>BP</td>
</tr>
</tbody>
</table>

The fuzzy control rule has been completed in MATLAB FIS, the detailed design will be presented in appendix 3.3 and the surface of the designed fuzzy control rule is shown in figure 3.15.
Fig. 3.15 FIS surface of fuzzy logic control MPPT strategy

The FLC MPPT control strategy is implemented in MATLAB SIMULINK and the control model is illustrated in figure 3.16.

Fig. 3.16 control model of FLC MPPT strategy
First of all, similar to analysis of both P&O MPPT strategy and IncCond MPPT strategy, the test of FLC MPPT strategy begins under the constant condition where temperature maintains at 290K and irradiation level maintains at 1100W/m². The MPPT tracking results are shown as figure 3.17(a) and 3.17(b).

![Fig. 3.17(a) output I-V curve of FLC MPPT strategy under constant condition T=290K, G=1100W/m²](image1)

As it is shown in figure 3.17(a) and 3.17(b), the operating voltage stops at the maximum power point voltage, which shows FLC MPPT strategy achieves the goal under constant condition. In contract with constant condition, the following test will be processed under varying conditions.

The first test is taken under varying temperature condition, the variation parameters are the same as those in table 3.1 which considered both condition of temperature increase and decrease. The results are illustrated in figure 3.18(a) and 3.18(b).
As it shows in figure 3.18(a) and 3.18(b), when temperature changes the FLC MPPT strategy could track along the PV array characteristic curves to detect the maximum power point. However, instead of jumps from current curve to new at exact jump time which has already been defined in table 3.1, the operating point usually jumps
about 0.14s earlier which is because of FLC’s characteristic of approximation and ability of prediction.

Taking irradiation variation into consideration, the variation parameters are the same as those in table 3.2. The results are illustrated in figure 3.19(a) and 3.19(b).

As shown in figure 3.19(a) and 3.19(b), the tracking result of FLC MPPT strategy under varying irradiation condition is not satisfactory. The reason is that PV array power
variation grade defined in MATLAB FIS is set too small to judge the large variation such as this test shows, however if the power grade has been set largely enough to match this test, it will reduce the accuracy of final result. Another solution is to add more grades to define the large jump brought by sharply change of irradiation which leads to the difficulty of designing of fuzzy rules. When the irradiation variation is changed into a smaller range such as from $1050\, \text{W/m}^2$ to $1100\, \text{W/m}^2$ and lastly to $1150\, \text{W/m}^2$, the results are shown in figure 3.20(a) and 3.20(b).

First group  
Second group

Fig. 3.20(a) output I-V curve of FLC MPPT strategy under small range varying irradiation condition
As it is shown in figure 3.20(a) and 3.20(b), when the range of irradiation variation is set smaller the first group clearly performs better than the previous one. However, for the second group, the current range is still not smaller enough to meet the goal. For example, as shown in figure 3.21, when irradiation firstly changes from G3 to G2, the actual power variation is about negative 0.06 (BN) with voltage variation of
positive 0.004 (SP). Based on table 3.3, the next reference photovoltaic voltage variation $\Delta V_{PV,ref}$ is means negative (MN), which means that the operating point returns to G3 curve. When the operating point takes a new step to move forward and need to jump from G3 curve to G2 curve, it will go through same process again and again, that’s the reason why the operating point stops around the first jump time.

Based on previous analysis, FLC MPPT strategy could achieve the goal of tracking maximum power point under constant condition. But when it comes to the varying condition the large variation of temperature and in most cases of irradiation level could lead to excessive power variation grade ranges set in MATLAB FIS, and cause the tracking result unable to achieve the goal.

3.5 Comparison between P&O MPPT Strategy, IncCond MPPT Strategy and FLC MPPT Strategy

The comparison model is completed in MATLAB SIMULINK which will be presented in Appendix 3.4. The comparison results of P&O MPPT strategy, IncCond MPPT strategy and FLC MPPT strategy under constant condition are shown in figure 3.22(a) and 3.22(b).
As it is shown in figure 3.22(a) and 3.22(b), the effect of tracking results are the same between P&O MPPT strategy and IncCond MPPT strategy when the reference value \( \varepsilon \) of IncCond MPPT strategy is set too large (0.001 in this case). All strategies could achieve the goal of tracking maximum power point. However, the FLC MPPT
strategy has less iteration in steady state. This advantage could also be shown in figure 3.23. The code will be provided in appendix 3.5

As shown in figure 3.23, the responding speed of FLC MPPT strategy to achieve the steady state is about 0.05s slower than both P&O MPPT strategy and IncCond MPPT strategy. However, the FLC MPPT strategy could eliminate iteration at steady state which could not be achieved by P&O MPPT strategy or IncCond MPPT strategy.
Another benefit of FLC MPPT strategy is the flexibility. By re-regulating the FIS rule which will be presented in appendix 3.6, the new responding speed of comparison between P&O MPPT strategy, IncCond MPPT strategy and FLC MPPT strategy in constant condition is shown in figure 3.24, which provides that FLC MPPT strategy could largely speed up the tracking process, however with the increase of responding speed, the stability of FLC MPPT strategy in steady state will also be reduced.

Fig. 3.24 new responding results of comparison between P&O MPPT strategy, IncCond MPPT strategy and FLC MPPT strategy in constant condition

Comparing IncCond MPPT strategy with P&O MPPT strategy, IncCond MPPT strategy should have a faster responding speed as long as the reference parameter $\varepsilon$ has been set suitably. In this test the reference parameter $\varepsilon$ should between

0.00500000000000000400750099999999999999 and 0.005000000000000004007501
As shown in figure 3.25(a), the reference value $\varepsilon$ is still too large to distinguish the responding speed between IncCond MPPT strategy and P&O MPPT strategy. While in figure 3.25(b), the reference value $\varepsilon$ is too small that after all leads to an unstable result even though it presents a faster response than P&O MPPT strategy at very beginning.
3.6 Conclusion

In this chapter, three MPPT strategies have been carefully studied, analyzed and compared including P&O MPPT strategy, IncCond MPPT strategy and FLC MPPT strategy. All three strategies have been modeled in MATLAB SIMULINK to get tested and compared. Based on the results, conclusion can be drawn that each of three strategies has their own advantage as well as drawbacks.

First of all, P&O MPPT strategy could achieve the goal of tracking maximum power point under both constant and varying conditions. The advantage of P&O MPPT
strategy is the simple concept which makes it the easiest one to design. In this case, P&O MPPT strategy has lower cost. However, the fixed perturbation step makes it harder for P&O MPPT strategy to promise both faster responding speed and more accurate result, that is why P&O MPPT strategy always has a large iteration around the maximum power point and thus oscillates in steady state.

Compared with P&O MPPT strategy, IncCond MPPT strategy is more flexible. IncCond MPPT strategy could easily achieve MPPT under both constant and varying condition. The complexity of IncCond MPPT strategy design is caused by an unknown reference value $\varepsilon$. Because the fast responding speed would not result if the reference value is set too large, nor could the accuracy be promised if $\varepsilon$ is too small. The range of a suitable reference value $\varepsilon$ is usually less than $10^{-20}$.

In the last, FLC MPPT method could also achieve the tracking goal in constant condition. The design of FLC MPPT strategy is easier than IncCond MPPT method for the reason that it eliminates the reference value design. Moreover, FLC MPPT strategy has the highest flexibility with little iteration around maximum power point in steady state. However, the FLC MPPT method is not suitable under a varying condition especially when the variation of environmental condition is in a large range. Because PV array power variation grade defined in MATLAB FIS will be too small to judge these large variation and will finally lead to a failure of tracking.
4.1 Introduction of Shading Effects [4-1]

Shading effects is a serious problem for the photovoltaic array distributed generation system, especially for large scale installation of PV array. This phenomenon can be caused by the shadow of buildings and trees, passing cloud or sometimes dust or aging. In those cases partial shading usually occurs, which means that parts of the PV array are shaded. Partial shading could lead to hot spot problem. If excessive heating problem exists for a certain time, the PV cell would be burned out and causes the open circuit in the shaded string. This is a severe damage to the PV array. With application of bypass diode, the hot spot problem could be avoided. Functioned by bypass diode, partially shaded PV array usually has several local maximum power points with a single global maximum power point which is the actual MPP that need to be tracked. Generally speaking, the total efficiency of photovoltaic generation conversion will be reduced due to partial shading. This kind of energy waste could bring very serious economic problems considering the high cost of PV investment.
As it is shown in figure 4.1, when a PV string operates in non-uniform condition, in order to support the common string current, the shaded cells must operate at the reversed voltage $V_{bias}$. The polarity of $V_{bias}$ is negative, meaning that the shaded cells consume energy and thus the maximum extractable power from the shaded PV array would be decreased.

In the meantime, high bias voltage $V_{bias}$ may lead to an avalanche break down of the p-n junction diode which causes thermal break-down of the cell. This is the reason for the so called “hot spot”. The conventional method to avoid hot spot problem is by applying bypass diodes, which are connected in parallel to the PV cells to limit reverse voltage and power loss as shown in figure 4.2.
From figure 4.2, the bypass diode begins to conduct when $V_2 - \sum_{i=1}^{n} V_i \geq V_D$ (i ≠ 2) is satisfied, where $V_D$ is the forward voltage drop of the diode. Based on previous analysis, the bypass diode provides an alternative current path, when partial shading occurs, the un-shaded cells on longer carry the same current as they used do. In this case, applying bypass diode could help increase utility efficiency of the photovoltaic conversion. That is the reason why comparing with shaded PV array without bypass diode which has a single maximum power point, the one with bypass diode has several local maximum power point and a global one.

4.2 Study and Analysis to reduce Shading Effects

In order to improve efficiency of PV array and avoid damage, there are basically four topics to study and analyze on partial shading problem: (1) PV array configurations, (2) System architectures, (3) MPPT strategies, (4) Converter circuit topologies.

4.2.1 PV Array Configuration

The goal to develop different PV array configuration is to alleviate the power loss under partial shading condition. As it is shown in figure 4.3, there are three conventional PV array configurations.
Compared with traditional series-parallel (SP) configuration, total-cross-tie (TCT) configuration and bridge-linked (BL) configuration have interconnections between PV strings, which enable different current flowing through the PV strings. That is how TCT and BL configurations decrease current which flows through shaded cells and keep those cells in forward bias region. With same function of the bypass diode, TCT and BL configurations could improve the maximum power point efficiency under partial shading condition.

Another solution to compensate power loss due to partial shading is reconfiguration, as it is shown in figure 4.4.
In figure 4.4, the adaptive bank of PV modules is used for energy compensation, when shading has been detected, the switching matrix will reconfigure the PV modules so that the shaded modules in the fixed part are compensated by the modules in the adaptive bank. In this case, the PV array system could be able to produce constant output power even being shaded. However, reconfiguration methods have several drawbacks: if most of the PV modules are under shading condition, reconfiguration method cannot compensate for all shaded cells with low number of modules in adaptive bank. On the other hand, with high number of modules in adaptive bank, the project investment will be increased and a complicated control algorithm will be required.

4.2.2 System Architectures
As it presented in figure 4.5, there are four basic grid-connected PV system architectures [4-1] which includes centralized architecture, series-connected Micro-converter, parallel-connected Micro-converter and Micro-inverter. The centralized architecture cannot achieve global MPPT for each individual module, therefore would cause mismatching loss. To avoid the shortage of the centralized architecture, both series-connected micro-converter and parallel-connected micro-converter apply DC-DC converter to track global MPP of each module, and then fed the resulting power to a central inverter. These two methods increase the cost to some extent for the reason of the cost brought by application of large amount of power electronics. Alternatively, the
micro-inverter architecture eliminates the central inverter and permits global MPPT for individual modules.

4.2.3 MPPT Strategies

In this thesis, three conventional MPPT strategies for shaded photovoltaic array will be introduced. The first method is unique global power maximum algorithm [4-1].

As it is shown in figure 4.6, the unique global power maximum strategy starts performing on vicinity of the previously-stored maximum power point which was found under the uniform insolation and temperature condition. Then it begins search from both right and left side. During this process it may detect several local maximum power points: if the slope’s polarity of the power-voltage curve changes from positive to negative, it means the existence of a local maximum power point on the left side. In contrast, if the polarity goes from negative to positive, it indicates that an existence of a maximum on the right side. After a local maximum power point is found, it will be compared with the previously-stored maximum power point. If the newly detected point is larger than the
stored one, the new point will be updated to be the global maximum power point and is stored in the memory. Otherwise, the stored global maximum power point will not be changed and the search on the previous direction is immediately terminated.

The major drawback of the unique global power maximum algorithm is the possibility of missing actual global maximum power point. When the operating point tracks from current-stored global MPP to the actual global MPP, if there is a small local MPP in between, the tracking process will miss the actual MPP. Also, this algorithm’s responding speed is not fast enough, especially under rapidly changing condition of partial shading.

The second method is called “Load-line maximum power point tracking strategy” [4-1].

As presented in figure 4.7, the process of load-line maximum power point tracking algorithm is as follows. Firstly, open circuit voltage $V_{OC}$ and short circuit current $I_{SC}$ of the PV array under uniform situation are measured, then the approximate maximum power point voltage $V_{MPP}$ and current $I_{MPP}$ are calculated based on equations $V_{MPP} = 80\%V_{oc}$ and $I_{MPP} = 80\%I_{sc}$. Then a load line is generated by connecting the
original point with the calculated maximum power point. When partial shading occurs, the voltage-current curve changes to the curve with shading which was shown in figure 4.7. There exists an intersection of the new voltage-current curve with the load curve which is in vicinity of actual global maximum power point. The last step is to apply conventional MPPT strategy in this vicinity to detect global maximum power point.

The application of load-line maximum power point tracking algorithm is limited by partial shading condition. Even though the load-line maximum power point tracking algorithm could help land the estimated operating point in vicinity of the global maximum power point, the partial shading could result in more complex multimodal voltage-current curves. Therefore, this method can only track the global maximum power point under certain shading conditions, in which the range of partial shading is small and simple, the number of local maximum should be no more than 2.

The third method is power increment technique [4-2] as illustrated in figure 4.8.
The strategy of power increment technique is to draw a constant PV load line in successive manner by control of power converter. Power converter, in this case, operates as an adjustable constant input-power load. The PV load line begins at zero with open circuit voltage \( V_{OC} \) as the intersection of it and P-V curve, with increase of PV load line, new intersection should be updated and stored as global maximum power point. If there is no intersection part between power-voltage curve and PV load line such as \( B_6 \) shown in figure 4.8, the last updated value is used as the operating point \( (B_5) \), which is in vicinity of global maximum power point. Then conventional MPPT strategy is applied to detect the global MPP.

4.2.4 Converter Circuit Topologies

The corresponding DC-DC converter technology in a PV system will be introduced in the following sections.

4.3 Modeling of Partially Shaded PV array

The shading model can be used to test the performances of different group under different partial shading conditions and their effects to the output of entire PV array.
In figure 4.9, the test PV array consists of 1000 (10*100) modules and receives insolation and temperature at several different levels. We separate the PV array into three different groups based on the different partial shading conditions. For example, there are 40 strings (assemblies in other words) connected in parallel, which could be classified as a group because all of them operate under same partial shading condition: every assembly has 5 modules operate under uniform insolation where $\lambda = 1$ and $45^\circ C$ of temperature, while other 5 modules under partial shading condition where $\lambda = 0.75$ and temperature $T = 40^\circ C$.

