Finite Element Analysis of Thermal Buckling in Automotive Clutch and Brake Discs

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FINITE ELEMENT ANALYSIS OF THERMAL BUCKLING IN AUTOMOTIVE CLUTCH AND BRAKE DISCS

A Thesis

Presented to

the Faculty of the Daniel Felix Ritchie School of Engineering and Computer Science

University of Denver

In Partial Fulfillment

of the Requirements for the Degree

Master of Science

by

Huizhou Yang

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ABSTRACT

Thermal buckling behavior of automotive clutch and brake discs is studied by making the use of finite element method. It is found that the temperature distribution along the radius and the thickness affects the critical buckling load considerably. The results indicate that a monotonic temperature profile leads to a coning mode with the highest temperature located at the inner radius. Whereas a temperature profile with the maximum temperature located in the middle leads to a dominant non-axisymmetric buckling mode, which results in a much higher buckling temperature. A periodic variation of temperature cannot lead to buckling. The temperature along the thickness can be simplified by the mean temperature method in the single material model. The thermal buckling analysis of friction discs with friction material layer, cone angle geometry and fixed teeth boundary conditions are also studied in detail. The angular geometry and the fixed teeth can improve the buckling temperature significantly. Young’s Modulus has no effect when single material is applied in the free or restricted conditions. Several equations are derived to validate the result. Young’s modulus ratio is a useful factor when the clutch has several material layers. The research findings from this paper are useful for automotive clutch and brake discs design against structural instability induced by thermal buckling.
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CHAPTER ONE: INTRODUCTION

1.1 Clutch System Introduction

1.1.1 System, components and operation

Clutch is a mechanical device that engages and disengages the power transmission. In the basic application, clutches connect and disconnect driving shafts and driven shafts. One shaft is typically attached to an engine or other power source while the other shaft provides output power for work. A typical automotive friction clutch system is illustrated in Figure 1-1.

![Typical friction clutch system and components](image)

Figure 1-1: Typical friction clutch system and components

In general, a friction clutch system consists of a rotor clutch disc, which rotates with the gearbox input shaft. When the pedal is gradually released, the pressure plates are
forced inward and press the clutch disc against the flywheel which is driven by the motor.

With the friction between the surfaces, the clutch connects the two shafts so they may be locked together but spinning at different speeds (slipping), and finally they spin at the same speed (engaged). The power will output from the driven shaft.

1.1.2 Classification and materials

According to the demand of industry, clutches can be classified into four categories: electromagnetic clutch, magnetic powder clutch, hydraulic clutch and friction clutch, where friction clutches are the vast majorities which ultimately rely on frictional forces for their operations. The purpose is to connect a moving member to another one that is stationary or moving at a different speed, often to synchronize the speeds and transmit the power. Clutches can also be classified as dry clutches and wet clutches by the cooling principle.

Various materials have been used for the disc-friction facings, including asbestos in the past. Modern clutches typically use a compound organic resin with copper wire facing or a ceramic material. Ceramic materials are typically used in heavy applications such as racing or heavy-duty hauling[1].

1.2 Brake System Introduction

1.2.1 System, components and operation

A brake is a mechanical device which stops or slows the movement of the objective[2]. According to the operation method, brakes can be divided into several types.
A most common brake system used in vehicles and the components are shown in Figure 1-2.

![Brake system and components](image)

Figure 1-2: Typical brake system and components

A friction brake usually consists of three main components. The rotor disc which rotates with the wheel is fixed on the hub. A brake caliper is mounted above the disc. Two friction pads are fixed on the caliper with one on each side of the disc. Brake pads can be inspected for wear via an inspection hole. If a brake pad is worn it is important to get it replaced as a worn brake pad can damage the brake disc.

The working principle of a friction brake system is shown in Figure 1-3. When the brake pedal is pushed down, a piston inside the brake master cylinder is also pushed
which causes the brake fluid forward into the slave cylinder along the brake line. A piston in the slave cylinder is forced by the fluid to push the brake pad against the brake disc which slows down the wheel\cite{3-4}.

![Figure 1-3: Working principle of a friction brake system](image)

### 1.2.2 Classification and materials

In generally, the brake disc is made of gray iron\cite{5}, a kind of cast iron due to the excellent resistance to wear. The design of the disc varies by the weight and the power of the vehicles. Ventilated discs\cite{6} are introduced to facilitate the heat dissipation process. They are used typically on the heavily-loaded front discs.

Asbestos was used many years for brake pads. Recently, the materials capable of resisting brake fade and providing smooth contact\cite{7} are considered. Four principal categories are mainly used nowadays. Non-metallic materials are gentle but have a short life. Semi-metallic materials last longer but are hard to generate braking torque. Fully
metallic materials last longer but work very loudly. Ceramic materials are quieter but have relatively poor heat dissipation capabilities\textsuperscript{[8]}. The selections are determined by the vehicle operating conditions.

1.3 **Background and Significance of the Study**

1.3.1 **Related thermal problems in automotive clutch and brake system**

An automotive brake system slows or stops the vehicle by converting the kinetic energy into thermal energy through the friction between the disc and brake pads. The heat generated in the procedure is absorbed by the friction surfaces and then dissipates into the environment by conduction and convection. During the friction procedure, the brake pads are forced against the brake disc. A vehicle driving at 55 mph can be stopped in 200 feet by a 15 feet/s\textsuperscript{2} deceleration in less than 4 seconds. A test reported by Halderman\textsuperscript{[3]} showed that the peak temperatures across the brake disc can reach 980°C. Another investigation\textsuperscript{[9]} shows that about 95\% of the heat produced by friction is absorbed by the brake disc which causes the temperature to rise quickly. The brake disc can bear a maximum 800°C in general.

The function of the clutch system is to engage and disengage the power transmission. When engaged, the clutch system transmits the power from the motor to the gearbox, finally drives the vehicle. However, during the engagement process, the clutch system can be considered as a brake system which converts kinetic energy into thermal energy through the friction between the clutch disc and the flywheel. The generated heat is
absorbed by the friction disc and the metal plates on the flywheel. This energy is partly
dissipated to the environment through heat convection by air or liquid and partly remains
in the system by thermal conduction.

In both of the clutch and brake friction procedures, the heat generated between the
friction surfaces is not uniform. That because with the same angular velocity, the sliding
speed varies with the radius, with a lower speed at the inner radius and a higher speed at
the outer radius. This non-uniform heat generation produces a high non-uniform
temperature distribution across the friction disc. Combined with the physical constraints,
thermal stresses will be generated. The clutch and brake discs are sensitive to a series of
failure modes due to the high temperature and the stress, e.g., thermoelastic instability
(TEI) and hot spots\cite{10,11}, thermal distortion\cite{12}, and thermal buckling\cite{13}.

In clutch or brake sliding bodies, a disturbance might change the uniform pressure
distribution and hence the frictional heat generation. This produces non-uniform
thermoelastic distortion and further non-uniformity in the contact pressure distribution. If
the sliding speed is sufficiently high, the thermal-mechanical feedback process is unstable,
leading eventually to the localization of the load in a small region of the nominal contact
area of the sliding surfaces\cite{14}. This phenomenon is known as TEI, which was first
reported by Parker and Marshall (1948) and explained by Barber (1967, 1969). Burton\cite{15}
first proposed the friction contact model and introduced the critical speed. The results
show that TEI only occurs when the sliding speed exceeds the critical speed. Fec et al.\cite{16}
first found that TEI leads to thermal fatigue in train brakes. Anderson et al.\cite{17} found that the unsatisfactory performance in the braking operation is induced by an excess of local temperature distribution. Barber and Lee\cite{18} found that the antisymmetric mode dominate the TEI. They also explained the predicted critical speed difference between the theory and experiments.

Thermal distortion mainly leads to the following problems: lateral runout, disc thickness variation (DTV) and thermal coning. Runout is the lateral deviation of the disc surface, caused by extreme heat or rapid temperature variations. DTV is the variation in the thickness of the disc. It is also named as ‘hot judder’ which is a thermal case of ‘brake judder’\cite{19-21}. Thermal coning occurs because the inner plates are constrained and the outer is displaced in the normal direction due to the non-uniform distribution.

Thermal buckling is due to thermal stress caused by non-uniform temperature distribution. Among these failures, the buckling phenomenon is one of the most important modes which will be introduced separately in the next section.

1.3.2 Introduction of thermal buckling

Among these failure modes, many of them are directly related to the heat input including TEI, hot spots and thermal distortion. However, the induced thermal stresses in the disc may reach a critical level which will lead to thermal buckling. Briefly a sufficiently large temperature gradient in the radial direction can result in the
circumferential or “hoop” thermal stress that leads to the loss of structural stability in some extreme cases\textsuperscript{22}.

In science, buckling is a mathematical instability, leading to a failure mode. Theoretically, buckling is caused by a bifurcation in the solution to the equations of static equilibrium. At a certain stage under an increasing load, further load is able to be sustained in one of two states of equilibrium: a purely compressed state (with no lateral deviation) or a laterally-deformed state. Buckling is characterized by a sudden failure of a structural member subjected to high compressive stress. This stress is lower than the loading capability stress at that point\textsuperscript{23}. It is usually caused by pressure, concentrated force, and temperature gradient. Buckling due to thermal stresses has been concerned significantly in the engineering applications, such as brake systems in automotive engineering, nuclear engineering and aerospace engineering.

The buckling phenomenon in vehicle transmission systems often occurs in two modes: “coning” mode and “potato chip” mode\textsuperscript{24}, which are shown in Figure 1-4. When a temperature gradient exists in the direction of the radius, the thermal stress can lead to the occurrence of thermal buckling in extreme cases. Consequently, the primary research of the thermal buckling problem in vehicle brakes and clutches is to find the critical temperature gradient when thermal buckling occurs.
Even if the thermal stresses are not large enough to cause buckling, they could produce a reduction in the apparent bending stiffness of the disc, which will lower the buckling loads as a result\cite{25}. Therefore, thermal buckling is a phenomenon that may work alone or be coupled with other thermomechanical problems to influence the stability of clutch and brake systems.

1.3.3 **Significance and objective**

The sliding process in friction systems generates heat from friction. The heat generated is usually distributed non-uniformly, which produces a non-uniform temperature gradient across the disc. This temperature distribution coupled with the boundary constraints, produces thermal stresses which cause the disc failure. Vehicle
maintenance is usually expensive and complained frequently. Therefore, it is important to understand the mechanisms of the thermomechanical problems. Then the designs can be optimized.

The objective of this thesis is first to study and understand the buckling behavior of friction discs, including the buckling modes and critical temperatures, then to develop methodologies to study thermal buckling of clutch and brake discs with realistic geometry shapes, material properties, temperature distributions and boundary conditions. The ultimate goal is to establish a way to predict the critical buckling load of a given disc plate, which will contribute to the design of clutches and brakes required in the realistic operating situations. This will help minimize the possibility of the buckling failures.

1.4 Overview of Thesis

This thesis consists of seven chapters. Chapter 1 is devoted to introducing the automotive clutch and brake systems, components, and how they work. Classification and materials are also introduced. The relative thermal problems are briefly discussed and thermal buckling is discussed separately. This is followed by an explanation on the significance of thermal buckling. Finally, the objectives of this work are presented.