Based on above introduction, same analysis could be developed for the entire PV array. The analysis results are illustrated in table 4.1 as follows.
Table 4.1 Partially Shaded PV array data analysis

<table>
<thead>
<tr>
<th>Number of groups</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data of group NO.1</td>
<td>Data of group NO.2</td>
</tr>
<tr>
<td><strong>Number of subassemblies in an assembly</strong></td>
<td>2</td>
</tr>
<tr>
<td><strong>Number of modules in Subassembly</strong></td>
<td>[5,5]</td>
</tr>
<tr>
<td><strong>Temperature in Subassembly</strong></td>
<td>[45,40]</td>
</tr>
<tr>
<td><strong>Insolation in Subassembly</strong></td>
<td>[1,0.75]</td>
</tr>
<tr>
<td><strong>Number of such assemblies in a group</strong></td>
<td>40</td>
</tr>
</tbody>
</table>

Based on previous analysis, the MATLAB result are shown as follows, the M.file is presented in Appendix 4.1

Fig. 4.10(a) I-V characteristics for subassemblies of Group 1

Fig. 4.10(b) P-V characteristics of subassemblies of Group 1
From figure 4.10 to figure 4.12, taking assembly into consideration, it’s obvious to see that the characteristic of subassemblies under different partial shading condition performs differently even in a same assembly.
As shown in figure 4.13, taking assembly performance into consideration, the different assembly performs differently under different partial shading conditions; it shows the multi-maximum power point characteristics.

As shown in figure 4.14, taking group performance into consideration, the different group performs differently under different partial shading conditions; it shows the multi-maximum power point characteristics.
As it shown in figure 4.15, taking the PV array as consideration, there are several local maximum power point and only one global maximum power point exist in PV array output caused by partial shading.

For data saved using MAT file versions prior to 7.3, the SIMULINK does not support loading the import data in structure with time. In this case, corresponding MPPT strategies for partial shaded PV array could not be analyzed in MATLAB SIMULINK. The data processing method is illustrated in Appendix 4.2.

4.4 Introduction of DC-DC Converter [4-4]

The effect of DC-DC converter is to control the PV array’s output voltage following the MPPT voltage in order to achieve the optimum power efficiency for the system.
Assume that the boost converter is operating on continuous mode, while the current flows through the inductor never falls to zero. The boost converter’s transfer function can be obtained by considering its steady-state operation.

The inductor average voltage is:

\[ V_{L\text{avg}}(t) = \frac{1}{T} \int_{t}^{1+T} v_L(t) \, dt \]  \hspace{1cm} (4.1)

The relationship between output and input voltage under continuous conduction mode is:

\[ \frac{V_o}{V_i} = \frac{1}{1 - D} \]  \hspace{1cm} (4.2)

where \( D \) represents the duty cycle. If the duty cycle equals zero the output voltage of boost converter has the minimum value and goes to infinity as the duty cycle goes to one.

Considering the lossless converter, for which \( P_i = P_o \) and in this case:

\[ \frac{I_i}{I_o} = \frac{1}{1 - D} \]  \hspace{1cm} (4.3)

The ripple at the inductor current can be obtained from the following equation, where \( V_L \) stands for the voltage across the inductor during switch ‘S’ is closed.
Assume $I_L$ increasing linearly then we have:

$$V_L = L \frac{di}{dt}$$  \hspace{1cm} (4.4)$$

where $\Delta I$ is the inductor current ripple of the boost converter where

$$\Delta I = \frac{V_i D}{fL}$$  \hspace{1cm} (4.6)$$

The output voltage ripple of the boost converter is caused by the charge and discharge process of the shunt capacitor over a switching cycle. When the switch is closing, the inductor begins to store energy and the capacitor supplies the load current $I_o$, while during the switch is opening, the energy stored in the inductor will transfer to the capacitor and the load.

The calculation of the output voltage ripple of the boost inverter is:

$$\Delta V_c = \frac{1}{C} \int_0^{t_{on}} I_o \, dt = \frac{I_o}{C} \int_0^{t_{on}} \, dt = \frac{I_o t_{on}}{C}$$  \hspace{1cm} (4.7)$$

Then:

$$\Delta V_c = \frac{I_o D}{fC}$$  \hspace{1cm} (4.8)$$

The maximum and minimum inductor currents are determined using the average value and the change in current:

$$I_{max} = I_L + \frac{\Delta I_L}{2} = \frac{V_i}{(1 - D)^2 R} + \frac{V_i DT}{2L}$$  \hspace{1cm} (4.9)$$
As long as the inductor current $I_{\text{min}}$ to be positive, the boost converter operate in the continuous mode. Hence, the boundary between continuous and discontinuous inductor current is determined by:

$$I_{\text{min}} = 0 = \frac{V_i}{(1-D)^2} - \frac{V_i DT}{2L}$$ \hspace{1cm} (4.11)

In this case,

$$L \geq \frac{(1-D)^2 DR}{2f}$$ \hspace{1cm} (4.12)

The capacitor should be large enough to limit the output voltage ripple $V_r = \Delta V_c/V_c$, the calculation of filter capacitor is:

$$V_r = \frac{\Delta V_c}{V_c} = \frac{I_o D}{V_c fC} = \frac{I_o fC}{V_c} = \frac{D}{R fC}$$ \hspace{1cm} (4.13)

Thus:

$$C \geq \frac{D}{R fV_r}$$ \hspace{1cm} (4.14)

The regulation result of DC-DC converter is presented in figure 4.17.

![Fig. 4.17 Regulation result of the DC-DC converter in PSCAD](image)
As it is shown in figure 4.17, the red line represents the reference value of DC-DC converter output voltage, the DC-DC converter regulates the output voltage to work around the reference one. The horizontal axis stands for simulation time which in this figure ranges from 0s to 100s.
5.1 Basic Analysis of Grid-Connected VSI (Voltage Source Inverter)

Power converter is the technology that enables the efficient and flexible interconnection of different components such as renewable energy generation and grid or controllable loads. The grid converter requires advanced semiconductor technology and signal processing techniques [5-1]. In conventional grid connected VSI (Voltage Source Inverter), a three-phase bridge circuit consisting of IGBTs, operates according to the control signal generated by control algorithm. Each IGBT works as a controllable switch to be turned on or off and thus controls the magnitude and the phase angle of the output waveform. In this case, the regulated output voltage or current curve according to grid-side’s requirements will be generated by the grid-connected VSI. The three phase grid connected VSI provides suitable three-phase voltage with the right frequency and phase angle for interconnection to the grid. The overall VSI structure in relation to LCL filter is illustrated in figure 5.1.
The equivalent circuit of grid connected VSI with LCL filter in the PV array system is presented as below:

Based on the DC link circuit analysis, the equivalent circuit equation is given below:
\[ C_1 \frac{dV_{dc}}{dt} + i_{dc} = \frac{V - V_{dc}}{R} \]  

(5.1)

For the grid connected side, the circuit analysis of the three-phase LCL filter is presented as follows [5-9]:

\[ \frac{di_{abc1}}{dt} = \frac{V_{abc}}{L_1} - \frac{R_c}{L_1}(i_{abc1} - i_{abc2}) - \frac{V_{c,abc}}{L_1} - \frac{i_{abc1}R_1}{L_1} \]  

(5.2)

\[ \frac{di_{abc2}}{dt} = \frac{V_{c,abc}}{L_2} + \frac{R_c}{L_2}(i_{abc1} - i_{abc2}) - \frac{V_{s,abc}}{L_2} - \frac{i_{abc2}R_2}{L_2} \]  

(5.3)

5.2 Control Strategy Design for Three-Phase Grid Connected VSI

For the reason that PI controller cannot eliminate steady-state error for the alternative current (AC) control as in three-phase AC grid connected PV system. To solve this problem, a mathematical transformation between three-phase abc stationary frame system and dq0 synchronous frame system has been universally applied to three-phase grid-connected system analysis and will be introduced in this section.

Dq0 transformation is short for Direct quadrature zero transformation which is usually applied to the three-phase AC circuit analysis. The main reason to apply such a mathematical transformation method is for simplification of analysis. In the case of a balanced three-phase system, the dq0 transformation reduces three alternative current quantities to two directive current quantities. Simplified calculations can then be carried out on these imaginary DC quantities. The results can also be inversely transformed back to original three-phase abc quantities and be feedback to the real system. In this case, the application of dq0 transformation not only reduces the complexity of system analysis but also provides an available DC operation condition for PI controller.
The principle of abc to dq0 transformation is shown in figure 5.3:

As it is shown in figure 5.3, after dq0 transformation, the three separate sinusoidal phase quantities are projected onto two rotating axes with the same angular speed as the three-phase sinusoidal quantities. The two axes are called the d-axis and the q-axis, with the q-axis leading d-axis at an angle of 90 degrees.

The transformations can be described in the following equations:

\[
I_{dq0} = T_{abc} = \frac{2}{3} \begin{bmatrix}
\sqrt{2} & \sqrt{2} & \sqrt{2} \\
\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}
\end{bmatrix} \begin{bmatrix}
\cos(\theta) & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\
\cos(\theta - \frac{2\pi}{3}) & \cos(\theta) & \sin(\theta - \frac{2\pi}{3}) \\
\cos(\theta + \frac{2\pi}{3}) & \sin(\theta) & \cos(\theta + \frac{2\pi}{3})
\end{bmatrix} \begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix}
\]  

\[ (5.4) \]

\[
I_{abc} = T^{-1}_{dq0} = \frac{2}{3} \sqrt{2} \begin{bmatrix}
\cos(\theta) & \sin(\theta) & \frac{\sqrt{2}}{2} \\
\cos(\theta - \frac{2\pi}{3}) & \sin(\theta - \frac{2\pi}{3}) & \frac{\sqrt{2}}{2} \\
\cos(\theta + \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) & \frac{\sqrt{2}}{2}
\end{bmatrix} \begin{bmatrix}
I_d \\
I_q \\
I_0
\end{bmatrix}
\]  

\[ (5.5) \]
In this case \( \theta = \omega t \),

\[
i_d = \sqrt{\frac{2}{3}} \left[ i_a \cos(\omega t) + i_b \cos \left(\omega t - \frac{2\pi}{3}\right) + i_c \cos \left(\omega t + \frac{2\pi}{3}\right) \right] \tag{5.6}
\]

\[
i_q = -\sqrt{\frac{2}{3}} \left[ i_a \sin(\omega t) + i_b \sin \left(\omega t - \frac{2\pi}{3}\right) + i_c \sin \left(\omega t + \frac{2\pi}{3}\right) \right] \tag{5.7}
\]

Taking derivative of \( i_d \) and \( i_q \), we get (5.8) and (5.11):

\[
\frac{di_d}{dt} = \sqrt{\frac{2}{3}} \left[ \frac{di_a}{dt} \cos(\omega t) + \frac{di_b}{dt} \cos \left(\omega t - \frac{2\pi}{3}\right) + \frac{di_c}{dt} \cos \left(\omega t + \frac{2\pi}{3}\right) \right] 

- \sqrt{\frac{2}{3}} \omega \left[ i_a \sin(\omega t) + i_b \sin \left(\omega t - \frac{2\pi}{3}\right) + i_c \sin \left(\omega t + \frac{2\pi}{3}\right) \right] \tag{5.8}
\]

Insert (5.2) and (5.7) into equation (5.8), we will have equation (5.9):

\[
\frac{di_{d1}}{dt} = \frac{V_d}{L_1} - \frac{R_c}{L_1} (i_{d1} - i_{d2}) - \frac{V_{cd}}{L_1} - \frac{R_1}{L_1} i_{d1} + \omega i_{q1} \tag{5.9}
\]

Insert (5.3) and (5.7) into equation (5.8), we will have equation (5.10):

\[
\frac{di_{d2}}{dt} = \frac{V_{cd}}{L_2} + \frac{R_c}{L_2} (i_{d1} - i_{d2}) - \frac{V_{sd}}{L_2} - \frac{R_2}{L_2} i_{d2} + \omega i_{q2} \tag{5.10}
\]

\[
\frac{di_q}{dt} = -\frac{2}{3} \left[ \frac{di_a}{dt} \sin(\omega t) + \frac{di_b}{dt} \sin \left(\omega t - \frac{2\pi}{3}\right) + \frac{di_c}{dt} \sin \left(\omega t + \frac{2\pi}{3}\right) \right] 

- \frac{2}{3} \omega \left[ i_a \cos(\omega t) + i_b \cos \left(\omega t - \frac{2\pi}{3}\right) + i_c \cos \left(\omega t + \frac{2\pi}{3}\right) \right] \tag{5.11}
\]

Insert (5.2) and (5.6) into equation (5.11), we will have equation (5.12):

\[
\frac{di_{q1}}{dt} = \frac{V_q}{L_1} - \frac{R_c}{L_1} (i_{q1} - i_{q2}) - \frac{V_{cq}}{L_1} - \frac{R_1}{L_1} i_{q1} - \omega i_{d1} \tag{5.12}
\]

Insert (5.3) and (5.6) into equation (5.11), we will have equation (5.13):
From above analysis, there exists coupling between the model equations in d-axis and q-axis. The coupling part need to be decoupled in controller design.

\[
\frac{di_{q2}}{dt} = \frac{V_{c,q}}{L_2} + \frac{R_c}{L_2} (i_{q1} - i_{q2}) - \frac{V_{s,q}}{L_2} - \frac{R_2}{L_2} i_{q2} - \omega i_{d2} \quad (5.13)
\]

Equation (5.14) and (5.15) are equivalent transformation from equation (5.9) and (5.10). By adding equation (5.14) and (5.15) together, Equation (5.16) is obtained with all controllable quantities in d-axis presented on the right hand side.

\[
\begin{align*}
L_1 \frac{di_{d1}}{dt} &= V_d - R_c (i_{d1} - i_{d2}) - V_{c,d} - R_1 i_{d1} + \omega L_1 i_{q1} \\
L_2 \frac{di_{d2}}{dt} &= -V_{s,d} + R_c (i_{d1} - i_{d2}) + V_{c,d} - R_2 i_{d2} + \omega L_2 i_{q2}
\end{align*}
\quad (5.14)
\]

Equation (5.17) and (5.18) are equivalent transformation from equation (5.12) and (5.13). By adding equation (5.17) and (5.18) together, Equation (5.19) is obtained with all controllable quantities in q-axis presented on the right hand side.

\[
\begin{align*}
V_d - V_{s,d} &= L_1 \frac{di_{d1}}{dt} + L_2 \frac{di_{d2}}{dt} + R_1 i_{d1} + R_2 i_{d2} - \omega L_1 i_{q1} - \omega L_2 i_{q2} \\
L_1 \frac{di_{q1}}{dt} &= V_q - R_c (i_{q1} - i_{q2}) - V_{c,q} - R_1 i_{q1} - \omega L_1 i_{d1} \\
L_2 \frac{di_{q2}}{dt} &= -V_{s,q} + R_c (i_{q1} - i_{q2}) + V_{c,q} - R_2 i_{q2} - \omega L_2 i_{d2}
\end{align*}
\quad (5.16)
\]

Equation (5.16) and (5.19) provide the relationship between each circuit element and the corresponding current flow through or voltage drop in d-q frame. They also reflect how d-axis and q-axis’ parameter are coupled with each other.