Chapter 2 presents a review of the prior work in thermal buckling research. Some of the buckling theories and equations are also reviewed in this chapter. These reviews show the importance of studying thermal buckling in automotive systems. From reviewing the
previous work, some disadvantages are concluded. The finite element approach is introduced in this chapter.

In chapter 3, a convergence study is presented first to compare the different elements can be used in clutch disc buckling research. A realistic clutch (with the detailed the dimension and the material) are chosen to be the uniform standard used in the study. The effects of different patterns of temperature distributions (uniform in the thickness direction) are studied. The critical buckling loads and the buckling modes are discussed.

In chapter 4, a parameter study of the disc material properties is introduced. An analytic validation is derived from the buckling theories. The effects of different patterns temperature distributions along the thickness direction of the brake disc are studied. The results are discussed and concluded in the end of this chapter.

In chapter 5, a clutch disc with a manufactured cone angle is studied. Axisymmetric elements are used to validate the results. A conclusion is presented at the end.

In chapter 6, a clutch disc with friction material layer at the surface is studied. Non-free boundary conditions are applied to this model as well. A detailed conclusion is presented.

Finally, the overall review of the thesis and the conclusion are presented in chapter 7, also with the further study in this field.
CHAPTER TWO: LITERATURE REVIEW

2.1 Review of Thermal Buckling Study

2.1.1 Review of buckling history

The study on the buckling phenomenon can be traced back to the 18th century. The instability problem was first raised by Euler Bernoulli. The famous Euler formula theory was obtained and used nowadays to predict the buckling load in linear elastic problem. In the Euler buckling model\[^{26-27}\], the applied load is gradually increased to exceed the critical load, and a buckled structure is obtained from the original configuration. Timoshenko offers the unique perspective of one who lived much of the history of buckling in his book\[^{28}\]. Karman\[^{29}\] built non-linear large deflection buckling theory in 1941 which greatly contributed to the development of the buckling study. Lindberg et al\[^{30}\] studied the dynamic buckling which is caused by suddenly loaded or released load. Bigoni\[^{31}\] studied the flutter instability which is caused by a follower load. Recently, they proved that flutter instability can be directly induced by dry friction\[^{32}\]. They also showed that the prescribed movement of the end of the structure against the curvature of the constraints strongly affects the buckling of an elastic structure\[^{33}\]. Often, the buckling is related to compression. Zaccaria et al\[^{34}\] recently found that dead tensile load can also
cause buckling and instability. In addition the weight of the structure can lead to buckling when its height exceeds the critical value, and this type is named as self-buckling\cite{35-37}.

The buckling studies based on the beam problems have been reviewed above. The first buckling study on circular plate was in 1891 by Bryan\cite{38}. A thin circular plate without a central hole was assumed in his paper. The result shows that the critical buckling load was related to a radially symmetric buckling mode.

The pioneering research of thermal buckling problem was studied by Gossard\cite{39} who outlined an approximate method, based on the large-deflection plate theory, for calculating the deflections of flat or initially imperfect plates subjected to thermal buckling. A simply supported panel was used as the model. In the study, a tentlike temperature distribution was applied across the structure. An experimental result obtained from the literature was compared with the work presented in the paper. The results showed a good agreement.

A summary of the previous work (before 2004) in this field has been presented in the review articles by Ma\cite{40} including the history of buckling study, thermal buckling study, imperfection buckling problems, and TEI study.

All of the studies reviewed above and those reviewed by Ma mainly were related to plate buckling. Some of them were based on the thermal buckling problem. In these studies, simple geometries and boundary conditions were assumed.
2.1.2 Review of recent works in thermal buckling

In recent years, there exist some studies in the field of thermal buckling. Murphy et al.\textsuperscript{[41]} presented a combined theoretical and experimental study of the buckling characteristics of thermally loaded, rectangular plates. An energy-based theoretical solution was used to calculate the critical buckling load. Through a series of parametric studies, the results indicated that $a/h$, $b/h$ (ratio of length to thickness and width to thickness) are independent. This means that the ratio $a/b$ is not sufficient to characterize this problem. An experimental validation was also studied in the paper. The results showed that the critical buckling temperature was a function of the edge length, which agrees well with the theoretical predictions.

Some of the researches are on laminated composite geometries and the finite element method was used in the study. Kuo et al.\textsuperscript{[42]}, studied the thermal buckling behavior of composite laminated plates by using the finite element method. Triangular plate elements were applied to a simple square plate. The thermal buckling mode shapes of cross-ply and angle-ply with various parameters and ratios were studied in detail. The results indicated that the high $E_1/E_2$ and $\alpha_1/\alpha_2$ ratios of two specific materials produce higher bending rigidity along the fiber direction, which means that the higher thermal buckling mode shapes are formed. The fiber angles also had some effects on the critical buckling temperature.
Shaterzadeh et al\cite{Shaterzadeh_2023}. presented the thermal buckling analysis of symmetric and anti-symmetric laminated composite plates with a cut-out. In the study, a MATLAB code was developed to simulate the problem. A simulation based on ABAQUS was also performed to make a comparison. A uniform temperature rise was applied for different boundary conditions. The results showed that the anti-symmetric composite plates were more difficult to buckle in comparison with the symmetric plates. An important result was that the presence of a hole in the center of the plate significantly increases the critical buckling temperature. The influence is inversely proportional to the aspect ratios due to local buckling occurrence.

In the previous studies, the geometries used were mainly rectangles or simple circular plates, which are still too fundamental in comparison with the clutch disc. There are some studies based on the annular geometries in the literature. Sadighi et al\cite{Sadighi_2023}. studied the buckling behavior of moderately thick heated annular plates made of FGMs. FGM is functional graded materials in which properties are graded in one direction. The temperature distributions or the thermal loads used in the work were defined along the thickness only. The material properties were considered temperature dependent. The stability equations were derived based on the adjacent-equilibrium criterion. The results indicated that the inner and outer boundary conditions, radii and thickness of the plate affect the buckling temperature significantly. The results also showed a disagreement between the solution from the derived stability equations and numerical solutions.
Joubaneh et al\textsuperscript{[45]}, studied the thermal buckling problem of solid circular plate made of porous material bounded with piezoelectric sensor-actuator patches. The general nonlinear equilibrium and linear stability equations were derived instead of the energy method used in others’ work. The results indicated that with the porosity increment, the critical buckling temperature will decrease and the structure will be unstable. The critical buckling temperature will increase with the increment of the ratios $h/a$ when the piezoelectric patches are not applied. The study was based on theoretical derivations only, with no experimental verification or numerical analysis involved in the validations.

Goo et al\textsuperscript{[46]}, studied the thermal buckling behavior of a circular aluminum plate. In the study, the experimental results were compared with the finite element results from ABAQUS. An aluminum plate was placed in a titanium ring and both heated to 160°C. The aluminum plate was restricted at the outer radius by the ring due to the different thermal expansion coefficients and buckling temperatures. The deformation was measured from the experiment and the phenomenon was also simulated by ABAQUS. The results showed a good agreement in the deformation curves. The critical buckling temperature from the finite element method was slightly higher than measured from the experiment due to the initial imperfections. The results indicated that ABAQUS is a good numerical tool for investigating the thermal buckling problems for engineering design and analysis. Therefore the results provide a solid groundwork for using the finite element method in thermal buckling.
2.1.3 Review of thermal buckling in automotive systems

Thermal buckling is a very important research field in both mechanical engineering materials engineering. For realistic applications, thermal buckling in automotive systems is particularly important. However, it has not been thoroughly investigated especially for clutches, which is a little surprising. Audebert et al.\textsuperscript{[24]} studied buckling of automotive clutch plates due to thermoelastic residual stresses. The study was purely theoretical, based on a tent-like temperature distribution in the structure. The results showed that clutch plates subjected to axisymmetric temperature excursions can develop residual in-plane bending moments of sufficient magnitude to cause buckling during unloading. The coning mode occurred when the residual stress at the outer radius was tensile and the potato chip mode occurred when it was compressive.

However, the variation in the thermal stress along the radial direction was not considered in the theory. Ma extended the method to automotive disk brakes and investigated the effect of geometric and material parameters on the critical buckling loads via the finite element method\textsuperscript{[47]}. These studies revealed that both axisymmetric and non-axisymmetric buckling modes can be caused by a uniformly distributed thermal loading in the circumferential direction. In addition, a slightly changed temperature profile in the radius can greatly affect the deformed shape of the buckling mode. However, Ma’s work did not consider the case when the disk is subjected to a temperature variation in both radial and circumferential directions.
2.2 Disadvantages in Previous Work

Despite the importance in automotive applications, thermal buckling has not been systematically studied in clutch and brake systems. The only existing studies were performed by Audebert and Ma, who used very basic geometries and simply assumed temperature distribution. The temperature variations in both radial and circumferential directions were not considered in either of the studies. The temperature distributions in the previous work were assumed uniform in the thickness, which is not quite realistic. Although the thickness of the clutch discs is relatively thin and the temperature is always considered uniform along thickness direction, brake discs are much thicker than clutch discs. The sliding process in a brake system usually lasts several seconds. In such a situation, the temperature cannot be uniform along the thickness. A non-uniform temperature profile can cause an underestimation in the critical thermal buckling load.

Another serious shortcoming is that only one material was used in the studies. In a realistic application, a friction layer is always applied on the surface of the friction discs to increase the efficiency of friction. The material properties of the friction layer are entirely different from the main part of the disc. The thickness ratio can be close to 1:2 in a clutch disc. This effect must be considered to analyze the buckling modes and to predict the critical buckling load. A study on the friction material effects was performed by Yi et al.\textsuperscript{[48]} who investigated the effects of friction material properties on TEI (thermoelastic instability) and critical speed. Both analytical and numerical models were used for
deducing the solutions and for investigating the critical speed changes, respectively. The results indicated that the symmetric TEI mode can become predominant in conditions where the friction material has either a large elastic modulus or a small thermal conductivity. The increase of friction material thermal conductivity or thermal expansion leads to an increase in the critical speed under the symmetric mode. However, there exist no such studies on thermal buckling.

In the previous works, the boundary conditions were always assumed free, which may lead to significant errors in some cases. For example, a set of teeth distributed around the outer edges of the discs can change the buckling load substantially. These teeth are fixed by the physical restrictions. The boundary conditions in that case are more like free-fixed, which would prevent the disc from buckling. In some situations the discs are manufactured with a cone angle to have a better contact between each other when the discs are slightly deformed. In conclusion, there are still a number of aspects to explore in the buckling study on clutch and brake discs.

2.3 Summary

In this chapter, the studies on buckling, thermal buckling have been reviewed. In the previous works, rectangle, plate, beam and annular plates were assumed as the geometries. The various techniques including the theoretical, experimental, and numerical methods were used to analyze the buckling phenomenon. However, there are
few studies associated with the applications in clutch systems. Thus, this thesis is devoted to advancing the research in this area.

The present thesis will incorporate the methodologies used in Ma’s work and target investigating thermal buckling in brakes and clutches in a more systematic way by taking into account the effects of various temperature profiles, friction materials and boundary conditions. We are particularly interested in the effect of those profiles resembling thermoelastic instability induced “banding modes” and “hot spots”, and we believe that the two phenomena can be coupled together at certain temperature ranges. An annular ring model will be assumed in the majority of the chapters because this geometry is a representative geometry in automotive clutch applications. The finite element modeling has been proved an efficient way to analyze the thermal buckling phenomenon. Therefore, the finite element method has been chosen as the primary numerical tool employed in this work to investigate various aspects of thermal buckling mentioned above.
CHAPTER THREE: FINITE ELEMENT ANALYSIS ON TEMPERATURE VARIATION IN RADIAL AND CIRCUMFERENTIAL DIRECTION OF CLUTCHES

In this chapter, only the radial and circumferential temperature distributions are studied. The temperature variation in the thickness direction is considered to be uniform. Thus, clutch discs are chosen in this study due to the thin thickness comparing with the brake discs. Also the results obtained from the clutch model should be very close to those obtained for as the brake model without considering the temperature variation in the thickness direction due to the geometric similarities.

3.1 Finite Element Method

Buckling is caused by the geometric nonlinearity in a deformed structure, which is equivalent to adding an additional stiffness term to the system. The general finite element formulation of the buckling problems thus involves the construction of stress stiffness matrix \( [k_\sigma] \) that is added to the stiffness matrix \([k]\). This stress stiffness matrix is then used to develop an eigenvalue equation for the buckling load \([11]\). More specifically, the structure is first loaded with a reference load to find the reference stress stiffness matrix, \([K_\sigma]_{\text{ref}}\). Another load \([R] = \lambda [R]_{\text{ref}}\) will then be applied to the structure, where \(\lambda\)
represents an arbitrary multiplier to the original reference load. Correspondingly the stress stiffness matrix is related to the applied load and becomes \(K_\sigma = \lambda [K_\sigma]_{ref}\).

Therefore the following two equations can be established.

\[
\begin{align*}
[K] + \lambda_{cr} [K_\sigma]_{ref} \{D\}_{ref} &= \lambda_{cr} \{R\}_{ref} \\
[K] + \lambda_{cr} [K_\sigma]_{ref} \{(D) + \delta D\} &= \lambda_{cr} \{R\}_{ref}
\end{align*}
\]

(1) 

(2)

where \(\{\delta D\}\) is the buckling displacement. The subtraction of them leads to

\[
\begin{align*}
[K] + \lambda_{cr} [K_\sigma]_{ref} \{\delta D\} &= \{0\}
\end{align*}
\]

(3)

This equation defines an eigenvalue problem, in which the term \(\lambda_{cr}\) is the eigenvalue and \(\{R\}_{cr} = \lambda_{cr} \{R\}_{ref}\) is the buckling load, which can be caused by any type of mechanical loads including thermal stresses. The above formulation has been fully integrated into the commercial code ABAQUS\(^{[49]}\). The leading eigenvalues and the corresponding eigenvectors are determined iteratively from a generalized eigenvalue equation. These eigenvalues are the load multipliers and the eigenvectors are expressed in terms of the nodal displacement components.

As a clutch disk has an annular shape with dimensions in both thickness and diameter, a three-dimensional (3-D) geometry is the first and natural choice to develop a numerical model. However, a clutch plate has a thin thickness compared to the other dimensions, and the stress/strain distributions across the thickness are usually negligible. We can therefore focus our attention on the temperature gradient in the radial and circumferential directions only. Due to this reason we may also use a reduced
two-dimensional (2-D) shell model. Furthermore, if the thermal load and the buckling shape are both axisymmetric, using an axisymmetric element type can considerably improve the numerical accuracy and efficiency. We will therefore develop three different finite element models, namely 3-D, 2-D and axisymmetric models, to investigate and compare the effects of dimensionality on the numerical solutions. Schematic of these models are shown in Figure 3-1 and Figure 3-2. A clutch plate can be partially constrained especially at the inner radius in some situations. These constraints may raise the buckling load significantly. For simplicity we assume free boundary conditions here for all cases and therefore the results obtained actually represent lower bounds to the solutions.

![Figure 3-1: Schematic of a clutch plate](image.png)
In ABAQUS there are various types of elements available in the element library. To make a comparison of the convergence performance in the solution we use some representative types and numbers of elements. It is well known that linear interpolating shape functions can cause shear locking in the solution\textsuperscript{50}, therefore we select the quadratic element C3D20R in the 3-D model, representing 20-node quadratic brick elements with reduced integration. For the 2-D model, we use the general shell element S4R (4-node general-purpose shell, reduced integration with hourglass control, finite membrane strains). For the axisymmetric model, we use the quadratic element type CAX8R (i.e. 8-node bi-quadratic, reduced integration). Examples of element types are illustrated in Figure 3-2.

3.2 Analytical Approximation

For the purpose of comparison and validation, an analytical approach has been developed on the basis of Timoshenko’s beam theory. The fundamental assumption here
is to approximate the annular ring geometry as a curved beam. According to Audebert et al.\cite{24}, the critical moments can be expressed as

\[
M^*_c = \frac{1 + K}{2} \pm \sqrt{\frac{(1 - K)^2}{2} + KN^2} \quad (4)
\]

\[
K = \frac{1 + \nu}{2} \quad (5)
\]

\[
M^*_c = \frac{2M^*_c G(R_o - R_i)\alpha^3}{3(R_o + R_i)} \quad (6)
\]

where $M^*_c$, $M_c$, $K$, $N$, $\nu$ are the dimensionless critical buckling moments, the critical buckling moments, the stiffness ratio, the buckling wave number and Poisson’s ratio, respectively. $G$, $\alpha$, $R_o$, $R_i$ represent the shear modulus, the cross section thickness, the outer radius and the inner radius of the annular disk, respectively. According to the critical moment equation Eq. (4), when the wave number $N=1$, the critical moment $M^*_c = 0$, which indicates that it is a rigid body mode that has no effect on the elastic stability. Consequently, we can predict that the first two dominant buckling modes are the “coning” mode ($N=0$) and the “potato chip” mode ($N=2$). When $N=0$, the critical moment equation becomes,

\[
M^*_c = K \quad (7)
\]

When $N=2$, the critical moment is,

\[
M^*_c = \frac{1 + K}{2} \pm \sqrt{\left(\frac{1 - K}{2}\right)^2 + 4K} \quad (8)
\]
The thermal stress distribution in the beam can be solved from the temperature distribution, which is then used to find the bending moment by integration. The in-plane moment caused by the circumferential stress can be expressed as\[^{28}\],

\[
M = \int_{r_i}^{r_o} \sigma(r)(r - \frac{R_o + R_i}{2})dr
\]  \hspace{1cm} (9)

where \(\sigma(r)\) is the radial thermal stress. When the free boundary conditions are applied at the inner and outer radii, \(\sigma(r)\) can be expressed as,

\[
\sigma(r) = \frac{\alpha E}{r^2} \int_{r_i}^{r} rTdr - \alpha ET + \frac{EB_1}{1 - \nu} + \frac{EB_2}{(1 + \nu)r^2}
\]  \hspace{1cm} (10)

\[
C_1 = \frac{\alpha}{2(1 - \nu)} \frac{1}{R_o^2 - R_i^2} \int_{R_i}^{R_o} Trdr
\]  \hspace{1cm} (11)

\[
C_2 = \frac{\alpha(1 + \nu)R_i^2}{R_o^2 - R_i^2} \int_{R_i}^{R_o} Trdr
\]  \hspace{1cm} (12)

If we assume a linear temperature gradient in radius,

\[
T = \Delta T \frac{r - R_i}{R_o - R_i}
\]  \hspace{1cm} (13)

Based on the above equations, the critical temperature gradient \(\Delta T\) can be determined.

### 3.3 Computational Simulation

#### 3.3.1 Convergence study

A set of convergence studies have been performed via the finite element analysis. Here we use the same material and geometric parameters as in Ma’s research\[^{40}\] for the purpose of comparisons. These parameters are shown in Table 3-1.
A linear reference temperature gradient is assumed along the radius direction as follows:

$$T = 50 \frac{r - 0.08}{0.13 - 0.08}$$  \hspace{1cm} (14)$$

where $T$ and $r$ represent the temperature and the radial position, respectively.

According to Table 3-1, the inner radius is 0.08 m and the outer radius is 0.13 m. Therefore the temperature varies from zero at the inner radius to 50 at the outer radius.

No constraints are placed at either inner or outer radius.

Three different element types, S4R, CAX8R, C3D20R have been investigated and compared. Meanwhile different numbers of elements have been used in the three dimensions, along the thickness, the radius and the circumference, respectively. The different combinations of element types and element numbers used are explained in Table 3-2. The eigenvalues obtained from the thermal buckling analysis are presented in Figure 3-3.
<table>
<thead>
<tr>
<th>Element type</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>S4R</td>
<td>4×16</td>
<td>6×24</td>
<td>8×32</td>
<td>10×48</td>
<td>12×64</td>
<td>14×96</td>
</tr>
<tr>
<td>C3D20R</td>
<td>1×4×16</td>
<td>1×6×24</td>
<td>1×8×32</td>
<td>1×10×48</td>
<td>1×12×64</td>
<td>14×96</td>
</tr>
<tr>
<td>C3D20R (4 elements along thickness)</td>
<td>4×4×16</td>
<td>4×6×24</td>
<td>4×8×32</td>
<td>4×10×48</td>
<td>4×12×64</td>
<td>4×14×96</td>
</tr>
<tr>
<td>CAX8R</td>
<td>2×4</td>
<td>2×6</td>
<td>2×8</td>
<td>2×10</td>
<td>2×12</td>
<td>2×14</td>
</tr>
</tbody>
</table>

Table 3-2: Different element types and numbers used in the convergence study

(a) the 1st dominant mode
According to Figure 3-3, we can have the following conclusions: (1) Axisymmetric element type CAX8R yields an accurate result for mode 1 (coning mode) with even a very coarse mesh (2 × 4); (2) With a medium-size mesh (1 × 10 × 48), 3-D element C3D20R leads to a converged result consistent with that of CAX8R; (3) The shell element S4R provides a relatively small error of approximately 0.33% with a fine mesh (14 × 96); (4) The element number in the thickness direction has little effect on the result. It should be pointed out that although the axisymmetric element type is the most efficient one, it can only be used for the study of coning modes in which the mode shape is also axisymmetric at the same time.
Ma\textsuperscript{[40]} constructed analytical models based on the beam theory and the Rayleigh Ritz method. To validate our finite element model, here we compared the critical buckling temperatures computed from the finite element analysis with Ma’s analytical solutions. The comparison is shown in Table 3-3. According to the table, the largest discrepancy between the finite element result using the S4R elements and the analytical solution is merely 2.7%, which can be considered acceptable. This comparison has shown that the shell element S4R is appropriate for the current study.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Beam theory</th>
<th>Rayleigh Ritz</th>
<th>FE (S4R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-557.5</td>
<td>-575</td>
<td>-576.2</td>
</tr>
<tr>
<td>2</td>
<td>682.5</td>
<td>687.5</td>
<td>668.8</td>
</tr>
</tbody>
</table>

Table 3-3: A comparison of the critical buckling temperature computed by the finite element method and the analytical method

Based on the above conclusions, we will use the shell element S4R with a relatively fine mesh (14 × 96) throughout the following discussions in this chapter.