In order to simplify the relationship we write it as follows:
For the reason that R is very small, it could be ignored in the following analysis.

Based on above analysis, the design for control strategy of a three phase synchronous frame PI controlled VSI is presented as figure 5.4.

\[
\begin{align*}
V_d &= V_{ld} - \omega L i_q + V_{s,d} \\
V_q &= V_{lq} + \omega L i_d + V_{s,q}
\end{align*}
\]  
(5.20)

where

\[
\begin{align*}
V_{ld} &= L \frac{di_d}{dt} + R i_d \\
V_{lq} &= L \frac{di_q}{dt} + R i_q
\end{align*}
\]  
(5.22)

(5.23)

For the reason that R is very small, it could be ignored in the following analysis.

Based on above analysis, the design for control strategy of a three phase synchronous frame PI controlled VSI is presented as figure 5.4.

Fig. 5.4 control strategy of a three-phase synchronous frame PI controlled VSI

As it is shown in figure 5.4, the control loop including decoupled loops design is achieved based on equation (5.20) and (5.21). The references of $i_d^*$ and $i_q^*$ are dependent on the desired outputs of the PV system. In this case, the expected power factor equals to
1, which means that reactive power should be controlled to zero. The outputs of PI controller which helps achieving this goal will be considered as $i_{q}^{*}$, while $i_{d}^{*}$ is the PI controller outputs for regulating PV array working at maximum power point.

5.3 Analysis and Design of PI Controller’s parameters

The difficulty of designing the PI controller’s parameters is that only two current feedback loops cannot provide complete information of a third-order LCL filter. In this section three different methods will be applied to design the PI controller’s parameters, which includes (1) self-tuning method by MATLAB SIMULINK, (2) trial and error method [5-3] by PSCAD and (3) mathematical analysis method [5-4] by MATLAB m-file.

5.3.1 Self-tuning method by MATLAB SIMULINK

The ideal way to select PI controller’s parameters is applying MATLAB self-tuning program. The decoupled system control loop for tuning PI controller is shown in figure 5.5, which consists of control loop and plant loop.
Fig. 5.5 Self-tuning method of three-phase grid connected VSI PI controller parameter design

However, for the reason of MATLAB tuning program’s limitation, self-tuning method cannot be used in PID controller design of this non-linear system. Based on the explanation of error message block in MATLAB: the loops containing PID controller 1 and PID controller 3 shown in above figure are “not physically closed”, and “the plant model in the PID loop linearizes to zero”.

For PID controller 2, the tuning result is shown in Appendix 5.1, from which the closed loop system with controller gains defined in PID block is unstable.

In this case, the assumption to apply self-tuning program in MATLAB to get the PI parameters is not applicable. Considering the complication to analyze state function for a non-linear system such as the grid-connected VSI with LCL filter system, the “trial and error method” is widely used in designing PI controller’s parameters.
5.3.2 Trial and error method by PSCAD

The system modeling is shown in Appendix 5.2, the control parameters and results are listed in Table 5.1:

<table>
<thead>
<tr>
<th>Table 5.1 Control Parameters of Trial and error method in PSCAD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PWM Switching Frequency</strong></td>
</tr>
<tr>
<td>2000Hz</td>
</tr>
<tr>
<td>Maximum/minimum Value of triangle curve</td>
</tr>
<tr>
<td>25/-25</td>
</tr>
<tr>
<td>PI Parameters for Reactive Power Control</td>
</tr>
<tr>
<td>$K_p = 7.78, K_i = 0.47$</td>
</tr>
<tr>
<td>PI Parameters for Inner Loop Current Control</td>
</tr>
<tr>
<td>$K_p = 10, K_i = 500$</td>
</tr>
</tbody>
</table>

The first condition is when the environmental condition is constant which means both environmental temperature and irradiation level doesn’t change, the results are shown as follows:

![Detected Maximum Power Point Voltage](image)

Fig. 5.6 Detected Maximum Power Point Voltage under Constant Environmental Condition

As it is shown in figure 5.6, the detected Maximum Power Point Voltage maintains at the constant value of 354V after 5s for the reason that the temperature and illumination level is constant thus the maximum power point wouldn’t change with time.
variation. Before 5s, it shows the tracking process of PV array’s maximum power point from a lower operating voltage which is randomly set in PSCAD before simulation.

![Fig. 5.7 DC side Voltage under constant environmental condition](image)

As it is shown in figure 5.7, the DC link voltage of the PV array has been controlled to a constant level by DC to DC converter. The decrease from MPPT voltage is caused by the energy consumption of the DC link capacitor.
Figure 5.8 shows that the grid-connected three-phase voltage and current are controlled to operate in same phase which would also be verified in figure 5.10, the result of grid-connected reactive power. For the reason that if the grid-connected three-phase voltage and current are in same phase, the reactive power equals to zero.
Figure 5.9 and figure 5.10 shows that the grid-connected reactive power control strategy has achieved the goal, which control the reactive power to be zero and thus keep the output of real power at a high level.

The second condition is to consider the system working under the non-uniform environmental condition, as this condition has already been studied in detail and analyzed in chapter 3. The test in this part will only consider the situation of varying illumination level, a system model is illustrated in appendix 5.2 with the same PI control strategy discussed above.
The results are shown as follows.

Figure 5.11 shows that under randomly changing illumination level, the I-V curve and the P-V curve are changing correspondingly which is reflected by the blue points. The MPPT algorithm can still detect the actual MPPT points of different characteristic curves under varying environmental condition, which is reflected as the red points in this figure.
As we can see from figure 5.12, the detected maximum real power and the maximum power point voltage is changing corresponding to the variation of illumination level. The PWM generator sends on and off pulses to the random number generation device which generates random values from 0 to 1200 to represent a changing illumination level and these values is applied as the input of the PV array, and it also results in a corresponding large variation of detected maximum power points.
As it is shown in figure 5.13, being affected by variable illumination level the dc-side voltage changes according to the variation of detected MPPT voltage.

Fig. 5.13 DC Side Voltage when Illumination Level is changing

Fig. 5.14 Grid-connected Three-phase Current and Voltage Curves when Illumination Level is changing
For the same reason, the grid-connected three-phase current and voltage curve tend to reflect the random variation of illumination level yet still maintain the characteristics of three-phase sinusoidal waveform as it is shown in figure 5.14.

Figure 5.15 and 5.16 shows that the output real power is affected by the variation of illumination level which also brings some influence to the reactive power control.
However the influence could not change the control result and the reactive power is regulated to zero which proves the practical of previously designed control strategy.

Based on previous tests, it takes long time for trial and error method to get a suitable set of control parameters. However, the selection is random and in most cases depends on the experience of the researcher. For this reason, a new PI controller parameter design method which is developed based on the automatic control theory will be introduced as follows.

5.3.3 Mathematical Analysis Method [5-4] by MATLAB

As it is shown in figure 5.17, the single-phase control loop of grid connected VSI with LCL filter is used to analyze the transfer function of single-phase grid connected system. The right sides represents LCL filter and the left side stands for controller which in this case consists of PI controller and system control gain. \( V_i^+ \), the output of controller part is used as the signal waveform for PWM generator control. By applying Mason function to the control loop analysis, the open loop and closed loop transfer function could be achieved as follows.
The following parameters \([5-4]\) are assumed: \(L_1 = 5.5\text{mH}\), the grid-side inductance \(L_2 = 1\text{mH}\), filter capacitance \(C_f = 20\mu\text{F}\) and the equivalent resistors in both inverter-side and grid-side are same \(R_{l1} = R_{l2} = 0.01\text{ohm}\). In the following, appropriate control parameters including \(K_p, K_i\) and \(K_c\) are designed mathematically based on the predefined system parameters including \(L_1, L_2, C_f, R_{l1}, R_{l2}\).

First of all, the effect of \(K_p\) is analyzed while \(K_i = 10, K_c = 100\) which are selected based on experience. The results are shown as follows.
Fig. 5.18 Zero-pole figure of control loop with variation of $K_p$ in overall view

The location of zeros and poles of the closed-loop system for different $K_p$ are listed in table 5.2 as follows:

<table>
<thead>
<tr>
<th></th>
<th>$K_p = 0.1, K_i = 10, K_c = 100$</th>
<th>$K_p = 0.3, K_i = 10, K_c = 100$</th>
<th>$K_p = 0.6, K_i = 10, K_c = 100$</th>
<th>$K_p = 0.9, K_i = 10, K_c = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero</td>
<td>-100</td>
<td>-33.3333</td>
<td>-16.6667</td>
<td>-11.1111</td>
</tr>
<tr>
<td>Pole1</td>
<td>-14546</td>
<td>-15503</td>
<td>-16601</td>
<td>-17479</td>
</tr>
<tr>
<td>Pole2</td>
<td>-1770+j1645</td>
<td>-1328+j3964</td>
<td>-788+j5673</td>
<td>-352+j6831</td>
</tr>
<tr>
<td>Pole3</td>
<td>-1770-j1645</td>
<td>-1328-j3964</td>
<td>-788-j5673</td>
<td>-352-j6831</td>
</tr>
<tr>
<td>Pole4</td>
<td>-107</td>
<td>-34.0000</td>
<td>-17.0000</td>
<td>-11.0000</td>
</tr>
</tbody>
</table>
As it is shown in figure 5.18 and table 5.2, the grid connected VSI system with LCL filter has one zero and four poles, which are all located on the left side of the s-domain. With the increase of $K_p$, pole 1 will move towards the imaginary axis, yet the distance between pole 1 and imaginary axis is still more than 5 times the distance between pole 2 or pole 3 to imaginary axis, which promises pole 2 and pole 3 the dominant poles. The dominant poles also move closer to the imaginary axis, which will make it harder for the system to remain stable as verified in figure 5.20, in the meanwhile, the fourth pole will get closer to the zero as illustrated in figure 5.19.

Fig. 5.19 Zero-pole figure of control loop with variation of $K_p$ in zoomed in view
Second, the effect of $K_i$ is analyzed while $K_p = 0.1, K_c = 100$. The results are shown as follows.
Fig. 5.21 Zero-pole figure of control loop with variation of $K_i$

<table>
<thead>
<tr>
<th></th>
<th>$K_i = 0.1$, $K_p = 0.1$, $K_c = 100$</th>
<th>$K_i = 1$, $K_p = 1$, $K_c = 100$</th>
<th>$K_i = 10$, $K_p = 0.1$, $K_c = 100$</th>
<th>$K_i = 100$, $K_p = 0.1$, $K_c = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero</td>
<td>-1</td>
<td>-10</td>
<td>-100</td>
<td>-1000</td>
</tr>
<tr>
<td>Pole1</td>
<td>-14550</td>
<td>-14550</td>
<td>-14546</td>
<td>-14512</td>
</tr>
<tr>
<td>Pole2</td>
<td>-1821+j1715</td>
<td>-1817+j1708</td>
<td>-1770+j1645</td>
<td>-2327</td>
</tr>
<tr>
<td>Pole3</td>
<td>-1821-j1715</td>
<td>-1817-j1708</td>
<td>-1770-j1645</td>
<td>-678+j1495</td>
</tr>
<tr>
<td>Pole4</td>
<td>-1</td>
<td>-10</td>
<td>-107</td>
<td>-678-j1495</td>
</tr>
</tbody>
</table>

As it is shown in figure 5.21 and table 5.3, one zero and four poles are all located on the left side of the s-domain. With the increase of $K_i$, pole 1 will move towards the
imaginary axis in a very small range, it wouldn’t effects the performance of dominant poles. The dominant poles also move closer to the imaginary axis, which will make it harder for the system to remain stable. The zero and the rest pole will get far away from each other.

As it is shown in figure 5.22, larger $K_i$ leads to longer settling time. However, smaller $K_i$ causes lower overshoot. Thus, $K_i$ should be chosen to be a medium value between 1 and 10.

Lastly, the effect of $K_c$ is analyzed while $K_p = 0.1, K_i = 10$. The results are shown as follows.
Fig. 5.23 Zero-pole figure of control loop with variation of $K_c$

Table 5.4 Location of zeros and poles with variation of $K_c$

<table>
<thead>
<tr>
<th>Pole</th>
<th>$K_c = 0.1, K_i = 10, K_p = 0.1$</th>
<th>$K_c = 1, K_i = 10, K_p = 0.1$</th>
<th>$K_c = 10, K_i = 10, K_p = 0.1$</th>
<th>$K_c = 100, K_i = 10, K_p = 0.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero</td>
<td>-100</td>
<td>-100</td>
<td>-100</td>
<td>-100</td>
</tr>
<tr>
<td>Pole1</td>
<td>-112.7+j7687.0</td>
<td>-87.6+j7684.6</td>
<td>-836.4+j7624.4</td>
<td>-14550</td>
</tr>
<tr>
<td>Pole2</td>
<td>-112.7-j7687.0</td>
<td>-87.6-j7684.6</td>
<td>-836.4-j7624.4</td>
<td>-1770+j1645</td>
</tr>
<tr>
<td>Pole3</td>
<td>-2.3+j12.2</td>
<td>-9.2+j38.1</td>
<td>-78.6+j96.3</td>
<td>-1770-j1645</td>
</tr>
<tr>
<td>Pole4</td>
<td>-2.3-j12.2</td>
<td>-9.2-j38.1</td>
<td>-78.6-j96.3</td>
<td>-107</td>
</tr>
</tbody>
</table>

As it is shown in figure 5.23 and table 5.4, the variation of $K_c$ will affects the dominant poles. Figure 5.24 illustrates that with the increase of $K_c$, the responding speed
will be improved which means the system will be able to achieve the steady-state faster. The m file code is presented in Appendix 5.3.

![Fig. 5.24 step response figure of control loop with variation of $K_c$](image)

Based on previous analysis, if we want to get a system with stable and fast response, a large $K_c$, a medium $K_i$ and a small $K_p$ is needed. In this case, the system usually has one zero with pole 4 near to it, a pair of dominated poles which includes pole 2 and pole 3, and after all pole 1 on the negative real axis which is located far away from imaginary axis. In such situation, an ideal method is to assign pole 4 and the zero to the same location to weaken their influence and to achieve pole-zero cancellation for pole 4.

The following equation should be established:

$$Y_3 - X_1 (Y_2 - X_1 (Y_1 - X_1)) = 0$$  \hspace{1cm} (5.26)

where
\[ X_0 = \frac{A_0}{B_0}, X_1 = \frac{A_1}{A_0}, Y_1 = \frac{B_1}{B_0}, Y_2 = \frac{B_2}{B_0}, Y_3 = \frac{B_3}{B_0} \]

After pole-zero cancellation, the open loop and closed loop transfer functions are obtained as

\[ G_{ol2}(s) = \frac{X_0}{s^3 + (Y_1 - X_1)s^2 + (Y_2 - X_1(Y_1 - X_1))s} \quad (5.27) \]

\[ G_{cl2}(s) = \frac{X_0}{s^3 + (Y_1 - X_1)s^2 + (Y_2 - X_1(Y_1 - X_1))s + X_0} \quad (5.28) \]

In order to include damping ratio \( \zeta \) and natural frequency \( \omega \) into the system analysis, the characteristic equation is derived as

\[ D = (s^2 + 2\zeta\omega s + \omega^2)(s + m\zeta\omega) \quad (5.29) \]

where \(-m\zeta\omega\) is the position of pole 1. Comparing equation (5.28) and (5.29), we can get the following equations:

\[ Y_1 - X_1 = (2 + m)\zeta\omega \quad (5.30) \]

\[ Y_2 - X_1(Y_1 - X_1) = (1 + 2m\zeta^2)\omega^2 \quad (5.31) \]

\[ X_0 = m\zeta\omega^3 \quad (5.32) \]

Considering equation (5.26), (5.30), (5.31) and (5.32) together, we can get

\[ X_1(2m\zeta^2 + 1)\omega^2 = 0. \]

For the reason that \( X_1 = \frac{Y_1 - X_1}{X_1} = \frac{Y_2 - (2m\zeta^2 + 1)\omega^2}{(2+m)\zeta\omega}, \) the previous equation could be rearranged as

\[ Y_2 - \frac{Y_2 - (2m\zeta^2 + 1)\omega^2}{(2+m)\zeta\omega} (2m\zeta^2 + 1)\omega^2 = 0. \]

The final equivalent equation is expressed in (5.33)

\[ (2 + m)\zeta B_3 - (2m\zeta^2 + 1)\omega B_2 + (2m\zeta^2 + 1)^2\omega^3 B_0 = 0 \quad (5.33) \]
m = 5 is assigned to make sure pole 1 to be a non-dominated pole. The relationship between $\zeta$ and $\omega$ is shown in figure 5.25 which is plotted according to equation (5.33). The M-code is included in appendix 5.4

![Relationship between damping ratio $\zeta$ and natural frequency $\omega$](image)

**Fig. 5.25 Relationship between damping ratio $\zeta$ and natural frequency $\omega$**

In order to select suitable damping ratio, natural frequency and controller parameters, the relationship between damping ration $\zeta$ and natural frequency $\omega$ will be inserted back into equation (5.26), (5.30), (5.31) and (5.32) to find corresponding control parameters: $K_p$, $K_i$ and $K_c$. The calculation process is presented in Appendix 5.5.

\[
K_c = \frac{(R_{l1} + R_{l2})}{(2m\zeta^2 + 1)\omega^2} + (m + 2)\zeta\omega (L_1L_2C_f) - (R_{l1}L_2C_f + R_{l2}L_2C_f) \frac{L_2C_f}{L_1L_2C_f} \] (5.34)

\[
K_p = \frac{L_1L_2C_f m \zeta \omega^3}{K_c} \] (5.35)

\[
K_i = \left( \frac{R_{l1}L_2C_f + R_{l2}L_1C_f + L_2C_f K_c}{L_1L_2C_f} - (2 + m)\zeta\omega \right) K_p \] (5.36)
Select damping ratio $\zeta=0.6$, the natural frequency $\omega = 3582.7$ Hz, the controller parameters are calculated as:

$$\begin{align*}
K_c &= 83.000 \\
K_p &= 0.1834 \\
K_i &= 2.0650
\end{align*}$$

The designed parameters have been tested and the results are listed as follows:

![Zero-pole figure of control loop](image)

**Fig. 5.26 Zero-pole figure of control loop with designed parameters**

As it is shown in figure 5.26, with the designed parameters, pole 4 and zero are assigned in the same position which is -11.3 in real axis. Pole 1 is located far away from imaginary axis to make pole 2 and pole 3 dominated poles, and all poles and zero are located in the left side of S-domain, which satisfies the system stability requirement.
In figure 5.27 the system achieves steady-state at about 0.002s, which is fast. The fast responding speed is dependent on a comparatively large value of $\zeta \omega$. 
Fig. 5.28 Root locus figure of control loop with designed parameters

Fig. 5.29 Bode Diagram of the system with designed parameters
As it shown in figure 5.29, the designed system has a large phase margin of 110° and gain margin of 14.7 dB. The parameter design and system analysis process in m file code are presented in Appendix 5.6.

5.4 Introduction of P+ Resonant Controller

The reason to apply P+ Resonant controller [5-11] in PV inverter design is to offset the inability to eliminate steady-state error for most stationary frame based linear current regulation systems. As introduced previously, the traditional PI controller in synchronous dq0 frame is the common solution to solve this problem. However the complexity of shifting the ac frame back to dc where the PI controller can be used is also brought into the control system design. In this case, a transformed regulator known as P+ Resonant controller is preferred considering the advantage of avoiding modulation and demodulation process between abc stationary frame and dq0 synchronous frame. That is because P+ Resonant controller is to transform the dc type regulator network into an equivalent ac regulator, which could directly achieve zero steady-state error in stationary frame.

The principle of stationary frame ac regulator (P+ Resonant controller) is to transform a dc system into an equivalent ac system, while still maintain the same frequency response characteristic. Before applying P+ Resonant controller to the system, an analysis of a conventional hybrid compensation system is introduced as follows to illustrate how to make the transformation from dc system to ac system achievable.
The product demodulation method [5-5] is applied by multiplying a signal by reference sinusoidal and co-sinusoidal waveforms. This method could shift any harmonic content at reference frequency to dc and to the double-frequency.

\[
\begin{align*}
x_a(t) &= x(t) \cdot \cos(\omega_0 t) = x(t) \cdot \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} \\
x_b(t) &= x(t) \cdot \sin(\omega_0 t) = x(t) \cdot \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2}
\end{align*}
\] (5.37) (5.38)

After the Fourier transformation, these equations change to

\[
\begin{align*}
X_a(\omega) &= \frac{1}{2} [X(\omega + \omega_0) + X(\omega - \omega_0)] \\
X_b(\omega) &= \frac{1}{2j} [-X(\omega + \omega_0) + X(\omega - \omega_0)]
\end{align*}
\] (5.39) (5.40)

If the signals go through a low-pass filter, the output signals become

\[
\begin{align*}
Y_a(\omega) &= \frac{1}{2} X_a(0) \\
Y_b(\omega) &= \frac{1}{2j} X_b(0)
\end{align*}
\] (5.41) (5.42)

Fig. 5.30 demodulating single-phase integral block [5-11]
Figure 5.30 is realized based on the previous analysis. It could be considered as a replacement block for the integral term of PI block working under a single phase synchronous frame. The corresponding mathematical expression of this figure is shown as below:

\[ v_{AC}(t) = \left[ [e_{AC}(t) \cdot \cos(\omega_0 t)] * h_{DC}(t) \right] \cdot \cos(\omega_0 t) + \left[ [e_{AC}(t) \cdot \sin(\omega_0 t)] * h_{DC}(t) \right] \cdot \sin(\omega_0 t) \tag{5.43} \]

where \( \omega_0 \) is fundamental frequency and ‘*’ denotes convolution product.

The goal is to find out the transfer function of \( H_{AC}(s) \), which maintains the same frequency responses as equation (5.43), but in the meantime avoid the modulation and demodulation processes. With the application of \( H_{AC}(s) \) the system’s expression could be changed to

\[ V_{AC}(s) = H_{AC}(s) \cdot E_{AC}(s) \tag{5.44} \]

where \( E_{AC}(s) \) represents the transfer function of AC system.

Equation (5.44) can be expressed in time domain as below:

\[ V_{AC}(t) = e_{AC}(t) * h_{AC}(t) \tag{5.45} \]

In order to simplify the analysis, the following two functions are included:

\[ \begin{align*}
  (h_{DC}(t) * (e_{AC}(t) \cdot \cos(\omega_0 t))) &= f_1(t) \tag{5.46} \\
  (h_{DC}(t) * (e_{AC}(t) \cdot \sin(\omega_0 t))) &= f_2(t) \tag{5.47}
\end{align*} \]

After Laplace transformation, equation (5.46) and (5.47) are converted to:

\[ F_1(s) = \ell\{h_{DC}(t) * (e_{AC}(t) \cdot \cos(\omega_0 t))\} = H_{DC}(s) \cdot \ell\{(e_{AC}(t) \cdot \cos(\omega_0 t))\} \]

\[ F_2(s) = \ell\{h_{DC}(t) * (e_{AC}(t) \cdot \sin(\omega_0 t))\} = \frac{1}{\pi} H_{DC}(s) \{\left[ E_{AC}(s + j\omega_0) + E_{AC}(s - j\omega_0) \right] \} \tag{5.48} \]
Equation (5.43) is extended into two functions and Laplace transformation is taken for each of them separately:

\[ A = \ell\{e_{AC}(t) \cdot \cos(\omega_0 t) \} \cdot \cos(\omega_0 t) = \ell\{f_1(t) \cdot \cos(\omega_0 t)\} \]
\[ = \frac{1}{2}\{F_1(s + j\omega_0) + F_1(s - j\omega_0)\} \]
\[ = \frac{1}{4}\{H_{DC}(s + j\omega_0)(E_{AC}(s + 2j\omega_0) + E_{AC}(s)) + H_{DC}(s - j\omega_0)(E_{AC}(s) + E_{AC}(s - 2j\omega_0))\} \]

\[ B = \ell\{e_{AC}(t) \cdot \sin(\omega_0 t) \} \cdot \sin(\omega_0 t) = \ell\{f_1(t) \cdot \sin(\omega_0 t)\} \]
\[ = \frac{1}{2}\{F_1(s + j\omega_0) - F_1(s - j\omega_0)\} \]
\[ = -\frac{1}{4}\{H_{DC}(s + j\omega_0)(E_{AC}(s + 2j\omega_0) - E_{AC}(s)) - H_{DC}(s - j\omega_0)(E_{AC}(s) - E_{AC}(s - 2j\omega_0))\} \]

Therefore, equation (5.43) could be transferred into the following function:

\[ V_{AC}(s) = A + B = \frac{1}{4}\{2[H_{DC}(s + j\omega_0) + H_{DC}(s - j\omega_0)]\} \cdot E_{AC}(s) \]
\[ = \frac{1}{2}\{H_{DC}(s + j\omega_0) + H_{DC}(s - j\omega_0)\} \cdot E_{AC}(s) \]

Insert equation (5.44) into (5.52), we can get:

\[ H_{AC}(s) = \frac{1}{2}[H_{DC}(s + j\omega_0) + H_{DC}(s - j\omega_0)] \]
In equation (5.53), the DC component $H_{DC}(s)$ and double frequency error component $H_{DC}(s + 2j\omega_0)$ which generated by demodulation process are both avoided. In this case, equation (5.53) achieves the corresponding frequency response of AC transfer function $H_{AC}(s)$ for any DC controller $H_{DC}$’s transfer function has been given. In the meanwhile, it avoids the effect of demodulation.

When the reference signal bandwidth is small in comparison to the reference frequency, an alternative could be made to equation (5.53):

$$H_{AC}(s) = H_{DC}\left(\frac{s^2 + \omega_0^2}{2s}\right)$$  \hspace{1cm} (5.54)

The transfer function of PI controller is:

$$H_{DC}(s) = K_p + \frac{K_i}{s}$$  \hspace{1cm} (5.55)

Insert equation (5.55) into equation (5.54), an equivalent ac compensator could be achieved as the following function, which could be viewed as a stationary frame controller having the same frequency response as the PI controller.

$$H_{AC}(s) = K_p + \frac{K_i}{s^2 + \omega_0^2}$$  \hspace{1cm} (5.56)

Equation (5.56) is considered as the transfer function of P+ Resonance controller.

5.5 Control Strategy Design for Three Phase P+ Resonant Controller

Based on above analysis, P+ Resonance controller could be directly applied to the stationary frame current control system. A double current control loop has been introduced as follows in figure 5.31:
Figure 5.31 shows the single phase equivalent control loop of grid-connected VSI system with LCL filter. The inner current regulation loop is completed by the feedback of the capacitor current $i_c$, which determines the output voltage $V_c$ of the LCL filter. This part of regulation maintains the system’s inner loop stability. The outer current control loop achieves the goal of regulating grid side current $i_2$ to a reference current $i'_2$, whose magnitude and phase angle are determined by the regulation of reactive power flow. By maintaining reactive power to be zero which means that the grid side current should be regulated in the same phase as the grid side voltage, the control goal would be achieved.

As it was calculated previously, the transfer function of the P+R controller is

$$G_c(s) = K_p + \frac{K_i s}{s^2 + \omega_0^2}$$

where $\omega_0 = 2\pi f_0$ represents the fundamental frequency; $K_p$ is defined as the proportional gain and $K_i$ as integral gain. The test of the P+R controller’s parameters will be introduced in the following part.

The open loop and closed loop transfer function of the system with P+Resonant controller are illustrated as follows [5-4].

$$G_{olPR}(s) = \frac{i_2}{i'_2 - i_2} = \frac{A_0 s^2 + K_i K_c s + A_0 \omega_0^2}{B_0 s^5 + B_1 s^4 + (B_2 + B_0 \omega_0^2) s^3 + (B_3 + B_1 \omega_0^2) s^2 + B_2 \omega_0^2 s + B_3 \omega_0^2}$$ (5.57)
\[ G_{clPR}(s) = \frac{i_2}{l_2} = \frac{A_0 s^2 + K_1 K_c s + A_0 \omega_0^2}{B_0 s^5 + B_1 s^4 + (B_2 + B_0 \omega_0^2) s^3 + (A_0 + B_3 + B_1 \omega_0^2) s^2 + (K_1 K_c + B_2 \omega_0^2) s + (A_0 + B_3) \omega_0^2} \]  \hspace{1cm} (5.58)

where \( A_0 = K_p K_c, A_1 = K_1 K_c, B_0 = L_1 L_2 C_f, B_1 = R_{L1} L_2 C_f + R_{L2} L_1 C_f + L_2 C_f K_c, B_2 = L_1 + L_2 + R_{L1} R_{L1} C_f + R_{L2} C_f K_c, B_3 = R_{L1} + R_{L2} \)

Fundamental frequency is assumed at \( \omega_n = 2 \times \pi \times 60 = 376.98 \) rad/sec

Previous designed controller parameters are applied: \( K_c = 83.000, K_p = 0.1834 \) and \( K_1 = 2.0650 \), and the results are presented as follows:

Fig. 5.32 zero-pole figure of control loop with P+ Resonant Controller
Comparing with figure 5.26, the above figure shows that P+ Resonant Control provides an additional pair of zero and pole. With the appropriate selection of controller parameters such as $K_p$, $K_i$ and $K_c$, two zeros and two poles brought by P+ Resonant controller could be canceled, the rest of poles’ positions are as the same as those generated by PI controller. In this case, system performs almost the same as it does under PI control, which could also be verified by following result shown in figure 5.33, figure 5.34 and figure 5.35. The corresponding test code in MATLAB is presented in appendix 5.7.