3.3.2 Different temperature distributions analysis

Automotive transmission clutches undergo different operating conditions, such as the initial engagement, full contact and plate separation stages. As a result, various temperature distributions in the clutch plates are possible. Linear, exponential and sinusoidal distributions have been found among the most representative temperature profiles. The linear distribution can be caused by the sliding speed as a linear function of
the radius and by the fact that the frictional heat generation rate is a linear function of the sliding speed. The sinusoidal distribution is related to the local high temperature regions known as hot spots that could be excited by thermoelastic instability. An exponential distribution can be caused by the non-uniform convective cooling on the plate surface or a non-uniform contact pressure during the engagement and separation of plates. Additionally, the temperature gradient may exist in both radial and circumferential directions. It is possible that the temperature patterns have some effects on the thermal buckling mode and the buckling temperatures, so we choose various temperature patterns and compute the eigenvalues of thermal buckling. To obtain a more accurate result, we use the parameters from a realistic clutch that are shown in Table 3-4.

<table>
<thead>
<tr>
<th>Inner Diameter ($D_i$)</th>
<th>Outer Diameter ($D_o$)</th>
<th>Thickness ($a$)</th>
<th>Young’s Modulus ($E$)</th>
<th>Poisson’s Ratio ($\nu$)</th>
<th>Thermal Expansion Coefficient ($\alpha$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>172 (mm)</td>
<td>250 (mm)</td>
<td>3 (mm)</td>
<td>160 (GPa)</td>
<td>0.29</td>
<td>$1.27 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Table 3-4: Geometric and material parameters of a realistic clutch plate

### 3.3.3 Effect of monotonic temperature distribution along the radius

Here we assume a temperature distribution monotonically changing from the inner to the outer radius. We investigate three different scenarios:

$$T_1 = 39 \cdot \frac{r - 0.086}{0.125 - 0.086}$$  \hspace{1cm} (15)

$$T_2 = 39 \sin(40.2768(R - 0.086))$$  \hspace{1cm} (16)
\[ T_3 = \frac{39(\exp(60(R - 0.086)) - 1)}{240.5436(0.125 - 0.086)} \]  \hspace{1cm} (17)

where \( T_1, T_2, T_3 \) represent, respectively, linear, sinusoidal and exponential temperature distributions that are also illustrated in Figure 3-4. We chose the constants in these functions in such a way that all three functions have the same temperature difference of 39°C across the radius.

![Figure 3-4: Monotonic temperature profiles along radius](image)

The eigenvalues and the buckling temperatures computed by ABAQUS are shown in Table 3-5 and Figure 3-5. It is noticed that some of the eigenvalues and temperature gradients are negative, representing that the temperature decreases from the inner to outer radius. According to Table 3-5, the buckling temperature is typically very low for a monotonic temperature profile. The lowest buckling temperature is around 178°C, which is well below the maximum operating temperature of a clutch plate.
<table>
<thead>
<tr>
<th>Mode</th>
<th>Eigenvalue</th>
<th>Mode</th>
<th>Buckling temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Linear Sinusoidal Exponential</td>
<td>Linear Sinusoidal Exponential</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-4.556 -4.838 -4.916</td>
<td>-177.7 -188.7 -191.7</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5.444 5.332 5.741</td>
<td>212.3 208.0 223.9</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10.92 10.82 11.32</td>
<td>425.7 421.9 441.4</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-11.73 -11.09 -12.99</td>
<td>-457.6 -432.5 -506.7</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>16.49 16.54 16.81</td>
<td>643.0 645.0 655.7</td>
<td></td>
</tr>
</tbody>
</table>

Table 3-5: Eigenvalues and buckling temperatures of the monotonic radial temperature profiles

According to Figure 3-5, the temperature distribution pattern has a limited effect on the buckling temperature. As the buckling temperatures of Mode 1 and Mode 2 are the lowest, these two modes are the leading modes. This is consistent with the conclusions in
the previous researches. The first two buckling shapes of thermal distribution are shown in Figure 3-6.

Figure 3-6: Buckling modes of the monotonic radial temperature profiles
In Figure 3-6, the first two thermal buckling modes are always the axisymmetric mode (coning mode) and the non-axisymmetric mode (potato chip mode) in the condition that the temperature increases or decreases with the radius monotonically. Meanwhile, the coning mode can appear only if the temperature at the inner radius is the highest. In addition, it is obvious that the deformed shape changes with different thermal patterns. Therefore, we can conclude that the different thermal patterns that increase or decrease monotonically along the radius direction have significant effects on the deformed shape rather than the thermal buckling temperature.

3.3.4 Effect of non-monotonic temperature distribution along the radius

To explore the phenomenon further, we assumed a different thermal profile pattern along the radius, in which the highest temperature is located at the mean radius, instead of the inner or outer radius. We made three different assumptions on the temperature distributions as expressed in the following functions,

\[
T_4 = \frac{78(r - 0.086)}{0.125 - 0.086} \quad (0.086 \leq r \leq 0.1055) \quad (18)
\]

\[
T_5 = 78 - \frac{78(r - 0.086)}{0.125 - 0.086} \quad (0.1055 \leq r \leq 0.125)
\]

\[
T_6 = 39 \sin(80.5536(R - 0.086)) \quad (19)
\]

\[
T_6 = \frac{39(\exp(120(R - 0.086)) - 1)}{240.5436(0.125 - 0.008)} \quad (0.086 \leq r \leq 0.1055) \quad (20)
\]

\[
T_6 = \frac{39(\exp(120(-R + 0.125)) - 1)}{240.5436(0.125 - 0.008)} \quad (0.1055 \leq r \leq 0.125)
\]

where \( T_4, T_5, T_6 \) are, respectively, linear, sinusoidal and exponential temperature distributions that are shown in Figure 3-7.
Figure 3-7: Non-monotonic radial temperature profiles

We chose the constants in these functions in such a way that all three functions have the same temperature difference of 39°C between the inner and the mean radii, which is consistent with the previous case of monotonic distribution. We follow the same procedure to perform the finite element analysis. The obtained eigenvalues and buckling temperatures are shown in Table 3-6 and Figure 3-8.
<table>
<thead>
<tr>
<th>Mode</th>
<th>Eigenvalue Linear</th>
<th>Eigenvalue Sinusoidal</th>
<th>Eigenvalue Exponential</th>
<th>Buckling temperature Linear</th>
<th>Buckling temperature Sinusoidal</th>
<th>Buckling temperature Exponential</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-46.21</td>
<td>-42.51</td>
<td>-53.25</td>
<td>-1802</td>
<td>-1658</td>
<td>-2077</td>
</tr>
<tr>
<td>2</td>
<td>-56.14</td>
<td>-51.69</td>
<td>-64.59</td>
<td>-2190</td>
<td>-2016</td>
<td>-2519</td>
</tr>
<tr>
<td>3</td>
<td>-61.88</td>
<td>-56.94</td>
<td>-71.23</td>
<td>-2413</td>
<td>-2221</td>
<td>-2778</td>
</tr>
<tr>
<td>4</td>
<td>-67.05</td>
<td>-61.63</td>
<td>-77.29</td>
<td>-2615</td>
<td>-2404</td>
<td>-3014</td>
</tr>
<tr>
<td>5</td>
<td>69.62</td>
<td>64.56</td>
<td>79.17</td>
<td>2715</td>
<td>2518</td>
<td>3087</td>
</tr>
</tbody>
</table>

Table 3-6: Eigenvalues and buckling temperatures of the non-monotonic radial temperature profiles

Figure 3-8: Temperatures of the non-monotonic radial temperature profiles

According to Figure 3-8, when the highest temperature is located at the mean radius, the effect of thermal patterns on the critical buckling temperature is insignificant. Compared to Tables 3-5 and 3-6, it is clear that the eigenvalues of this case are much larger than those under monotonic temperature distributions. For example, when the
temperature distribution is a piecewise linear function, the eigenvalue of Mode 1 here is approximately 10 times greater than that of monotonic temperature profile. In terms of the buckling temperature, we have found that the difference is close to 16 times, which is even greater. In fact, the typical value of the buckling temperature for non-monotonic temperature profiles is well above the operating temperatures of automotive clutches. Hence thermal buckling is more probable to take place under monotonic radial temperature profiles as opposed to non-monotonic temperature distributions.

We also investigated the deformed shapes of the first two dominant modes that have the lowest buckling eigenvalues, as shown in Figure 3-9. In the figure it is clear that Mode 1 of all three thermal patterns has a “potato chip” shape and Mode 2 is a higher order one with more waves in the circumference. Meanwhile, the buckling shape varies with different thermal patterns. Consequently, we can conclude that the shape of the thermal pattern mainly affects the deformed shape rather than the buckling temperature. Finally, the axisymmetric coning mode cannot take place in a condition that the highest temperature is located in the middle of the radius.
Figure 3-9: Buckling modes of the non-monotonic radial temperature profiles
3.3.5  Effect of circumferential temperature distribution

The temperature gradient may also exist in the circumferential direction. Here we assume a sinusoidal thermal distribution that has the same maximum temperature difference in the following form:

\[ T_\gamma = 39\sin(N\theta) \quad N=0.5, 1, 1.5, 2 \]  

(21)

where \( \theta \) and \( n \) represent, respectively, the angular position and the wave number.

The buckling eigenvalues and temperatures computed from the finite element analysis are reported in Table 3-7.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Eigenvalue</th>
<th>Buckling temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n=0.5</td>
<td>n=0.8</td>
</tr>
<tr>
<td>1</td>
<td>27.46</td>
<td>22.45</td>
</tr>
<tr>
<td>2</td>
<td>43.43</td>
<td>35.46</td>
</tr>
<tr>
<td>3</td>
<td>60.36</td>
<td>49.27</td>
</tr>
<tr>
<td>4</td>
<td>63.03</td>
<td>51.67</td>
</tr>
<tr>
<td>5</td>
<td>89.75</td>
<td>73.74</td>
</tr>
</tbody>
</table>

Table 3-7: Eigenvalues and buckling temperatures of the sinusoidal temperature profile in the circumference

According to the table, roughly speaking there is a trend in the buckling temperature to increase with the wave number. For example, the buckling temperature of mode 1 is 1785°C at \( n=1.5 \), and 1071°C at \( n=0.5 \). However, the relationship is not linear. For
example, it is noticed that the buckling temperature decreases slightly from n=0.5 to n=1, then increases from n=1 to n=1.5. In general the buckling temperature here is much higher than those of the monontonic radial profile. However, some values are still below 1000°C and this can come close to the vicinity of the maximum operating temperature in some extreme cases. The eigenvectors of the first two modes are shown in Figure 3-10.
Figure 3-10: Dominant buckling modes of the sinusoidal temperature distributions along the circumference
For an integer wave number greater than 1, it is found that the structure will not buckle at any temperature. It can be explained by the fact that the material expands in the hot region and contracts in the cold region. These thermal effects will counteract each other and eventually cancel out, leading to an unconditional stable situation. Meanwhile, a temperature distribution of non-integer wave number may lead to buckling, although this scenario is not common in practical applications.

3.4 Conclusions

In this chapter, the effects of different temperature distribution patterns are studied. It has been found that the coning mode only occurs when the temperature distribution is monotonic along radius. Also, the critical buckling temperature under this temperature pattern is the lowest. The type of the monotonic temperature distribution has limit effect on thermal buckling temperature. However, it affects the buckling shape.

The non-monotonic temperature distribution shows a great resistance on thermal buckling due to the high critical buckling temperature. The results agree well with Timoshenko’s beam theory.

The circumferential temperature distribution is a rare pattern but shows a complicated result. When the wave number is integer, buckling will never happen due to the counteraction of the hot and cold material expansion.