![Root Locus Figure of Control Loop with P+ Resonant Controller](image1.png)

![Step Response Figure of Control Loop with P+ Resonant Controller](image2.png)

Fig. 5.33 Root locus figure of control loop with P+ Resonant Controller

Fig. 5.34 Step response figure of control loop with P+ Resonant controller
Fig. 5.35 Bode Diagram of control loop with P+ Resonant controller

Fig. 5.36 Bode diagram of PI and P+ Resonant Controller
As it shown in figure 5.36, comparing with PI controller, P+ Resonant Controller has bigger magnitude and phase gain at the fundamental frequency. This quality ensures P+ Resonant controller to track without steady-state error. The corresponding M-code is included in appendix 5.8

Based on above analysis, an experience to test the performance of PI controller and P+ Resonant controller under fundamental frequency is taken, a sudden disturbance of grid voltage $V_s = 3000V$ was applied beginning at 0.2s, the testing results are shown as follows:

As it is shown in figure 5.37, when PI controller and P+ Resonant Controller working under the fundamental frequency, PI controller is more sensitive to the disturbance which means that it is easier to lose the stability, while P+ Resonant controller has a good performance of closely tracking the reference current all the time.
The P+ Resonant controller’s performance of dynamic tracking is shown in figure 5.38. The reference current is changed from 25A, 377 rad/s to 45A, 200 rad/s at 0.3s, in such situation P+ Resonant controller could still achieve the goal of tracking reference current.

5.6 Analysis of the LCL Filter

The connected filter in a grid inverter has a crucial role to play in transient behavior. On one hand, grid connected PWM converter generates low harmonic current distortion at PWM frequency. On the other hand, low frequency harmonic could be produced due to grid voltage background distortion and grid current harmonic distortion. In this case, the low pass filter needs to have high disturbance rejection capability and dynamic performance.

Three commonly used low-pass filters including L filter, LC filter and LCL filter are introduced as follows:
The advantage of L filter is easy to apply because of the single inductor structure. Universally, L filter maintains a high inductance value and thus has low attenuation. The disadvantage of L filter is to cause a poor system dynamics and in this case makes the system response for a long time. According to the properties of L filter, the design of inverter switching frequency requires a high value to reduce current harmonics.

In order to compensate the disadvantage of L filter, the shunt capacitor is applied to the filter to further attenuate the current harmonics caused by inverter switching frequency. Compare with L filter, LC filter has low reactance and high magnitude impedance. Normally, LC filter is applied to the system where load has high impedance. The disadvantage of LC filter is the resonance frequency varies over time as the inductance value of the grid varies.

The advantages to apply LCL filter are: it has better performance to attenuate current harmonics at switching frequency since a lower ripple current distortion is achievable across the grid-side inductor while the current ripple has already been reduced by the capacitor. For this reason, LCL filter can attenuate the ripple current harmonic with small inductance and capacitance value.

In this case, LCL filters are conventionally applied to the grid-connected power electronic systems. Some studies focus on the filter optimization design based on power loss and efficiency [5-7]; some focus on developing new control loops to suppress the current distortion arising from the grid voltage harmonics [5-8]; while some others apply pole placement approach [5-7], almost the same strategy as previous PI controller parameter design, to get suitable system parameters. In this thesis, system plant has
already been fixed with well-developed control loop. Instead of developing new strategy of LCL filter design, the study will only focus on the effects brought by LCL filter to the system based on existing modeling and analysis.

First of all, with previously assumed LCL filter parameters and calculated controller parameters, the system is stable and the tracking goal is achieved. The FFT analysis is provided in figure 5.39.

![FFT analysis](image)

**Fig. 5.39 FFT analysis of grid current $i_2$**

As it is shown in figure 5.39, the harmonics current distortion ($THD = 2.32\%$) meets the requirement stated in IEC61727-2004 standard [5-9], which clarifies that the total harmonics current should below 5%, and $3^{\text{th}}$–$9^{\text{th}}$ odds harmonics below 4%, $11^{\text{th}}$–$15^{\text{th}}$ odds harmonics below 2%, $35^{\text{th}}$ harmonics upwards below 0.3% is acceptable.

Under steady state condition, when $L_1, L_2$ and $C_f$ are varied $\pm 50\%$ respectively, the influence on the system performance is shown as follows:
5.6.1 L1 Changing

As it is shown in figure 5.40, when L1 changes from 50%L1 to 150%L1, pole 4 and zero are always assigned in the same location; the pair of dominated poles (pole 2 and pole 3) move far away from imaginary axis, while pole 1 will move towards to the imaginary axis which may weaken the effect of dominant poles. All the poles and zero are located in the left side of S-domain, meaning that this variation of L1 will not change the system stability.
As it is shown in figure 5.41, with the increase of L1, system will become stable faster while the overshoot is reduced.
The detail data analysis of figure 5.42 is presented in table 5.5

Table 5.5 Data summary of Varying L1

<table>
<thead>
<tr>
<th></th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 Gm</td>
<td>6.4333</td>
<td>6.1038</td>
<td>5.8732</td>
<td>5.6922</td>
<td>5.5586</td>
</tr>
<tr>
<td>L1 Pm</td>
<td>48.2795</td>
<td>56.3685</td>
<td>65.5433</td>
<td>76.4704</td>
<td>90.3420</td>
</tr>
<tr>
<td>L1 Wcg</td>
<td>8255.0</td>
<td>8069.7</td>
<td>7937.4</td>
<td>7832.0</td>
<td>7753.2</td>
</tr>
<tr>
<td>L1 Wcp</td>
<td>3906.2</td>
<td>3772.1</td>
<td>3579.9</td>
<td>3360.1</td>
<td>3014.4</td>
</tr>
<tr>
<td>100% L1</td>
<td>110% L1</td>
<td>120% L1</td>
<td>130% L1</td>
<td>140% L1</td>
<td>150% L1</td>
</tr>
</tbody>
</table>
where the Gm stands for gain margin, Pm represents the phase margin, Wcg and Wcp are the corresponding crossover frequencies respectively.

As it is shown in figure 5.42 and table 5.5, when L1 changes from 50%L1 to 150% L1, the magnitude gain changes from 118.1% to 93.7% of the initial one, while the phase gain changes from 48.2795° to 170.5909°.

5.6.2 L2 changing
As it is shown in figure 5.43, when $L_2$ is varied from $50\%L_2$ to $150\%L_2$, pole 4 and zero are always assigned in the same location; the pair of dominated poles (pole 2 and pole 3) move toward to imaginary axis, while pole 1 will move far away from the imaginary axis which strengthen the effect of dominant poles. All the poles and zero are located in the left side of S-domain, indicating that the variation of $L_2$ will not change the system stability.
As it is shown in figure 5.44, with the increase of L2, system will get stable slower while the overshoot is increased.
The detailed data analysis of figure 5.45 is presented in table 5.6.

**Table 5.6 Data summary of Varying L2**

<table>
<thead>
<tr>
<th></th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>L2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gm</td>
<td>4.9627</td>
<td>5.0588</td>
<td>5.1554</td>
<td>5.2519</td>
<td>5.3488</td>
</tr>
<tr>
<td>Pm</td>
<td>171.7261</td>
<td>169.4466</td>
<td>164.2551</td>
<td>145.0948</td>
<td>122.6163</td>
</tr>
<tr>
<td>Wcg</td>
<td>10451</td>
<td>8618.0</td>
<td>8975.9</td>
<td>8462.2</td>
<td>8040.2</td>
</tr>
<tr>
<td>Wcp</td>
<td>365.3419</td>
<td>457.8718</td>
<td>669.7138</td>
<td>1422.5</td>
<td>2159.1</td>
</tr>
<tr>
<td>100%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L2</td>
<td>110%</td>
<td>120%</td>
<td>130%</td>
<td>140%</td>
<td>150%</td>
</tr>
<tr>
<td></td>
<td>L2</td>
<td>L2</td>
<td>L2</td>
<td>L2</td>
<td>L2</td>
</tr>
</tbody>
</table>
As it is shown in figure 5.45 and table 5.6, when $L_2$ changes from 50% of $L_2$ to 150% of $L_2$, the magnitude gain changes from 91.13% to 109.07% of the initial one, while the phase gain changes from 171.7261° to 85.0905°.

5.6.3 Cf changing

![Zero-pole figure of varying Cf](image)

Fig. 5.46 Zero-pole figure of varying Cf
As it is shown in figure 5.46, when Cf is varied from 50%Cf to 150%Cf, pole 4 and zero are always assigned in the same location; the pair of dominated poles (pole 2 and pole 3) move toward to imaginary axis, while pole 1 will move far away from the imaginary axis which strengthen the effect of dominant poles. All the poles and zero are located in the left side of S-domain, meaning that the variation of Cf will not change the system stability.

Fig. 5.47 Step response figure of varying Cf

As it is shown in figure 5.47, with the increase of Cf, system will get stable slower while the overshoot is increased.

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The detailed data analysis of figure 5.48 is presented in table 5.7

<table>
<thead>
<tr>
<th></th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gm</td>
<td>5.4466</td>
<td>5.4545</td>
<td>5.4478</td>
<td>5.4547</td>
<td>5.4459</td>
</tr>
<tr>
<td>Pm</td>
<td>172.4491</td>
<td>171.0055</td>
<td>168.2258</td>
<td>159.5020</td>
<td>129.9224</td>
</tr>
<tr>
<td>Wcg</td>
<td>1087.0</td>
<td>9929.2</td>
<td>9187.9</td>
<td>8599.1</td>
<td>8101.7</td>
</tr>
<tr>
<td>Wcp</td>
<td>308.3325</td>
<td>366.9520</td>
<td>479.4763</td>
<td>828.0762</td>
<td>1901.2</td>
</tr>
<tr>
<td>100%</td>
<td>110%</td>
<td>120%</td>
<td>130%</td>
<td>140%</td>
<td>150%</td>
</tr>
</tbody>
</table>
As it is shown in figure 5.48 and table 5.7, when $C_f$ is varied from 50% $C_f$ to 150% $C_f$, the magnitude gain changes from 100.01% to 99.97% of the initial one, while the phase gain changes from 172.4494° to 73.2453°.

Based on above analysis, the results manifests that the design method is practical. In the meantime, a little smaller $L_1$, and larger $L_2$ and $C_f$ may be preferred. In this case, 10% $L_1$, 1000% $L_2$ and 1000% $C_f$ are selected as the base case for the following analysis. The corresponding m file code is included as appendix 5.10.

Under disturbing condition which is caused by grid connected voltage $V_s$, the harmonic impedance of the system is shown in equation (5.59).

$$Z_{in} = \frac{V_s}{I_2} = \frac{B_0s^4 + B_1s^3 + B_2s^2 + (B_3 + A_0)s + A_1}{L_1C_3 + (R_1C + K_c)s^2 + s} \quad (5.59)$$

Two sets of LCL filter parameters are used to analyzed the LCL filter effects for the harmonic impedance, one of them is the initial value and another is the previously selected base case value. The results are shown as follows:
As it is shown in figure 5.49 and table 5.8, the compared LCL increases the harmonic impedance system’s stability.

The test results are shown in follows:

![Bode Diagram](image)

**Fig.5.49 Bode plot of Harmonic Impedance**

<table>
<thead>
<tr>
<th></th>
<th>Gm</th>
<th>Pm</th>
<th>Wcg</th>
<th>Wcp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial LCL</td>
<td>Inf</td>
<td>-1.0621</td>
<td>NaN</td>
<td>868600</td>
</tr>
<tr>
<td>Compared LCL</td>
<td>Inf</td>
<td>7.7907</td>
<td>NaN</td>
<td>1.4428</td>
</tr>
</tbody>
</table>
As it is shown in figure 5.50 and 5.51 the compared LCL could help reducing THD, which proves that the previous analysis is practical.
CHAPTER 6: CONCLUSION AND FUTURE WORK

6.1 Conclusion

As it has been proposed in chapter 1, reducing energy losses and improving reliability of the grid connected PV system is the major goal of this thesis during the study of solar photovoltaic system modeling and control.

Chapter 3 and chapter 4 are focused on reducing energy losses by taking maximum power point tracking strategy and partial shading effect into consideration. Based on the analysis in chapter 3, three different MPPT strategies have been analyzed, simulated and compared. All three MPPT strategies could achieve the tracking goal with different characteristics of each. P&O MPPT strategy is very simple which makes it the easiest one to design. However, the fixed perturbation step makes it hard to guarantee both faster responding speed and more accurate result at the same time. Compared with P&O MPPT strategy, IncCond MPPT strategy is more flexible. But the complexity of IncCond MPPT strategy design is brought by an unknown reference value ε. The design of FLC MPPT strategy is easier than IncCond MPPT. Moreover, FLC MPPT strategy has the highest flexibility with little oscillation around maximum power point in steady state. However, the FLC MPPT method is not applicable under a varying condition especially when environmental conditions vary in a large range. Chapter 4 provides a overview of
counteracting partial shading effect from several aspects including PV array configuration, MPPT strategies and converter topologies.

Chapter 5 is focused on the improving reliability of the grid-connected PV system. In order to design an optimum current control loop with suitable control parameters, three different methods are applied for PI controller parameter design, which include self-tuning method, trial and error method and mathematical analysis method. Because of the limitation of software, self-tuning method could not be achieved for a non-linear system. While trial and error method is commonly applied in industry, it usually takes a long time and a lot of experience to get the right controller parameters for a new researcher. The complexity will be increased with increased number of PI controllers and the order of the plant’s transfer function. Mathematical analysis method is based on zero-pole cancellation and pole replacement. By defining damping ratio and natural frequency, unknown controller parameters could be calculated and applicable for the predefined control loop. Considering the complexity brought by modulation and demodulation process between abc reference stationary frame and dq0 reference synchronous frame of PI control loop, P+ Resonant controller, with simple control loop structure is studied in chapter 5. Testing results show that P+ Resonant controller apparently eliminates steady-state error under fundamental frequency, which could not be achieved by PI controller. At last, LCL filter is analyzed and modeled under both steady state and disturbance condition. The effects of LCL filter with improved disturbance rejection capability and dynamic performance is verified by simulation case study.

6.2 Future work
Conventional MPPT methods may not work well under shading conditions, so further research of MPPT algorithms should be conducted to accommodate shading in PV array system. In aspect of power electronics, a further development and improvement of power electronics converters including DC to DC and DC to AC can be pursued to improve system efficiency and reduce. Also, more advanced topics related to power system analysis. First, the islanding issue should be studied. Islanding, can cause a safety hazard to utility service person and damage to equipment. So, anti-islanding protection application of PV system should be further studied based on the existing model. Second, the control strategy design for hybrid wind and solar generation system could also be considered as a study topic, for the reason that the photovoltaic generation and wind power generation can complement each other. In this case, the hybrid solar and wind system may perform better in week grids than wind or PV generation system alone. Applying the current PV system modeling for smart grid study can also be a potential topic.
REFERENCES


[5-1] Remus Teodorescu, Marco Liserre, Pedro Rodriguez, Grid Converters for photovoltaic and Wind Power Systems, IEEE WILEY.


APPENDIX

Appendix 2.1 PV Array System Modeling in PSCAD
PV cell modeling in M-file

```matlab
%T0=298
%T=301
%T1=T-T0
%Is=0.3  %assumed%
%Isc=Is*(1+5*1e-4*T1)
Isc=3;
T=298;
I0=6*1e-6;
Rs=0.9*1e-3;
Rp=1000;
q=6*1e-19;
k=1.38*1e-23;
A=1.08;
Ud=(2.7E-3:0.001:0.0971);
I=Isc-I0*(exp(q*Ud/(A*k*T))-1)-Ud/Rp;
U=Ud-I*Rs;
figure(1)
plot(U,I)
xlabel ('Value (V)')
ylabel ('Value (I)')
title('I-V curve')
P=U.*I;
figure(2)
plot(U,P)
grid
xlabel('Value (V)')
ylabel('Value (P)')
title('P-V curve')
```

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Appendix 3.1 P&O MPPT Strategy Model in Constant Condition

Subsystem
Appendix 3.1 P&O MPPT Strategy Model in Constant Condition (.m)

clc
I=evalin('base','I')
V=evalin('base','V')
P=evalin('base','P')

Ipv=evalin('base','Ipv')
Vpv=evalin('base','Vpv')
Ppv=evalin('base','Ppv')
figure (1)
plot(V,I)
hold on
plot(Vpv,Ipv,'r*')
title('MPPT V-I Curve')
xlabel('Voltage unit: V')
ylabel('Current unit: I')
grid on

figure (2)
plot(V,P)
hold on
plot(Vpv,Ppv,'r*')
title('MPPT V-P Curve')
xlabel('Voltage unit: V')
ylabel('Power unit: W')
grid on

Appendix 3.2 P&O MPPT Strategy Model in Temperature Varying Condition
Appendix 3.2 P&O MPPT Strategy Model in Temperature Varying Condition (.m)

clc
I1=evalin('base','I1');
I2=evalin('base','I2');
I3=evalin('base','I3');
V1=evalin('base','V1');
V2=evalin('base','V2');
V3=evalin('base','V3');
P1=evalin('base','P1');
P2=evalin('base','P2');
P3=evalin('base','P3');
Ipv=evalin('base','Ipv');
Vpv=evalin('base','Vpv');
Ppv=evalin('base','Ppv');

figure (1)
plot(V1,I1,'g-')
hold on
plot(V2,I2,'c-')
hold on
plot(V3,I3,'m-')
hold on
plot(Vpv,Ipv,'r.')
legend('T1=240K','T2=290K','T3=340K','P&O MPPT')
title('MPPT V-I Curve')
xlabel('Voltage unit: V')
ylabel('Current unit: I')
grid on

figure (2)
plot(V1,P1,'g-')
hold on
plot(V2,P2,'c-')
hold on
plot(V3,P3,'m-')
hold on
plot(Vpv,Ppv,'r.')
legend('T1=240K','T2=290K','T3=340K','P&O MPPT')
title('MPPT V-P Curve')
xlabel('Voltage unit: V')
Appendix 3.2 P&O MPPT Strategy Model in Irradiation Varying Condition

```matlab
clc
I1=evalin('base','I1');
I2=evalin('base','I2');
I3=evalin('base','I3');
V1=evalin('base','V1');
V2=evalin('base','V2');
V3=evalin('base','V3');
P1=evalin('base','P1');
P2=evalin('base','P2');
P3=evalin('base','P3');
Ipv=evalin('base','Ipv');
```

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Vpv=evalin('base','Vpv');
Ppv=evalin('base','Ppv');

figure (1)
plot(V1,I1,'g-')
hold on
plot(V2,I2,'c-')
hold on
plot(V3,I3,'m-')
hold on
plot(Vpv,Ipv,'r.')
legend('G1=100W/m^2','G2=600W/m^2','G3=1100W/m^2','P&O MPPT')
title('MPPT V-I Curve')
xlabel('Voltage unit: V')
ylabel('Current unit: I')
grid on

figure (2)
plot(V1,P1,'g-')
hold on
plot(V2,P2,'c-')
hold on
plot(V3,P3,'m-')
hold on
plot(Vpv,Ppv,'r.')
legend('G1=100W/m^2','G2=600W/m^2','G3=1100W/m^2','P&O MPPT')
title('MPPT V-P Curve')
xlabel('Voltage unit: V')
ylabel('Power unit: W')
grid on

Appendix 3.3 FIS Design of Fuzzy Logic Control MPPT Strategy
Appendix 3.4 Comparison Model of P&O MPPT Strategy, IncCond MPPT Strategy and FLC MPPT Strategy
Appendix 3.5 Responding Results of Comparison between P&O MPPT Strategy, IncCond MPPT Strategy and FLC MPPT Strategy in Constant Condition (.m)

clc
I=evalin('base','I')
V=evalin('base','V')
P=evalin('base','P')
T=evalin('base','t')

Ipv_f=evalin('base','Ipv_f')
Vpv_f=evalin('base','Vpv_f')
Ppv_f=evalin('base','Ppv_f')

Ipv_po=evalin('base','Ipv_po')
Vpv_po=evalin('base','Vpv_po')
Ppv_po=evalin('base','Ppv_po')

Ipv_inc=evalin('base','Ipv_inc')
Vpv_inc=evalin('base','Vpv_inc')
Ppv_inc=evalin('base','Ppv_inc')

figure (1)
plot(V,I)
hold on
plot(Vpv_f,Ipv_f,'r*')
hold on
plot(Vpv_po,Ipv_po,'bo')
hold on
plot(Vpv_inc,Ipv_inc,'k.'
legend('V-I curve','fuzzy mppt','P&O mppt','IncCond mppt')
title('MPPT V-I Curve')
xlabel('Voltage unit: V')
ylabel('Current unit: I')
grid on

figure (2)
plot(V,P)
hold on
plot(Vpv_f,Ppv_f,'r*')
hold on
plot(Vpv_po,Ppv_po,'bo')
hold on
plot(Vpv_inc,Ppv_inc,'k.'

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Appendix 3.6 Re-regulation of FIS
Appendix 3.7 The Output Curve of Comparison between P&O MPPT Strategy, IncCond MPPT Strategy and FLC MPPT Strategy in Both Temperature and Irradiation Varying Condition

```matlab
clc
I=evalin('base','I')
V=evalin('base','V')
P=evalin('base','P')
T=evalin('base','t')

Ipv_f=evalin('base','Ipv_f')
Vpv_f=evalin('base','Vpv_f')
Ppv_f=evalin('base','Ppv_f')

Ipv_po=evalin('base','Ipv_po')
Vpv_po=evalin('base','Vpv_po')
```
Ppv_po=evalin('base','Ppv_po')

Ipv_inc=evalin('base','Ipv_inc')
Vpv_inc=evalin('base','Vpv_inc')
Ppv_inc=evalin('base','Ppv_inc')

figure (1)
plot(V,I)
hold on
plot(Vpv_f,Ipv_f,'r.')
hold on
plot(Vpv_po,Ipv_po,'b.')
hold on
plot(Vpv_inc,Ipv_inc,'k.')
legend('V-I curve','fuzzy mppt','P&O mppt','IncCond mppt')
title('MPPT V-I Curve')
xlabel('Voltage unit: V')
ylabel('Current unit: I')
grid on

figure (2)
plot(V,P)
hold on
plot(Vpv_f,Ppv_f,'r.')
hold on
plot(Vpv_po,Ppv_po,'b.')
hold on
plot(Vpv_inc,Ppv_inc,'k.')
legend('P-V curve','fuzzy mppt','P&O mppt','IncCond mppt')
title('MPPT V-P Curve')
xlabel('Voltage unit: V')
ylabel('Power unit: W')
grid on

figure (3)
plot(T,Ppv_f,'r-')
hold on
plot(T,Ppv_po,'b-')
hold on
plot(T,Ppv_inc,'k-')
legend('fuzzy mppt','P&O mppt','IncCond mppt')
title('Responding Speed of different MPPT Algorithms')
xlabel('Time (s)')
ylabel('Power (W)')
grid on

Appendix 4.1 Shading Main Program: PVprog [4-2]

clc

% This is the main program which calls programs PVprog2 and PVprog3. Therefore
% you will be required to download all these three programs.
% Input should be provided in such a way that the group with lowest
% insolation should be entered first and then the data should be entered
% into the ascending order with respect to the insolation.
% i.e. the final
% entry should be of the group with uniform full or maximum insolation.

no_parallel_config=input('Number of groups : ');  
col_plot=['b' 'g' 'r' 'c' 'm' 'y' 'k']; 

for j=1:no_parallel_config
    fprintf('
');  
    fprintf('DATA OF GROUP NUMBER NO. %d
',j);  
    S{j,1}=input('Number of subassemblies in an assembly  : ');  
    S{j,2}=input('Modules in subassembly; Temp; Insolation : ');  
    S{j,3}=input('Number of such assemblies in a group : ');  
end

for i=1:no_parallel_config  
    N(i)=S{i,3}(1,1);                        % Number of assemblies in a group  
    Sub_assemblies(i)=S{i,1}(1,1);           % Number of subassemblies  
end

for k=1:no_parallel_config
    for i=1:Sub_assemblies(k)
        n(k,i)=S{k,2}(1,i);                   % Number of modules in subassemblies  
        g(k,i)=S{k,2}(3,i);                  % Insolation on the modules on subassemblies 
        Temp(k,i)=S{k,2}(2,i);               % Temp on the modules on subassemblies 
        PVprog2(g(k,i),Temp(k,i),n(k,i),i,k);  
    end
end
del=0.5;

for k=1:no_parallel_config
    j=0;
    cont=1;
    I=0;
    x=0;
    V=0.001;
    flg1(k)=1;
    while (V>0)
        I=I+0.001;
        x=x+1;

        for(j=1:1:Sub_assemblies(k)) % No. of modules in sub-assembly* voltage of module
            volt_subassembly(j)=n(k,j)* PVprog3(I,g(k,j),Temp(k,j));
        end

        V=sum(volt_subassembly); % Sum of the voltage of sub-assemblies in a series

        if (x>2)
            dev_V=Vsum_last-V;
            if (dev_V>4) I=I-0.0009975;
            for(j=1:1:Sub_assemblies(k)) % No. of modules in sub-assembly* voltage of module
                volt_subassembly(j)=n(k,j)* PVprog3(I,g(k,j),Temp(k,j));
            end
            V=sum(volt_subassembly);
        end

        Vsum_last=V;
    end
end

volt(1,x)=V; % voltage of a group
curr(1,x)=I*N(k); % total current from similar series assemblies i.e from group

power(1,x)=volt(1,x)*curr(1,x); % power from group
 volt1(k,x)=volt(1,x);
curr1(k,x)=curr(1,x);
power1(k,x)=power(1,x);
end

curradj(k)=curr(1,x);
curr_seriesassembly(1,:)=curr(1,:)/N(k);
power_seriesassembly(1,:)=power(1,:)/N(k);
figure(2*no_parallel_config+1);
hold on;
legend('G1','G2','G3') %FOR FIXED 3 GROUPS

for i=1:length(VIP)
    VIP(1,i)=round(VIP(1,i));
end

for i=1:(length(VIP)-1)
if(VIP(1,i)==VIP(1,i+1))
    VIP(2,i+1)=VIP(2,i);
    VIP(3,i+1)=VIP(3,i);
end

len(k)=length(volt); %Store the lengths of the voltage

% Minimising the number of points on final output curves
len_count=length(VIP);
for i=1:len_count;
VIP(1,i)=round(VIP(1,i));
end
for i=1:len_count-1
if(VIP(1,i)==VIP(1,i+1))
VIP(2,i+1)=VIP(2,i);
VIP(3,i+1)=VIP(3,i);
end
end
p=1;
z(1,p)=VIP(1,1);
z(2,p)=VIP(2,1);
z(3,p)=VIP(3,1);
for i=1:len_count-1
if ((i~=1) & ((VIP(1,i))~=VIP(1,i+1)))
p=p+1;
z(1,p)=VIP(1,i+1);
z(2,p)=VIP(2,i+1);
z(3,p)=VIP(3,i+1);
end
end;
for i=1:length(z)
Vs(k,i)=z(1,i);
Is(k,i)=z(2,i);
Ps(k,i)=z(3,i);
end
end
for k=1:no_parallel_config
toglflg=0;
indx=length(Vs);
for i=1:length(Vs)
if (Vs(k,i)==0 & toglflg==0)
toglflg=1;
indx=i;
init=Vs(k,indx-1);
inc=length(Vs)-indx;
delinc=0;
end
if (i>=indx & i<length(Vs))
delinc = delinc + 1;
Vs(k,i) = (init)*(inc - delinc + 1)/(inc + 1);
end
end
end

for k = 1:no_parallel_config
    for i = 1:length(Vs)
        if (i >= 1 && Is(k,i) == 0)
            Is(k,i) = Is(k,i-1);
            Ps(k,i) = Is(k,i)*Vs(k,i);
        end
    end
end

for i = 1:length(Vs)
    Parray(i) = 0;
    Varray(i) = 0;
    Iarray(i) = 0;
end

for i = 1:length(Vs)
    Vt(i) = Vs(no_parallel_config,i);
    for k = 1:(no_parallel_config)
        Vtemp = Vs(k,:);
        Ptemp = Ps(k,:);
        Itemp = Is(k,:);
        Pgr(k,i) = interp1(Vtemp,Ptemp,Vt(i),'linear','extrap');
        Igr(k,i) = interp1(Vtemp,Itemp,Vt(i),'linear','extrap');
        if Igr(k,i) <= 0
            Igr(k,i) = 0;
            Pgr(k,i) = 0;
        end
    end
    Parray(i) = Pgr(k,i) + Parray(i);
    Iarray(i) = Igr(k,i) + Iarray(i);
end
    if (i > 1)
        if (Iarray(i) < Iarray(i-1))
            Iarray(i) = Iarray(i-1);
        end
    end
end
Varray = Vt;
figure;
hold on;
title('I-V characteristics of an array');
plot(Varray,Iarray);
figure;
hold on;
title('P-V characteristics of an array');
plot(Varray,Parray);

VIP=[Varray;Iarray;Parray];
save VIP;
clear all;
clc;
return

Shading sub-program: PVprog2

function temp=PVprog2(Sun,T,No_of_modules,Subass_no,Gr_no)
%PVprog2.m model for the MAX-^0 solar array returns %current, voltage and power for the given illumination and temperature %
% I,V are array current and voltage %Sun=num of suns where 1 Sun =1000W/m^2
col=['b' 'g' 'r' 'c' 'm' 'y' 'k'];
k= 1.38e-23; %Boltzman's constant
q=1.6e-19; %charge of an electron
V=1; %Initialisation of the array voltage
I=0; % and array current so that one enters into while loop atleast once
x=0; %initialisation
while (V>0);
I=I+.001;
x=x+1;

% enter the following constants here, and the model will be % calculated based on these for 1000 W/m^2
A=1.2; % 'diode quality' factor = 2 for crystalline and <2 for amorphous
Vg=1.12; % band gap voltage , 1.12eV for xtal Si, 1.75 for amorphous Si
Ns=36; % number of series connected cells (diodes)
T1=273+25;
Voc_T1=21.06/Ns; % open ckt voltage per cell at temp T1
Isc_T1=3.8; % short ckt current per cell at temp T1
T2=273+75;
Voc_T2=17.05/Ns; % open ckt voltage per cell at temp T2