The negative values in the results indicate the contrary temperature distribution. For study convenience, the studies in the following chapters will ignore the effect.
It should be pointed out that the monotonic temperature distribution is the most realistic pattern due to the friction at the outer edge. Therefore, this temperature pattern has been used for further study.
CHAPTER FOUR: PARAMETRIC STUDY AND EFFECT OF TEMPERATURE DISTRIBUTION ALONG THE THICKNESS DIRECTION IN BRAKE DISCS

In this chapter, a study on material properties has been performed. The temperature distribution in the thickness direction is investigated, meanwhile the temperature variation in the radial direction is assumed monotonic. The brake discs have been chosen to the study due to the relatively larger thickness comparing with the clutch discs. Also, uniform temperature distribution along the thickness direction may no longer valid for brakes. In this study, the brake disc is assumed to be fully covered by the brake pad, which enables the smooth temperature distribution along the radius.

4.1 The Effect of Material Properties

Vehicles have been designed and manufactured with different velocity limits, loading capabilities and operating conditions. The design of the brake and clutch systems must be optimized to meet these requirements. It is essential to perform a parametric study on material properties to determine how these parameter variations affect the critical buckling temperature.

In this chapter, C3D20R element has been used instead of shell elements due to the relatively large errors generated by the shell elements when the temperature along the
thickness direction varies, which will be discussed later in this chapter. Here we use the same material and geometric parameters as in Ma’s research\cite{40} for the purpose of comparison. These parameters are shown in Table 3-1.

4.1.1 The effect of Young’s modulus

A set of studies with various Young’s modulus have been performed via finite element analysis. The coefficient of thermal expansion and Poisson’s ratio remain constant. A linear reference temperature gradient is assumed along the radius direction as follows:

\[ T = 250 \frac{r - 0.08}{0.13 - 0.08} \]  \hspace{1cm} (22)

The value of Young’s modulus changes linearly from 88GPa to 110GPa with an increment of 4.4GPa (5%), an overall increase of 25%. The mesh (3×10×100) used in the finite element analysis is shown in Figure 4-1. This model is used through this chapter.

Figure 4-1: Finite element model used in the study on material properties
The eigenvalues and the critical buckling temperatures computed by ABAQUS are reported in Table 4-1.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\nu=0.3$</th>
<th>$\alpha=1.25\times10^{-5}$(K$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E=88</td>
<td>E=92.4</td>
</tr>
<tr>
<td></td>
<td>(GPa)</td>
<td>(GPa)</td>
</tr>
<tr>
<td></td>
<td>E=96.8</td>
<td>E=101.2</td>
</tr>
<tr>
<td></td>
<td>(GPa)</td>
<td>(GPa)</td>
</tr>
<tr>
<td></td>
<td>E=105.6</td>
<td>E=110</td>
</tr>
<tr>
<td></td>
<td>(GPa)</td>
<td>(GPa)</td>
</tr>
<tr>
<td>Mode1</td>
<td>2.2971</td>
<td>2.2971</td>
</tr>
<tr>
<td>T(°C)</td>
<td>574.275</td>
<td>574.275</td>
</tr>
<tr>
<td>Mode2</td>
<td>2.6555</td>
<td>2.6555</td>
</tr>
<tr>
<td>T(°C)</td>
<td>663.875</td>
<td>663.875</td>
</tr>
<tr>
<td>Mode3</td>
<td>5.3013</td>
<td>5.3013</td>
</tr>
<tr>
<td>T(°C)</td>
<td>1325.325</td>
<td>1325.325</td>
</tr>
</tbody>
</table>

Table 4-1: Eigenvalue and buckling temperatures of different Young’s modulus

An interesting result is that the eigenvalue is independent of Young’s modulus, which means that the thermal buckling in this case is independent of Young’s modulus. Even when the modulus is set to a very small value, the result remains the same. We can prove it based on the following discussion.

There is no existing equation in the literature that can be directly used to determine the thermal buckling temperature. A set of equations (Eqs.9~12) can be used to establish the relationship between the in-plate bending moment and the temperature load from the theory of thermoelasticity. This can be simply expressed as:
Another set of equations (Eqs.4–6) can be used to compute the critical buckling moment without thermal effects based on Timoshenko’s buckling theory. Combining all these equations we obtain:

\[ M \sim M_{\text{critical}} \sim \Delta T_{\text{critical}} \] (24)

Thus, from Timoshenko’s theory we obtain the critical buckling moment. By applying this moment to Eq. (23) we can determine the critical buckling load (i.e. temperature). The critical buckling load is the product of eigenvalue and the reference temperature load in the finite element analysis:

\[ \Delta T_{\text{critical}} = \lambda \cdot \Delta T_{\text{reference}} \] (25)

Eq.(6) can be written as:

\[ M_c = \frac{E}{2(1+\nu)} \frac{2M^*(R_o - R_i)\alpha^3}{3(R_o + R_i)} \] (26)

By replacing the shear modulus G, where:

\[ G = \frac{E}{2(1+\nu)} \] (27)

Substituting Eq. (10) into Eq. (9) we obtain:

\[ M = Et \int_{R_i}^{R_o} \left[ \frac{\alpha}{r^2} \int_{R_i}^{r} rT \, dr \right] - \alpha T + \frac{C_1}{1-\nu} + \frac{C_2}{(1+\nu)r^2} \left( r - \frac{R_o + R_i}{2} \right) dr \] (28)

In this equation, \( C_1 \) and \( C_2 \) are uncorrelated with \( E \) in both free and non-free boundary conditions.
By combining Eq. (26) and (28), the critical buckling load can be found. It has been found that Young’s modulus disappears from the final result. Therefore the critical buckling load is independent of Young’s modulus. This conclusion shows a good agreement with the finite element simulation.

From the result, we can conclude that when single material is used, Young’s modulus has no effect on thermal buckling, which may benefit the clutch/brake design in real applications. It should be noted that in this case only a single material is used. The result based on models with multiple materials would not be the same. The discussion on this problem will be made in Chapter 6. Also, Young’s modulus does have effects on some mechanical problems other than thermal loads. The current study is based on the thermal effect only.

4.1.2 The effect of Poisson’s ratio

A set of studies with various value of Poisson’s ratio have been performed via the finite element analysis. The coefficient of thermal expansion and Young’s modulus remain constant. A linear reference temperature gradient is assumed along the radius direction as Eq. (22). Poisson’s ratio changes linearly from 0.24 to 0.3 with an increment of 0.012 (5%), an overall 25% increase.

The eigenvalues and the critical buckling temperatures computed by ABAQUS are reported in Table 4-2 and Figure 4-2.
\[ E = 110 \text{GPa} \quad a = 1.25 \times 10^{-5} \text{K}^{-1} \]

<table>
<thead>
<tr>
<th>V</th>
<th>Mode1</th>
<th>Mode2</th>
<th>Mode3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.24</td>
<td>2.2905, 572.625</td>
<td>2.7191, 679.775</td>
<td>5.4338, 1358.4</td>
</tr>
<tr>
<td>0.252</td>
<td>2.2918, 572.95</td>
<td>2.7061, 676.525</td>
<td>5.4065, 1351.6</td>
</tr>
<tr>
<td>0.264</td>
<td>2.2931, 573.275</td>
<td>2.6932, 673.3</td>
<td>5.3796, 1344.9</td>
</tr>
<tr>
<td>0.276</td>
<td>2.2944, 573.6</td>
<td>2.6805, 670.125</td>
<td>5.3532, 1338.3</td>
</tr>
<tr>
<td>0.288</td>
<td>2.2957, 573.925</td>
<td>2.668, 667</td>
<td>5.3271, 1331.8</td>
</tr>
<tr>
<td>0.3</td>
<td>2.2971, 574.275</td>
<td>2.6555, 663.875</td>
<td>5.3013, 1325.3</td>
</tr>
</tbody>
</table>

Max T. Diff.

<table>
<thead>
<tr>
<th>V</th>
<th>Mode1</th>
<th>Mode2</th>
<th>Mode3</th>
</tr>
</thead>
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<tr>
<td>0.24</td>
<td>2.2905</td>
<td>2.7191</td>
<td>5.4338</td>
</tr>
<tr>
<td>0.252</td>
<td>2.2918</td>
<td>2.7061</td>
<td>5.4065</td>
</tr>
<tr>
<td>0.264</td>
<td>2.2931</td>
<td>2.6932</td>
<td>5.3796</td>
</tr>
<tr>
<td>0.276</td>
<td>2.2944</td>
<td>2.6805</td>
<td>5.3532</td>
</tr>
<tr>
<td>0.288</td>
<td>2.2957</td>
<td>2.668</td>
<td>5.3271</td>
</tr>
<tr>
<td>0.3</td>
<td>2.2971</td>
<td>2.6555</td>
<td>5.3013</td>
</tr>
</tbody>
</table>

Table 4-2: Eigenvalues and buckling temperatures of different Poisson’s ratios

Figure 4-2: Buckling temperature variation of different Poisson’s ratios
From Table 4-2 we can see that the critical buckling temperature increases almost linearly with an increasing Poisson’s ratio for Mode 1. This effect is opposite for Mode 2 and Mode 3. This is caused by the different values of $N$ in Eq. (4), where $N$ is the order of the mode. This variation induces the changing of $M^*_c$ with different order of $K$, where $K = \frac{1+\nu}{2}$, which changes the expressions of $\nu$ related terms.

From Figure 4-2 we can see that the curves are almost flat. This means that the influence of Poisson’s ratio on the buckling temperature is very small. The maximum temperature difference is only 1.65°C for Mode 1 in the specified Poisson’s ratio region. Even for Mode 3, which rarely occurs in real applications (A test reported by Halderman[3] showed that the peak temperatures across the brake disc can reach 980°C.), the difference is 33.1°C, about 2.5% of the critical buckling temperature. This effect can be ignored as the range of Poisson’s ratio, 0.24–0.3, is found in most materials. Therefore, we can conclude that the effect of Poisson’s ratio is not significant in thermal buckling.