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Isc_T2=3.92;  % short ckt current per cell at temp T2
Tak = 273 + T;  %array working temp
Trk = 273+25;  %reference temp
%when V=0 light generated current Iph_T1= array short
circuit current
%constant 'a' can be determined from Iac vs T

Iph_T1=Isc_T1*Sun;
a=(Isc_T2-Isc_T1)/Isc_T1*1/(T2-T1);
Iph=Iph_T1*(1+a*(Tak-T1));
Vt_T1=k*T1/q;  % =A*kT/q

Ir_T1=Isc_T1/(exp(Voc_T1/(A*Vt_T1))-1);
Ir_T2=Isc_T2/(exp(Voc_T2/(A*Vt_T1))-1);
b=Vg*q/(A*k);

Ir=Ir_T1*(Tak/T1).^(3/A).*exp(-b.*(1./Tak-1/T1));
X2v=Ir_T1/(A*Vt_T1)*exp(Voc_T1/(A*Vt_T1));
dVdI_Voc=-1.15/Ns/2;  %dV/dI at Voc per cell from
manufacturer graph
Rs=-dVdI_Voc-1/X2v;  %series resistance per cell

Vt_T=A*1.38e-23*Tak/1.6e-19;  % =A*kT/q

bracket=(log((Iph-I)/Ir+1));
flg=(isreal(bracket)& (bracket>0));
if (flg)
Vtemp=Vt_T*bracket-I*Rs;
if (Vtemp>0) V=Vtemp; end
else
V=V-0.001;
I=I-0.0009;
end
if (V<0) V=0; end;

volt(x)=36*V*No_of_modules;
curr(x)=I;

power(x)=volt(x)*curr(x);
end

figure(2*Gr_no-1);
Grid on;
hold on;
plot(volt,curr,col(Subass_no));

char GR_NO=Gr_no;
titl1 = 'IV characteristics for subassemblies of Group-';
    GR_NO=num2str(Gr_no);
    tit = strcat(titl1,GR_NO);
    Title(tit);
    Xlabel('Voltage (V)');
    Ylabel('Current (A)');
    figure(2*Gr_no);
    Grid on;
    hold on;
    plot(volt,power,col(Subass_no));

titl1 = 'PV characteristics for subassemblies of Group-';
    tit = strcat(titl1,GR_NO);
    Title(tit);
    Xlabel('Voltage (V)')
    Ylabel('Power (W)');
    temp=[curr;volt;power];
    return

Shading sub-Program: PVprog3

function V=P
    %PVprog3.m model for the solar array
    %takes input as illumination, sun and temperature
    %V=PVprog3(G,T)returns the array voltage
    %I,V are array current and voltage
    %Sun=num of suns where 1 Sun =1000W/m^2
    k= 1.38e-23;    %Boltzman's constant
    q=1.6e-19;      %charge of an electron
    Vc=0.01;

    % enter the following constants here, and the model will be
    % calculated based on these for 1000 W/m^2
    A=1.2;         % 'diode quality' factor = 2for crystalline and
                    <2 for amorphous
    Vg=1.12;       % band gap voltage , 1.12eV for xtal Si, 1.75
                    for amorphous Si
    Ns=36;         % number of series connected cells (diodes)
    T1=273+25;
    Voc_T1=21.06/Ns;    % open ckt voltage per cell at temp T1
    Isc_T1=3.8;       % short ckt current per cell at temp T1

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\begin{verbatim}
T2=273+75;
Voc_T2=17.05/Ns;  % open ckt voltage per cell at temp T2
Isc_T2=3.92;        % short ckt current per cell at temp T2
Tak =273+T;        %array working temp
Trk =273+25;       %reference temp

%when V=0 light generated current Iph_T1= array short circuit current
%constant 'a' can be determined from Iac vs T

Iph_T1=Isc_T1*Sun;
a=(Isc_T2-Isc_T1)/Isc_T1*1/(T2-T1);
Iph=Iph_T1*(1+a*(Tak-T1));

Vt_T1=k*T1/q;       % =A*kT/q
Ir_T1=Isc_T1/(exp(Voc_T1/(A*Vt_T1))-1);
Ir_T2=Isc_T2/(exp(Voc_T2/(A*Vt_T1))-1);

b=Vg*q/(A*k);
Ir=Ir_T1*(Tak/T1).^(3/A).*exp(-b.*(1./Tak-1/T1));
X2v=Ir_T1/(A*Vt_T1)*exp(Voc_T1/(A*Vt_T1));
dVdI_Voc=-1.15/Ns/2;        %dV/dI at Voc per cell from manufacturer graph
Rs=-dVdI_Voc-1/X2v;         %series resistance per cell
Vt_T=A*1.38e-23*Tak/1.6e-19; % =A*kT/q

%%%%Cheaking The Condition That log((Iph-I+Ir)/Ir) Is Real
Or Not%%%%%%

po=log((Iph-I+Ir)/Ir);

if(isreal(po) & (po>0))
    Vc=(Vt_T*po)-I*Rs;
    if (Vc>0) Vc=Vc; end
    else
        Vc=Vc-0.01;
    end

if(Vc<0)
    Vc=0;
end
\end{verbatim}
\[ V = V_c N_s \]

Appendix 4.2 Processing Time Varying Data from MATLAB M.file to SIMULINK

In Time Series Tools design:
When importing the previous data to SIMULINK for future analysis and design, the goal could not be achieved for the version problem.

Appendix 5.1 Tuning Results of PID Controller 2
Appendix 5.2 System synchronous frame control with PI controller in PSCAD

System modeling for constant illumination level
Control strategy of a three-phase grid-connected PV array system

System modeling for randomly changing illumination level

the PWM generator sends on and off pulses to the random number generation device which generates random values from 0 to 1200 to represent a changing illumination level and these values is applied as the input of the PV array.

Appendix 5.3 Analysis of Each Control Parameter

```matlab
%% LCL filter parameters definition
Cf=20e-6; % FILTER CAPACITOR (F)
Rc=0; % DAMPING RESISTOR (Ohm)
L1=5.5e-3; % INVERTER SIDE INDUCOR (H)
RL1=0.01; % EQUIVALENT RESISTOR (Ohm)
L2=1e-3; % GRID SIDE INDUCOR (H)
RL2=0.01;

%% initial assumption of control parameters
```
for Kp=[0.1, 0.3, 0.6, 0.9]
    Ki=10
    Kc=100
%% Open loop and Close Loop Transfer Function without parameter design
    A0=Kp*Kc;
    A1=Ki*Kc;
    B0=L1*L2*Cf;
    B1=RL1*L2*Cf+RL2*L1*Cf+L2*Cf*Kc;
    B2=L1+L2+RL1*RL2*Cf+RL2*Cf*Kc;
    B3=RL1+RL2;
    num_ol1=[A0 A1];
    den_ol1=[B0 B1 B2 B3];
    Gol1=tf(num_ol1,den_ol1)
    num_cl1=[A0 A1];
    den_cl1=[B0 B1 B2 (B3+A0) A1];
    Gcl1=tf(num_cl1,den_cl1)
%% zero-pole
    figure (1)
    pzmap(Gcl1);
    legend('Kp=0.1','Kp=0.3','Kp=0.6','Kp=0.9')
    grid on;
    hold on;
    title( 'zero-pole figure of control loop with variation of Kp')
    [p1,z1]=pzmap(Gcl1)
%% step response
    figure (2)
    stepplot(Gcl1)
    legend('Kp=0.1','Kp=0.3','Kp=0.6','Kp=0.9')
    hold on
    title( 'step response figure of control loop with variation of Kp')
%% root locus
    figure (3)
    rlocus(Gcl1);
    legend('Kp=0.1','Kp=0.3','Kp=0.6','Kp=0.9')
    hold on
    title( 'root locus figure of control loop with variation of Kp')
xlabel('real axis')
ylabel('imaginary axis')
end
for Ki=[0.1, 1, 10, 100]
KP=0.1
Kc=100

%% Open loop and Close Loop Transfer Function without parameter design
A0=Kp*Kc;
A1=Ki*Kc;
B0=L1*L2*Cf;
B1=RL1*L2*Cf+RL2*L1*Cf+L2*Cf*Kc;
B2=L1+L2+RL1*RL2*Cf+RL2*Cf*Kc;
B3=RL1+RL2;
num_ol1=[A0 A1];
den_ol1=[B0 B1 B2 B3];
Gol1=tf(num_ol1,den_ol1)
num_cl1=[A0 A1];
den_cl1=[B0 B1 B2 (B3+A0) A1];
Gcl1=tf(num_cl1,den_cl1)

%% zero-pole
figure (4)
pzmap(Gcl1);
legend('Ki=0.1','Ki=1','Ki=10','Ki=100')
grid on;
hold on;
title( 'zero-pole figure of control loop with variation of Ki')
[p1,z1]=pzmap(Gcl1)

%% step response
figure (5)
stepplot(Gcl1)
legend('Ki=0.1','Ki=1','Ki=10','Ki=100')
hold on

title( 'step response figure of control loop with variation of Ki')

%% root locus
figure (6)
rlocus(Gcl1);
legend('Ki=0.1','Ki=1','Ki=10','Ki=100')
hold on

title( 'root locus figure of control loop with variation of Ki')
xlabel('real axis')
ylabel('imaginary axis')
end

for Kc=[0.1, 1, 10, 100]
Ki=10
Kp=0.1

%% Open loop and Close Loop Transfer Function without parameter design
A0=Kp*Kc;
A1=Ki*Kc;
B0=L1*L2*Cf;
B1=RL1*L2*Cf+RL2*L1*Cf+L2*Cf*Kc;
B2=L1+L2+RL1*RL2*Cf+RL2*Cf*Kc;
B3=RL1+RL2;
num_ol1=[A0 A1];
den_ol1=[B0 B1 B2 B3];
Gol1=tf(num_ol1,den_ol1)
num_cl1=[A0 A1];
den_cl1=[B0 B1 B2 (B3+A0) A1];
Gcl1=tf(num_cl1,den_cl1)

%% zero-pole
figure (7)
pzmap(Gcl1);
legend('Kc=0.1','Kc=1','Kc=10','Kc=100')
grid on;
hold on;
title( 'zero-pole figure of control loop with variation of Kc')
[p1,z1]=pzmap(Gcl1)

%% step response
figure (8)
stepplot(Gcl1);
legend('Kc=0.1','Kc=1','Kc=10','Kc=100')
hold on

%% root locus
figure (9)
rlocus(Gcl1);
legend('Kc=0.1','Kc=1','Kc=10','Kc=100')
hold on

title( 'root locus figure of control loop with variation of Kc')

Appendix 5.4 Relationship Between Damping Ratio $\zeta$ and Natural Frequency $\omega$
%% LCL filter parameters definition
Cf=20e-6;  \% FILTER CAPACITOR (F)
Rc=0;     \% DAMPING RESISTOR (Ohm)
L1=5.5e-3; \% INVERTER SIDE INDUCTOR (H)
RL1=0.01; \% EQUIVALENT RESISTOR (Ohm)
L2=1e-3;  \% GRID SIDE INDUCTOR (H)
RL2=0.01;

%% relationship between damping ratio 'e' and natural frequency 'w'
b3=RL1+RL2
b0=L1*L2*Cf
b2=L1+L2+RL1*RL2*Cf
m=5
for e=0:0.001:1;
    W=[(2*m*e^2+1)^2*b0 0 -(2*m*e^2+1)*b2 (m+2)*e*b3];
    w=roots(W);
    figure (10)
    plot(e,w)
    hold on
    grid on
end
e=0.707
W=[(2*m*e^2+1)^2*b0 0 -(2*m*e^2+1)*b2 (m+2)*e*b3];
    w=roots(W)

Appendix 5.5 Calculation of Controller Parameters

(1) Calculation for $K_c$
Insert $B_3 = R_{L1} + R_{L2}, B_2 = L_1 + L_2 + R_{L1}R_{L2}C_f + R_{L2}C_f K_c, B_0 = L_1L_2C_f$ into equation $Y_1 - X_1 = (2 + m)\omega$ (5.30) and $Y_3 - X_1(Y_2 - X_1(Y_1 - X_1)) = 0$ (5.26), the $\frac{K_i}{K_p}$ part could be deleted, the $K_c$ will be left as:

$$K_c = \frac{\frac{(R_{L1} + R_{L2})}{(2m\zeta^2 + 1)\omega^2} + (m + 2)\zeta\omega(L_1L_2C_f) - (R_{L1}L_2C_f + R_{L2}L_2C_f)}{L_2C_f}$$

(5.34)
Appendix 5.6 PI Parameter Design and System Analysis Process

%% Calculation of Controller Parameters

e=0.6
\[ W = \left(2 \cdot m \cdot e^2 + 1\right)^2 \cdot b_0 - \left(2 \cdot m \cdot e^2 + 1\right) \cdot b_2 (m+2) \cdot e \cdot b_3; \]
\[ w = \text{roots}(W) \]
\[ K_c = \left(\frac{R_{L1}L_2C_f + R_{L2}L_1C_f + L_2C_f K_c}{L_1L_2C_f} \right)^{-1} - (2 + m) \zeta \omega \]

%% System Stability Test based on

Kc=83
Kp=0.1834
Ki=2.0650
A0=Kp*Kc;
A1=Ki*Kc;
B0=L1*L2*Cf;
B1=RL1*L2*Cf+RL2*L1*Cf+L2*Cf*Kc;
B2=L1+L2+RL1*RL2*Cf+RL2*Cf*Kc;
B3=RL1+RL2;

\[ \text{num}_{\text{ol1}} = \left[ A0 \ A1 \right]; \]
\[ \text{den}_{\text{ol1}} = \left[ B0 \ B1 \ B2 \ B3 \right]; \]
\[ \text{Gol1} = \text{tf}(\text{num}_{\text{ol1}}, \text{den}_{\text{ol1}}) \]
\[ \text{num}_{\text{cl1}} = \left[ A0 \ A1 \right]; \]
\[ \text{den}_{\text{cl1}} = \left[ B0 \ B1 \ B2 \ (B3+A0) \ A1 \right]; \]
\[ \text{Gcl1} = \text{tf}(\text{num}_{\text{cl1}}, \text{den}_{\text{cl1}}) \]

%% zero-pole
figure (11)
pzmap(Gcl1);
grid on;
hold on;
title('zero-pole figure of control loop')
[pl,z1]=pzmap(Gcl1)
%%% step response
figure (12)
stepplot(Gcl1)
hold on
title('step response figure of control loop')
%%% root locus
figure (13)
rlocus(Gcl1);
hold on
title('root locus figure of control loop')
xlabel('real axis')
ylabel('imaginary axis')
%%% bode plot
figure(14)
bode(Gcl1);
grid on;
[Gm,Pm,Wcg,Wcp]=margin(Gcl1)
margin(Gcl1)