4.1.3 The effect of coefficient of thermal expansion

A set of studies with various coefficients of thermal expansion have been performed via the finite element analysis. Poisson’s ratio and Young’s modulus remain constant. The temperature distribution keeps the same as the previous study. The coefficient of thermal expansion changes linearly from $1\times10^{-5}\text{K}^{-1}$ to $1.25\times10^{-5}\text{K}^{-1}$ by the increment of $0.05\times10^{-5}\text{K}^{-1}$ (5%), with an overall increase of 25%.
The eigenvalues and the critical buckling temperatures computed by ABAQUS are shown in Table 4-3 and Figure 4-3.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\alpha$</th>
<th>$T(°C)$</th>
<th>$\alpha=1\times$</th>
<th>$\alpha=1.05\times$</th>
<th>$\alpha=1.1\times$</th>
<th>$\alpha=1.15\times$</th>
<th>$\alpha=1.2\times$</th>
<th>$\alpha=1.25\times$</th>
<th>Max T.</th>
<th>Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>$10^{-5}(K^{-1})$</td>
<td>$10^{-5}(K^{-1})$</td>
<td>$10^{-5}(K^{-1})$</td>
<td>$10^{-5}(K^{-1})$</td>
<td>$10^{-5}(K^{-1})$</td>
<td>$10^{-5}(K^{-1})$</td>
<td>Diff.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T(°C)</td>
<td>2.8714</td>
<td>2.7346</td>
<td>2.6103</td>
<td>2.4968</td>
<td>2.3928</td>
<td>2.2971</td>
<td>717.85</td>
<td>683.65</td>
<td>652.575</td>
<td>624.2</td>
</tr>
<tr>
<td>Mode 2</td>
<td>$10^{-5}(K^{-1})$</td>
<td>$10^{-5}(K^{-1})$</td>
<td>$10^{-5}(K^{-1})$</td>
<td>$10^{-5}(K^{-1})$</td>
<td>$10^{-5}(K^{-1})$</td>
<td>$10^{-5}(K^{-1})$</td>
<td>Diff.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T(°C)</td>
<td>3.3195</td>
<td>3.1614</td>
<td>3.0177</td>
<td>2.8865</td>
<td>2.7662</td>
<td>2.6555</td>
<td>829.875</td>
<td>790.35</td>
<td>754.425</td>
<td>721.625</td>
</tr>
<tr>
<td>Mode 3</td>
<td>$10^{-5}(K^{-1})$</td>
<td>$10^{-5}(K^{-1})$</td>
<td>$10^{-5}(K^{-1})$</td>
<td>$10^{-5}(K^{-1})$</td>
<td>$10^{-5}(K^{-1})$</td>
<td>$10^{-5}(K^{-1})$</td>
<td>Diff.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T(°C)</td>
<td>6.6269</td>
<td>6.3113</td>
<td>6.0244</td>
<td>5.7625</td>
<td>5.5224</td>
<td>5.3013</td>
<td>1656.7</td>
<td>1577.8</td>
<td>1506.1</td>
<td>1440.6</td>
</tr>
</tbody>
</table>

Table 4-3: Eigenvalues and buckling temperatures of different coefficients of thermal expansion
Figure 4-3: Buckling temperature variation of different coefficients of thermal expansion

According to Figure 4-3, the buckling temperature decreases almost linearly with the coefficient of thermal expansion. The decrement is significant, which shows the importance of thermal expansion in thermal buckling. The curve of Mode 3 has a larger slope than Modes 1 and 2 due to the effect of $N$ in Eq. (4). All the modes show the monotonic decrease with the coefficient of thermal expansion. This can be validated and derived from Eq. (28), which can be simply expressed as:

$$\Delta T_{critical} \sim \frac{1}{\alpha}$$  \hspace{1cm} (29)

This validation also proves the accuracy and the feasibility of ABAQUS in the study on thermal buckling. From Table 4-3, the maximum temperature difference is 143.58°C for Mode 1 in this study. This means that thermal expansion is the key factor in the
thermal buckling study. In practical applications, this material property must be measured carefully to make an accurate prediction on the thermal buckling load.

4.2 Effect of temperature distribution along the thickness direction

Brake disc is a mechanical device which frequently suffers from contact friction. Often, each contact lasts several seconds. Within this time period, the heat generated is usually distributed non-uniformly along the thickness direction, even if the thickness is about 6mm. Thus, in thermal buckling study, a consideration of temperature distribution along the thickness direction is practical. The study on this effect can also be used to determine whether the thermal stress along the thickness direction is negligible. This will also benefit the further study on the model with friction layers.

4.2.1 2-D and 3-D elements comparison

In the previous chapter, we have shown the accuracy and the efficiency of shell element (2-D). The temperature distribution is considered uniform along the thickness direction in that case. However, the study becomes a 3-D problem when temperature distribution along the thickness direction is assumed not uniform. Temperature now varies in r, θ, z directions in a cylindrical coordinate system. Therefore, a 3-D element is appropriate for this study. However, taking the consideration of computational efficiency, shell element is still a good choice if it shows a correct result. Thus, a comparison is performed between the shell element model and 3-D element model.
Shell element S4R and 3-D element C3D20R have been chosen in this study. The material properties and disc dimensions are shown in Table 3-1. A linear reference temperature gradient is assumed along the radius direction as Eq. (22). A linear temperature reduction by 20% is assumed along the thickness direction. The expression of the 3-D temperature field across the structure can be written as:

\[
T = 250 \left( \frac{r - 0.08}{0.13 - 0.08} \right) \left( \frac{0.2z}{0.006} + 0.8 \right) \quad (0 \leq z \leq 0.006)
\]  

(30)

The temperature profile along thickness direction is shown in Figure 4-4.

![Linear temperature profile along thickness direction](image)

Figure 4-4: Linear temperature profile along the thickness direction

In consideration of the change on the reference temperature load in this case (250°C at the friction surface and 200°C at the bottom, instead of constant 250°C in previous study), the computed critical buckling temperature cannot simply be the product of the
eigenvalue and 250°C as previous study. To figure out the new reference load, it is reasonable to assume the mean value of the temperature load along the thickness direction as the reference temperature load. By integrating the Eq. (30) with respect to z and taking the average, we can obtain:

\[ T = 225 \left( \frac{r - 0.08}{0.13 - 0.08} \right) \]  

(31)

The reference temperature load in Eq. (31) is now constant 225°C. Eq. (30) and (31) will be both used as the temperature fields in this simulation to perform the comparison. The mesh for S4R element model is 10×100, and 3×10×100 for C3D30R element model. The computational results of both models are shown in Table 4-4 and Table 4-5 respectively.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Original Function</th>
<th>Integrated Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>2.8951</td>
<td>651.3975</td>
</tr>
<tr>
<td>Mode 2</td>
<td>3.3609</td>
<td>756.2025</td>
</tr>
<tr>
<td>Mode 3</td>
<td>6.7127</td>
<td>1510.3575</td>
</tr>
</tbody>
</table>

Table 4-4: Eigenvalues and buckling temperatures of the linear temperature profile along thickness direction by using shell element model
Table 4-5: Eigenvalues and buckling temperatures of the linear temperature profile along thickness direction by using 3-D element model

According to Table 4-4, there is a significant difference in critical buckling temperatures between the results obtained from two different temperature fields by using shell element model. If the shell element is still suitable in this 3-D problem, we can have the following possible corollaries:

1. The simplified temperature distribution (Eq. 31) cannot displace the original 3-D temperature field (Eq. 30).

2. According to the data in the first group in Table 4-4, the critical buckling temperature of Mode 1 is 651°C. In consideration of the small stress along the thickness direction, the critical buckling temperature of Mode 1 should be near the result obtained from the model with uniform temperature distribution in the thickness direction (574.275°C). However, the difference is 77°C which is very significant. Therefore, we
can also think that the reference temperature load (225°C) cannot simply be the mean value of the temperature load along the thickness direction.

According to Table 4-5, we find that the eigenvalues in both groups are absolutely the same. This cannot be a coincident, which means that the simplified temperature field (Eq. 31) can substitute for the original one (Eq. 30). This result also negatives the corollaries 1 & 2. Thus, we can conclude that shell element is not appropriate in this 3-D case. One explanation is that the 3-D element model has a fixed origin of coordinates. The temperature field is established based on that coordinate. However, the shell element model does not have a clear coordinate corresponding relation along thickness direction due to the restriction of shell element itself. This causes the error in the simulation. Therefore, 3-D element model has been chosen in this study.

**4.2.2 Effect of different temperature distribution patterns along thickness direction**

It should be noted that the computed critical buckling temperatures must be the same by using Eq. (22) and (31) as the reference temperature gradient due to the same temperature distribution pattern. Combing this conclusion and Table 4-5, we can obtain that the critical buckling temperatures computed by using Eq. (30) and (22) as the temperature fields are the same, which indicates that the temperature distribution along the thickness direction has no effect on thermal buckling. Three further studies are investigated to validate this conclusion.
A basic study is to check the mesh precision. Two models with different mesh densities are investigated in this study. One of the models has 3 elements along the thickness direction and the other one has 9. The element densities are shown in Figure 4-5. The other simulation settings keep the same with those in previous study. The results are shown in Table 4-6.

![Figure 4-5: Mesh densities along the thickness direction](image)

<table>
<thead>
<tr>
<th>Mode</th>
<th>3 elements</th>
<th>9 elements</th>
<th>Temperature Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Eigenvalue</td>
<td>Buckling Temp. (°C)</td>
<td>Eigenvalue</td>
</tr>
<tr>
<td>Mode 1</td>
<td>2.5523</td>
<td>574.2675</td>
<td>2.5523</td>
</tr>
<tr>
<td>Mode 2</td>
<td>2.9507</td>
<td>663.9075</td>
<td>2.9505</td>
</tr>
<tr>
<td>Mode 3</td>
<td>5.8907</td>
<td>1325.4075</td>
<td>5.8904</td>
</tr>
</tbody>
</table>

Table 4-6: Eigenvalues and buckling temperatures of mesh precision study along the thickness direction
According to Table 4-6, the temperature difference can be ignored, which means that the 3 elements along the thickness direction are sufficient for this study. This also proves that the thermal stress along the thickness direction can be ignored.

In another study, the percentage of the linear temperature reduction along thickness direction becomes the variable. 10%, 30% and 40% reductions are assumed in this study. The expressions of the temperature fields are:

\[
T = 250 \left( \frac{r - 0.08}{0.13 - 0.08} \right) \cdot \left( \frac{0.1z}{0.006} + 0.9 \right) \\
T = 250 \left( \frac{r - 0.08}{0.13 - 0.08} \right) \cdot \left( \frac{0.3z}{0.006} + 0.7 \right) \quad (0 \leq z \leq 0.006) \\
T = 250 \left( \frac{r - 0.08}{0.13 - 0.08} \right) \cdot \left( \frac{0.4z}{0.006} + 0.6 \right)
\]

The temperature profiles are shown in Figure 4-6.

![Figure 4-6: Linear temperature profiles along the thickness direction](image)

Figure 4-6: Linear temperature profiles along the thickness direction
By integrating Eq. (32) with respect to $z$ and taking the average value, we can obtain:

$$T = 237.5 \left( \frac{r - 0.08}{0.13 - 0.08} \right)$$

$$T = 212.5 \left( \frac{r - 0.08}{0.13 - 0.08} \right)$$

$$T = 200 \left( \frac{r - 0.08}{0.13 - 0.08} \right)$$

Eq. (32) and (33) will be both used as the temperature fields in this simulation to perform the comparison. The other settings keep the same with those in previous study. The results are shown in Table 4-7.

<table>
<thead>
<tr>
<th>Mode</th>
<th>10% Reduction</th>
<th>30% Reduction</th>
<th>40% Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Original</td>
<td>Integrated</td>
<td>Original</td>
</tr>
<tr>
<td>Mode 1</td>
<td>2.4180</td>
<td>2.4180</td>
<td>2.7025</td>
</tr>
<tr>
<td>Mode 2</td>
<td>2.7954</td>
<td>1.7954</td>
<td>3.1243</td>
</tr>
<tr>
<td>Mode 3</td>
<td>5.5807</td>
<td>5.5807</td>
<td>6.2372</td>
</tr>
</tbody>
</table>

Table 4-7: Eigenvalues of the study on different temperature reduction percentages along the thickness direction

According to Table 4-7, we can see that the data in each group are absolutely the same, which validates that the simplified temperature field can substitute for the original one. Also, we have previously indicated that the critical buckling temperatures are the same with the same thermal load pattern, no matter what the values of the reference
temperature load are. Combining these two conclusions, we can conclude that the linear temperature distribution along thickness direction has no effect on thermal buckling.

An exponential temperature distribution is assumed along the thickness direction in the third study. This distribution is possible because within a several seconds’ friction process, the temperature near the friction surface will be higher and decrease rapidly along the thickness direction.

The temperature reduction is set to be 20%. The expression of the temperature field can be written as:

\[
T = 250 \left( \frac{r - 0.08}{0.13 - 0.08} \right) \times \left[ 0.01048(e^{500z} - 1) + 0.8 \right] \tag{34}
\]

When \( z = 0.006 \), \( T_{bottom} = 250 \left( \frac{r - 0.08}{0.13 - 0.08} \right) \tag{35} \]

When \( z = 0 \), \( T_{bottom} = 200 \left( \frac{r - 0.08}{0.13 - 0.08} \right) \tag{36} \]

The shape of the temperature profile is shown in Figure 4-7.
Figure 4-7: Exponential temperature profile along the thickness direction

By integrating the equation with respect of $z$ and taking the average value, we can obtain:

$$T_{\text{mean}} = \frac{1}{0.006} \int_0^{0.006} Tdz = 214.05 \times \left( \frac{r - 0.08}{0.13 - 0.08} \right)$$  \hspace{1cm} (37)

Eq. (34) and (37) will be both used as the temperature fields in this simulation to make the comparison. The other settings keep the same with those in previous study. The results are shown in Table 4-8.
<table>
<thead>
<tr>
<th>Mode</th>
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<th>Temperature</th>
<th>Eigenvalue</th>
<th>Temperature</th>
<th>Temp. Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.6826</td>
<td>574.211</td>
<td>2.6829</td>
<td>574.275</td>
<td>-0.064</td>
</tr>
<tr>
<td>2</td>
<td>3.1020</td>
<td>663.9831</td>
<td>3.1016</td>
<td>663.897</td>
<td>0.0861</td>
</tr>
<tr>
<td>3</td>
<td>6.1936</td>
<td>1325.740</td>
<td>6.1919</td>
<td>1325.376</td>
<td>0.364</td>
</tr>
</tbody>
</table>

Table 4-8: Eigenvalues and buckling temperatures of exponential temperature profile along the thickness direction

From Table 4-8 we can see that the eigenvalues in both groups are almost the same, which indicates that the exponential temperature profile along the thickness direction also has no effect on thermal buckling. The tiny difference is caused by taking approximations in the integration process.

4.3 Conclusion

From the studies in this chapter, we can conclude that the possible patterns of temperature distribution along the thickness direction have very small or no effect on thermal buckling. The thermal stress along the thickness direction can be ignored. Temperature distribution along the radial direction is the dominant factor to induce thermal buckling.

The thermal stress or the temperature distribution along the thickness direction in brake and clutch discs is always considered negligible. This study has validated that assumption via finite element method. This result will benefit the further study.
taking the consideration of temperature variation along the thickness direction, the computational efficiency and simplification can be obtained.
CHAPTER FIVE: THERMAL BUCKLING STUDY ON DISCS WITH A MANUFACTURED CONE ANGLE

The friction discs may have a slight deformation after using for a time period due to the thermal and physical problems. Sometimes, the friction discs are manufactured with a small cone angle. The manufactured cone angle will help in that situation by having a better contact surface. This non-flat geometry may have effect on thermal buckling. A study on this effect has been performed in this chapter.

5.1 Effect of Different Manufactured Cone Angles on Thermal Buckling

This chapter is devoted to studying the effect of different manufactured cone angles on thermal buckling. In this study, linear monotonic temperature distribution along the radial direction has been chosen as the reference temperature gradient due to the simple function and computational efficiency.

5.1.1 Study on the effect by using 3-D element model

A set of studies with various cone angles have been performed via finite element analysis. The schematic of the disc and the finite element model are shown in Figure 5-1 and Figure 5-2. The geometry dimensions and material properties are shown in Table 3-1.
A linear reference temperature gradient is assumed along the radius direction as Eq. (22). The mesh used in this model is $6 \times 10 \times 100$. The element type is C3D20R. The cone angle varies from $1^\circ$ to $5^\circ$ by the increment of $1^\circ$. The thermal buckling simulation results are shown in Table 5-1 and Figure 5-3.
<table>
<thead>
<tr>
<th>Angle</th>
<th>1°</th>
<th>2°</th>
<th>3°</th>
<th>4°</th>
<th>5°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>2.3917</td>
<td>2.6871</td>
<td>3.1353</td>
<td>3.5079</td>
<td>3.9860</td>
</tr>
<tr>
<td>Mode 2</td>
<td>2.7089</td>
<td>2.8689</td>
<td>3.2185</td>
<td>4.0449</td>
<td>5.2500</td>
</tr>
<tr>
<td>Mode 3</td>
<td>5.4073</td>
<td>5.7243</td>
<td>6.2516</td>
<td>6.9876</td>
<td>7.9302</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode 1</th>
<th>597.925</th>
<th>671.775</th>
<th>783.825</th>
<th>876.975</th>
<th>996.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 2</td>
<td>677.225</td>
<td>717.225</td>
<td>804.625</td>
<td>1011.225</td>
<td>1312.5</td>
</tr>
<tr>
<td>Mode 3</td>
<td>1351.825</td>
<td>1431.075</td>
<td>1562.9</td>
<td>1746.9</td>
<td>1982.55</td>
</tr>
</tbody>
</table>

Table 5-1: Eigenvalues and buckling temperatures of different cone angle models obtained by using 3-D element model

Figure 5-3: Buckling temperature variations of different cone angle models
From Table 5-1, we can find that the critical buckling temperature increases significantly with the increase of the cone angle for all modes. In Figure 5-3, the buckling temperature variation curves are almost linear for Mode 1 and 3. However, Mode 1 has the prior study significance than the others because it occurs first and has the lowest critical buckling temperature. The dominant buckling modes are shown in Figure 5-4.

![Mode 1 and Mode 2](a) Cone angle 1°~2°

![Mode 1 and Mode 2](b) Cone angle 3°~5°

Figure 5-4: Dominant buckling modes of different cone angle models

According to Figure 5-4, we find that for 1°~2° cone angle models, the coning modes occur first, which agrees the conclusion in chapter 3 (For a linear profile of temperature, the axisymmetric coning mode occurs first). However, for 3°~5° cone angle
models, the potato chip modes occur first, which shows a disagreement with the previous conclusion.

According to Table 5-1, the critical buckling temperature difference between 1° and 5° cone angle models for Mode 1 is about 400°C. The result indicates that the manufactured cone angle improves the thermal buckling temperature significantly. A study by using axisymmetric element model has been performed to validate this conclusion.

5.1.2 Validation on the conclusion by using axisymmetric element model

The axisymmetric element takes the advantage of the axisymmetric deformation. Thus, only the coning mode can be performed by axisymmetric element. However, this introduces an indirect validation by comparing the axisymmetric modes between the results obtained from 3-D and axisymmetric element models. The axisymmetric element CAX8R has been chosen in this study. The axisymmetric element model is shown in Figure 5-5.

Figure 5-5: Axisymmetric element model of manufactured cone angle discs
The simulation settings keep the same with previous study. The result is shown in Table 5-2.

<table>
<thead>
<tr>
<th></th>
<th>CAX8R</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0°</td>
<td>1°</td>
<td>2°</td>
<td>3°</td>
<td>4°</td>
<td>5°</td>
</tr>
<tr>
<td>Mode 1</td>
<td>2.297</td>
<td>2.391</td>
<td>2.687</td>
<td>3.218</td>
<td>4.045</td>
<td>5.250</td>
</tr>
<tr>
<td></td>
<td>C3D20R</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode 1</td>
<td>2.297</td>
<td>2.391</td>
<td>2.687</td>
<td>3.135</td>
<td>3.507</td>
<td>3.986</td>
</tr>
<tr>
<td>Mode 2</td>
<td>2.655</td>
<td>2.708</td>
<td>2.868</td>
<td>3.218</td>
<td>4.045</td>
<td>5.250</td>
</tr>
<tr>
<td>Mode 3</td>
<td>5.301</td>
<td>5.407</td>
<td>5.724</td>
<td>6.251</td>
<td>6.987</td>
<td>7.930</td>
</tr>
</tbody>
</table>

Table 5-2: Eigenvalues of different cone angle models obtained by using axisymmetric element model

According to Table 5-2, the eigenvalues obtained by using axisymmetric element model totally match the boldface data obtained by using 3-D element model. It should be noted that the axisymmetric element model can only perform the axisymmetric modes, which means that the modes of boldface data in Table 5-2 are axisymmetric. Therefore, we can indicate that Mode 1 is axisymmetric for 1° and 2° cone angle models and Mode 2 is axisymmetric for 3~5° cone angle models. This also infers that Mode 1 of 3~5° models is not axisymmetric, which agrees well with the result in last section. This study validates that the cone angle can improve the critical buckling temperature significantly.
One of the possible explanations for the result is that the first buckling mode of the flat disc is coning mode for the linear temperature distribution. This deformed mode has its own specific cone angle. We can name this angle $\theta_1$. On the other hand, the manufactured cone angle model can be considered as an initial conning model. We can name this cone angle $\theta_2$. When $\theta_2 < \theta_1$, the manufactured cone angle does not reach the level of the coning mode angle. The buckling Mode 1 for $1^\circ$ and $2^\circ$ cone angle models will be coning mode. When $\theta_2 > \theta_1$, the manufactured angle has already exceeded the coning mode angle. The buckling Mode 1 for $3^\circ$--$5^\circ$ cone angle models will automatically jump over the conning mode to the potato chip mode. This buckling mode is shown in Figure 5-6. Based on beam buckling theory, for an inclined beam, it requires a larger external force along the beam direction to buckle than a non-inclined beam in order to have the same component force in buckling direction. Similarly, the cone angle model requires a larger thermal load to buckle than the flat model. In order to have the same component thermal stress, this thermal load will increase with the cone angle, which explains the buckling temperature variation in Figure 5-3. Also, with the same inner and outer radius, the cone angle models have a relatively larger volume and weight, which also improves the buckling temperature.
5.2 Conclusion

The results indicate that the manufactured cone angle improves the thermal buckling temperature significantly. For the simulation settings used in this study, a 5° cone angle model can withstand 1000°C, which is rarely in real applications. This study also indicates that the buckling modes are different for different cone angle models. For the disc used in this study, the critical point is between 2° and 3°. The coning mode occurs first when the cone angle is below this threshold. Otherwise, the potato chip mode happens first.

It should be noted that this study is devoted to thermal buckling research which means that other mechanical problems are not considered. The cone angle geometry may cause the non-uniform stress distribution induced by mechanical problems other than thermal load. These problems associated with vibration, inclined plane contact and etc. may cause structure instability. The current study is based on the thermal effect only.
CHAPTER SIX: FINITE ELEMENT ANALYSIS OF THERMAL BUCKLING IN CLUTCH DISCS WITH FRICTON LAYER AND NON-FREE BOUNDARY CONDITIONS

Clutch discs are frequently subjected to friction between contact surfaces. In order to have better friction efficiency and less wearing, friction layers are applied to the surfaces of the discs. These layers are made of specific materials which have the entirely different properties comparing with the non-friction layer materials. No doubt the introduction of friction layers will influence thermal buckling. Also, the clutch discs are fixed by the teeth distributed around the outer edge in real applications. The non-free boundary condition should also be considered in thermal buckling study. This chapter has been performed to study these effects.