Appendix 5.7 P+ Resonant Controller Parameter Test and System Analysis Process

clc
%%% LCL filter parameters definition
Cf=20e-6; % FILTER CAPACITOR (F)
Rc=0; % DAMPING RESISTOR (Ohm)
L1=5.5e-3; % INVERTER SIDE INDUCOR (H)
RL1=0.01; % EQUIVALENT RESISTOR (Ohm)
L2=1e-3; % GRID SIDE INDUCOR (H)
RL2=0.01;
Kc=83
Kp=0.1834
Ki=2.0650
w0=2*pi*60
%%% open loop and close loop transfer function
A0=Kp*Kc;
A1=Ki*Kc;
B0=L1*L2*Cf;
\[ \begin{align*}
B_1 &= RL_1 \cdot L_2 \cdot C_f + RL_2 \cdot L_1 \cdot C_f + C_f \cdot K_c; \\
B_2 &= L_1 + L_2 + RL_1 \cdot RL_2 \cdot C_f + RL_2 \cdot C_f \cdot K_c; \\
B_3 &= RL_1 + RL_2; \\
\text{num}_{\text{ol}} &= [A_0 \ Ki \cdot K_c \ A_0 \cdot w_0^2]; \\
\text{den}_{\text{ol}} &= [B_0 \ B_1 \ B_2 + B_0 \cdot w_0^2 \ B_3 + B_1 \cdot w_0^2 \ B_2 \cdot w_0^2 \ B_3 \cdot w_0^2]; \\
G_{ol} &= \text{tf} (\text{num}_{\text{ol}}, \text{den}_{\text{ol}}) \\
\text{num}_{\text{cl}} &= [A_0 \ Ki \cdot K_c \ A_0 \cdot w_0^2]; \\
\text{den}_{\text{cl}} &= [B_0 \ B_1 \ B_2 + B_0 \cdot w_0^2 \ A_0 + B_3 + B_1 \cdot w_0^2 \ Ki \cdot K_c + B_2 \cdot w_0^2 \ (A_0 + B_3) \cdot w_0^2]; \\
G_{cl} &= \text{tf} (\text{num}_{\text{cl}}, \text{den}_{\text{cl}}) \\
\% \ zero-pole \\
\text{figure} (1) \\
pzmap(G_{cl}); \\
grid \ on; \\
hold \ on; \\
\text{title}( \ 'zero-pole \ figure \ of \ control \ loop' ) \\
[p,z] = pzmap(G_{cl}) \\
\% \ step \ response \\
\text{figure} (2) \\
stepplot(G_{cl}); \\
hold \ on \\
\text{title}( \ 'step \ response \ figure \ of \ control \ loop' ) \\
\% \ root \ locus \\
\text{figure} (3) \\
rlocus(G_{cl}); \\
hold \ on \\
\text{title}( \ 'root \ locus \ figure \ of \ control \ loop' ) \\
xlabel('real \ axis') \\
ylabel('imaginary \ axis') \\
\% \ bode \ plot \\
\text{figure}(4) \\
bode(G_{cl}); \\
grid \ on; \\
[G_{m}, P_{m}, W_{cg}, W_{cp}] = \text{margin}(G_{cl}) \\
\text{margin}(G_{cl}) \\
\end{align*} \]

Appendix 5.8 Bode Plot Diagram of Both PI Controller and P+ Resonant Controller at Fundamental Frequency

```matlab
clc 
\% LCL filter parameters definition 
C_f=20e-6; \% FILTER CAPACITOR (F) 
R_c=0; \% DAMPING RESISTOR (Ohm) 
L_1=5.5e-3; \% INVERTER SIDE INDUCTOR (H)
```

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RL1=0.01;      % EQUIVALENT RESISTOR (Ohm)
L2=1e-3;        % GRID SIDE INDUCTOR (H)
    RL2=0.01;
    Kc=83
    Kp=0.1834
    Ki=2.0650
    w0=2*pi*60

%% P+R and PI close loop transfer function
A0=Kp*Kc;
A1=Ki*Kc;
B0=L1*L2*Cf;
B1=RL1*L2*Cf+RL2*L1*Cf+L2*Cf*Kc;
B2=L1+L2+RL1*RL2*Cf+L2*Cf*Kc;
B3=RL1+RL2;

%% P+ Resonant
num_pr=[A0 Ki*Kc A0*w0^2];
den_pr=[B0 B1 B2+B0*w0^2 A0+B3+B1*w0^2 Ki*Kc+B2*w0^2 (A0+B3)*w0^2];
Gpr=tf(num_pr,den_pr)

%% PI
num_pi=[A0 A1];
den_pi=[B0 B1 B2 (B3+A0) A1];
Gpi=tf(num_pi,den_pi)

%% bode plot
figure(1)
bode(Gpr)
hold on;
bode(Gpi)
hold on;
grid on;

[Gm_pr,Pm_pr,Wcg_pr,Wcp_pr]=margin(Gpr)
margin(Gpr)

[Gm_pi,Pm_pi,Wcg_pi,Wcp_pi]=margin(Gpi)
margin(Gpi)

Appendix 5.9 Comparison Between PI Controller and P+ Resonant Controller under Disturbing Condition
Appendix 5.10 LCL Filter Parameter Analysis

(1) $L_1$ varying

```matlab
clc

%% LCL filter parameters definition
Cf=20e-6; % FILTER CAPACITOR (F)
Rc=0;    % DAMPING RESISTOR (Ohm)
l1=5.5e-3; % INVERTER SIDE INDUCTOR (H)
RL1=0.01; % EQUIVALENT RESISTOR (Ohm)
L2=1e-3; % GRID SIDE INDUCTOR (H)
RL2=0.01;
for L1=[0.5*l1, 0.6*l1, 0.7*l1, 0.8*l1, 0.9*l1, l1, 1.1*l1, 1.2*l1, 1.3*l1, 1.4*l1, 1.5*l1];
```
\[
\begin{align*}
K_c &= 83; \\
K_p &= 0.1834; \\
K_i &= 2.0650; \\
w_0 &= 2\pi \times 60;
\end{align*}
\]

% Open loop and Close Loop Transfer Function without
% parameter design
\[
A_0 = K_p \times K_c; \\
A_1 = K_i \times K_c; \\
B_0 = L_1 \times L_2 \times C_f; \\
B_1 = R_{L1} \times L_2 \times C_f + R_{L2} \times L_1 \times C_f + L_2 \times C_f \times K_c; \\
B_2 = L_1 + L_2 + R_{L1} \times R_{L2} \times C_f + R_{L2} \times C_f \times K_c; \\
B_3 = R_{L1} + R_{L2};
\]

\[
\begin{align*}
\text{num}_\text{ol1} &= [A_0 \ A_1]; \\
\text{den}_\text{ol1} &= [B_0 \ B_1 \ B_2 \ B_3]; \\
\text{G}_{\text{o}1} &= \text{tf}(\text{num}_\text{ol1}, \text{den}_\text{ol1}); \\
\text{num}_\text{cl1} &= [A_0 \ A_1]; \\
\text{den}_\text{cl1} &= [B_0 \ B_1 \ B_2 \ (B_3+A_0) \ A_1]; \\
\text{G}_{\text{c}1} &= \text{tf}(\text{num}_\text{cl1}, \text{den}_\text{cl1}); \\
\text{num}_\text{ol1} &= [A_0 \ A_1]; \\
\text{den}_\text{ol1} &= [B_0 \ B_1 \ B_2 \ B_3]; \\
\text{G}_{\text{o}1} &= \text{tf}(\text{num}_\text{ol1}, \text{den}_\text{ol1}); \\
\text{num}_\text{cl1} &= [A_0 \ A_1]; \\
\text{den}_\text{cl1} &= [B_0 \ B_1 \ B_2 \ (B_3+A_0) \ A_1]; \\
\text{G}_{\text{c}1} &= \text{tf}(\text{num}_\text{cl1}, \text{den}_\text{cl1}); \\
% \% \text{zero-pole} \\
% \text{figure (1)} \\
pzmap(\text{G}_{\text{c}1}); \\
\text{grid on}; \\
\text{hold on}; \\
\text{legend}(\text{'50\% L1', '60\% L1', '70\% L1', '80\% L1', '90\% L1', '100\% L1', '110\% L1', '120\% L1', '130\% L1', '140\% L1', '150\% L1'}) \\
\text{title( 'zero-pole figure of varying L1')} \\
[p1, z1] = \text{pzmap}(\text{G}_{\text{c}1}); \\
% \% \text{step response} \\
% \text{figure (2)} \\
\text{stepplot}(\text{G}_{\text{c}1}); \\
\text{grid on}; \\
\text{hold on}; \\
\text{legend}(\text{'50\% L1', '60\% L1', '70\% L1', '80\% L1', '90\% L1', '100\% L1', '110\% L1', '120\% L1', '130\% L1', '140\% L1', '150\% L1'}) \\
\text{title( 'step response figure of varying L1')} \\
% \% \text{root locus} \\
% \text{figure (3)} \\
rlocus(\text{G}_{\text{c}1}); \\
\text{grid on};
(2) L2 varying

clc

% LCL filter parameters definition
Cf=20e-6; % FILTER CAPACITOR (F)
Rc=0; % DAMPING RESISTOR (Ohm)
L1=5.5e-3; % INVERTER SIDE INDUCOR (H)
RL1=0.01; % EQUIVALENT RESISTOR (Ohm)
L2=1e-3; % GRID SIDE INDUCTOR (H)
RL2=0.01;

for L2=[0.5*L2, 0.6*L2, 0.7*L2, 0.8*L2, 0.9*L2, L2, 1.1*L2, 1.2*L2, 1.3*L2, 1.4*L2, 1.5*L2];
    Kc=83;
    Kp=0.1834;
    Ki=2.0650;
    w0=2*pi*60;

% Open loop and Close Loop Transfer Function without parameter design
A0=Kp*Kc;
A1=Ki*Kc;
B0=L1*L2*Cf;
B1=RL1*L2*Cf+RL2*L1*Cf+L2*Cf*Kc;
B2=L1+L2+RL1*RL2*Cf+RL2*Cf*Kc;
B3=RL1+RL2;
num_ol1=[A0 A1];
den_ol1=[B0 B1 B2 B3];
Gol1=tf(num_ol1,den_ol1);
num_cl1=[A0 A1];
den_cl1=[B0 B1 B2 (B3+A0) A1];
end
Gcl1=tf(num_cl1,den_cl1);
    num_ol1=[A0 A1];
    den_ol1=[B0 B1 B2 B3];
    Gol1=tf(num_ol1,den_ol1);
    num_cl1=[A0 A1];
    den_cl1=[B0 B1 B2 (B3+A0) A1];
    Gcl1=tf(num_cl1,den_cl1);

    % zero-pole
    figure (1)
    pzmap(Gcl1);
    grid on;
    hold on;

    legend('50\%L2','60\%L2','70\%L2','80\%L2','90\%L2','100\%L2','110\%L2','120\%L2','130\%L2','140\%L2','150\%L2')
    title('zero-pole figure of varying L2')
    [p1,z1]=pzmap(Gcl1)

    % step response
    figure (2)
    stepplot(Gcl1)
    hold on
    legend('50\%L2','60\%L2','70\%L2','80\%L2','90\%L2','100\%L2','110\%L2','120\%L2','130\%L2','140\%L2','150\%L2')
    title('step response figure of varying L2')

    % root locus
    figure (3)
    rlocus(Gcl1)
    hold on

    legend('50\%L2','60\%L2','70\%L2','80\%L2','90\%L2','100\%L2','110\%L2','120\%L2','130\%L2','140\%L2','150\%L2')
    title('root locus figure of varying L2')
xlabel('real axis')
ylabel('imaginary axis')

    % bode plot
    figure(4)
    bode(Gcl1)
    hold on;
    grid on;

    legend('50\%L2','60\%L2','70\%L2','80\%L2','90\%L2','100\%L2','110\%L2','120\%L2','130\%L2','140\%L2','150\%L2')

    [Gm,Pm,Wcg,Wcp]=margin(Gcl1)
    margin(Gcl1);
end

(3) Cf varying
clc

%% LCL filter parameters definition

\[ cf = 20 \times 10^{-6}; \] % FILTER CAPACITOR (F)
\[ Rc = 0; \] % DAMPING RESISTOR (Ohm)
\[ L1 = 5.5 \times 10^{-3}; \] % INVERTER SIDE INDUCTOR (H)
\[ RL1 = 0.01; \] % EQUIVALENT RESISTOR (Ohm)
\[ L2 = 1 \times 10^{-3}; \] % GRID SIDE INDUCTOR (H)
\[ RL2 = 0.01; \]

for \[ Cf = [0.5 \times cf, 0.6 \times cf, 0.7 \times cf, 0.8 \times cf, 0.9 \times cf, cf, 1.1 \times cf, 1.2 \times cf, 1.3 \times cf, 1.4 \times cf, 1.5 \times cf]; \]
\[ Kc = 83; \]
\[ Kp = 0.1834; \]
\[ Ki = 2.0650; \]
\[ w_0 = 2 \times \pi \times 60; \]

% Open loop and Close Loop Transfer Function without parameter design
\[ A_0 = Kp \times Kc; \]
\[ A_1 = Ki \times Kc; \]
\[ B_0 = L_1 \times L_2 \times Cf; \]
\[ B_1 = RL_1 \times L_2 \times Cf + RL_2 \times L_1 \times Cf + L_2 \times Cf \times Kc; \]
\[ B_2 = L_1 + L_2 + RL_1 \times RL_2 \times Cf + RL_2 \times Cf \times Kc; \]
\[ B_3 = RL_1 + RL_2; \]
\[ num_ol_1 = [A_0 A_1]; \]
\[ den_ol_1 = [B_0 B_1 B_2 B_3]; \]
\[ G_{ol1} = \text{tf}(num_ol_1, den_ol_1); \]
\[ num_cl_1 = [A_0 A_1]; \]
\[ den_cl_1 = [B_0 B_1 B_2 (B_3 + A_0) A_1]; \]
\[ G_{cl1} = \text{tf}(num_cl_1, den_cl_1); \]
\[ legend('50\%Cf','60\%Cf','70\%Cf','80\%Cf','90\%Cf','100\%Cf','110\%Cf','120\%Cf','130\%Cf','140\%Cf','150\%Cf') \]
\[ title('zero-pole figure of varying Cf') \]
\[ [pl, zl] = \text{pzmap}(G_{cl1}) \]
\[ \text{hold on}; \]
\[ \text{step response} \]

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figure (2)
    stepplot(Gcl1)
    hold on
legend('50%Cf','60%Cf','70%Cf','80%Cf','90%Cf','100%Cf','110%Cf','120%Cf','130%Cf','140%Cf','150%Cf')
title( 'step response figure of varying Cf')
    \% root locus
figure (3)
rlocus(Gcl1);
    hold on
legend('50%Cf','60%Cf','70%Cf','80%Cf','90%Cf','100%Cf','110%Cf','120%Cf','130%Cf','140%Cf','150%Cf')
title( 'root locus figure of varying Cf')
    xlabel('real axis')
    ylabel('imaginary axis')
    \% bode plot
figure(4)
bode(Gcl1);
    hold on;
grid on;
legend('50%Cf','60%Cf','70%Cf','80%Cf','90%Cf','100%Cf','110%Cf','120%Cf','130%Cf','140%Cf','150%Cf')
[Gm,Pm,Wcg,Wcp]=margin(Gcl1)
    margin(Gcl1);
end