6.1 Effect of Friction Layer on Thermal Buckling

6.1.1 Effect of different friction materials on thermal buckling under monotonic temperature distribution along the radial direction

A set of studies with two different friction materials have been performed via the finite element analysis. The friction materials used in Lee and Barber’s research\cite{18}, have been chosen in this study. The material of non-friction layer is assumed to be cast iron.
The properties of the materials used in friction layer and non-friction layer are shown in Table 6-1.

<table>
<thead>
<tr>
<th></th>
<th>Young’s Modulus</th>
<th>Poisson’s Ratio</th>
<th>Thermal Expansion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E (GPa)</td>
<td>v</td>
<td>α (10^5 K⁻¹)</td>
</tr>
<tr>
<td>Cast Iron</td>
<td>110</td>
<td>0.3</td>
<td>1.25</td>
</tr>
<tr>
<td>Material A</td>
<td>0.53</td>
<td>0.25</td>
<td>3</td>
</tr>
<tr>
<td>Material B</td>
<td>1</td>
<td>0.25</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 6-1: Material properties

Material A is asbestos-based and B is semi-metallic material. The detailed geometry dimensions are shown in Table 6-2. The ABAQUS model is shown in Figure 6-1.

<table>
<thead>
<tr>
<th></th>
<th>Outer Diameter</th>
<th>Inner Diameter</th>
<th>Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Do (mm)</td>
<td>Di (mm)</td>
<td>t (mm)</td>
</tr>
<tr>
<td>Friction layer</td>
<td>260</td>
<td>160</td>
<td>0.5</td>
</tr>
<tr>
<td>Non-friction layer</td>
<td>260</td>
<td>160</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Table 6-2: Geometric dimensions
The mesh density for the friction layer is $3 \times 8 \times 100$, and $6 \times 8 \times 100$ for the non-friction layer. A linear reference temperature gradient is assumed along the radius direction as Eq. (22). Both friction materials A and B are investigated. The eigenvalues and critical buckling temperatures are shown in Table 6-3 and Table 6-4, respectively.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>2.2971</td>
<td>574.275</td>
<td>1.9314</td>
<td>482.85</td>
<td>91.425</td>
</tr>
<tr>
<td>Mode 2</td>
<td>2.6555</td>
<td>663.875</td>
<td>2.2399</td>
<td>559.975</td>
<td>103.9</td>
</tr>
<tr>
<td>Mode 3</td>
<td>5.3013</td>
<td>1325.325</td>
<td>4.4728</td>
<td>1118.2</td>
<td>207.125</td>
</tr>
</tbody>
</table>

Table 6-3: Eigenvalues and critical buckling temperatures for clutch disc by using material A as friction material
According to Tables 6-3 and 6-4, the model with friction layer is easier to buckle under the same thermal load. The critical buckling temperature is quite lower comparing with the model without friction layer. For Mode 1, the critical buckling temperature difference is about 90°C which cannot be ignored. Therefore, it is essential to perform a study to determine how the friction layers affect the critical buckling temperature.

Here is the explanation for the critical buckling temperature difference. Although Young’s modulus has no effect on critical buckling temperature, it does affect the critical buckling moment. Comparing with non-friction layer, the friction layer has a relatively small critical buckling moment due to the smaller Young’s modulus and the thinner thickness. These buckling moments can be computed from Eq. (6). In consideration of the interaction between two material layers, the critical buckling moment of the whole structure must be higher than that of friction layer. However, it must be lower than the
critical buckling moment without the introduction of friction layer. This finally reduces the critical buckling temperature.

Comparing Tables 6-3 and 6-4, the critical buckling temperatures of both models are almost the same. Even the coefficients of thermal expansion of two friction materials are totally different. From Table 6-1, we can see that the Material A and B both have a very small Young’s modulus comparing with cast iron. From the previous study, we have proved that thermal buckling is independent of Young's modulus when single material is applied. Thus, we can speculate that Young’s modulus ratio between cast iron and friction material plays an important role in this study.

For study efficiency, only Material A has been chosen in further studies. An individual study has been performed to determine how the value of Young’s modulus affects the buckling temperature. Instead of 110GPa and 0.53GPa set in previous study, 110Pa and 0.53Pa are set as the new Young’s modulus to perform the same study in the last section. The simulation result is absolutely the same as those in Table 6-3, which validates that the value of Young’s modulus has no effect on thermal buckling.

A set of simulations have been performed to study the effect of Young’s modulus ratio. Eight Young’s modulus ratios, 1:1, 2:1, 4:1, 8:1, 16:1, 32:1, 64:1 and 128:1 (Young’s modulus of cast iron: Young’s modulus of friction material) are assumed to in this study. The other simulation settings keep the same with previous study. The results are shown in Table 6-5 and Figure 6-2.
<table>
<thead>
<tr>
<th>Young’s modulus ratios (Cast iron: friction material)</th>
<th>1:1</th>
<th>2:1</th>
<th>4:1</th>
<th>8:1</th>
<th>16:1</th>
<th>32:1</th>
<th>64:1</th>
<th>128:1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>2.3341</td>
<td>2.1513</td>
<td>2.0461</td>
<td>1.9896</td>
<td>1.9603</td>
<td>1.9454</td>
<td>1.9379</td>
<td>1.9342</td>
</tr>
<tr>
<td>Model 2</td>
<td>2.7094</td>
<td>2.4972</td>
<td>2.3743</td>
<td>2.3082</td>
<td>2.2738</td>
<td>2.2563</td>
<td>2.2475</td>
<td>2.2431</td>
</tr>
<tr>
<td>Model 3</td>
<td>5.4087</td>
<td>4.9856</td>
<td>4.7405</td>
<td>4.6085</td>
<td>4.5400</td>
<td>4.5051</td>
<td>4.4875</td>
<td>4.4786</td>
</tr>
</tbody>
</table>

| Critical Buckling Temperature (°C)                   |     |     |     |     |     |     |     |     |
| Model 1                                              | 583.5 | 537.8 | 511.5 | 497.4 | 490.1 | 486.4 | 484.5 | 483.5 |
| Model 2                                              | 677.4 | 624.3 | 593.6 | 577.1 | 568.5 | 564.1 | 561.9 | 560.8 |
| Model 3                                              | 1352.2 | 1246.4 | 1185.1 | 1152.1 | 1135.0 | 1126.3 | 1121.9 | 1119.7 |

Table 6-5: Eigenvalues and Buckling Temperatures of Young’s modulus ratio study

Figure 6-2: Effect of Young’s modulus ratio on thermal buckling
According to Figure 6-2, the buckling temperature variation curves show an exponential decrease. This result indicates that there exists a complicated interaction between two material layers. From Table 6-5, we can find that the critical buckling temperature decreases rapidly with the increase of Young’s modulus ratio at the initial and finally converges to about 483°C for Mode 1 in this case. The Young’s modulus ratios for Material A and Material B are 207 and 110. At these ratio points, the curve has already converged. This well explains why the critical buckling temperatures of both models are almost the same in last section.

6.1.2 Effect of different friction layer thickness on thermal buckling under monotonic temperature distribution along the radial direction

The variation of the thickness of friction layer will no doubt affect the critical buckling temperature. In this section, a set of studies have been performed via finite element method to determine how the thickness of friction layer affects the critical buckling temperature.

In this study, the thickness of friction layer ranges from 0.4mm to 0.8mm. The total thickness of the disc remains 6mm. Material A has been chosen as the friction material. The temperature distribution keeps monotonic as Eq. (22). The results are shown in Table 6-6 and Figure 6-3.
<table>
<thead>
<tr>
<th>Thickness</th>
<th>0.4mm</th>
<th>0.5mm</th>
<th>0.6mm</th>
<th>0.7mm</th>
<th>0.8mm</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mode1</strong></td>
<td>2.0019</td>
<td>1.9314</td>
<td>1.8622</td>
<td>1.7943</td>
<td>1.7278</td>
</tr>
<tr>
<td><strong>Mode2</strong></td>
<td>2.3202</td>
<td>2.2399</td>
<td>2.1610</td>
<td>2.0836</td>
<td>2.0075</td>
</tr>
<tr>
<td><strong>Mode3</strong></td>
<td>4.6326</td>
<td>4.4724</td>
<td>4.3150</td>
<td>4.1604</td>
<td>4.0087</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode</th>
<th>Critical buckling temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mode 1</strong></td>
<td>500.475</td>
</tr>
<tr>
<td><strong>Mode 2</strong></td>
<td>580.05</td>
</tr>
<tr>
<td><strong>Mode 3</strong></td>
<td>1158.15</td>
</tr>
</tbody>
</table>

Table 6-6: Eigenvalues and buckling temperatures of clutch disc with different thickness friction layers

![Figure 6-3: Buckling temperature variation of clutch disc with different thickness friction layers](image_url)
According to Figure 6-3, the buckling temperature variation curves show a linear decrease with the increase of thickness of friction layer for each mode. From Table 6-6, we can find that the maximum buckling temperature difference is about 69°C for Mode 1 in this case. The relationship of buckling temperature and the thickness of friction layer for Mode 1 can be fitted into following function:

$$ T = -171.4t + 568.7 \quad (38) $$

When t=0, which means that the disc is only made of cast iron, the buckling temperature is about 570°C, which agrees well with previous study in the same condition (574°C). However, the Eq. (38) also indicates that the buckling temperature could be negative with a large t. Thus, another study has been performed to find the answer.

In this study, the thickness of friction layer is set to be 6mm, which means that the disc is now only made of Material A. The other settings keep unchanged. The simulation result is shown in Table 6-7.

<table>
<thead>
<tr>
<th></th>
<th>t=6mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mode1</td>
</tr>
<tr>
<td>Eigenvalue</td>
<td>0.9548</td>
</tr>
<tr>
<td>Temperature (°C)</td>
<td>238.7</td>
</tr>
</tbody>
</table>

Table 6-7: Eigenvalues and buckling temperatures of clutch disc made of material A

According to Table 6-7, we can obtain that the critical buckling temperature is 238.7°C for Mode 1. If we substitute t=6mm into Eq. (38), the answer is -459.7°C which
is totally different from 238.7°C. This result indicates that the entire red curve in Figure 6-3 is not simply linear. However, we do not need to care about the whole curve of the buckling temperature variation, because the thickness of friction layer is normally around 0.7mm and the clutch discs mainly consist of non-friction material.

Therefore, we can conclude that the critical buckling temperature decreases linearly with the increase of the thickness of friction layer in this case.

**6.1.3 Effect of temperature distribution along the thickness direction on thermal buckling of clutch discs with friction layer**

The previous study in chapter 4 has proved that when single material is applied, the temperature distribution along the thickness direction has no effect on thermal buckling. For the discs with friction layer, the higher temperature will be distributed on the surface of friction layer and the lower temperature will occur at the bottom of the non-friction layer. With multiple materials applied, the temperature distribution along the thickness direction may affect the critical buckling temperature.

In the following study, the thickness of friction layer is set to be 0.5mm. Material A has been chosen as the friction material. A monotonic temperature gradient is assumed along the radial direction as Eq. (22). A linear temperature reduction by 10% to 40% is assumed along the thickness direction. The expressions of the 3-D temperature fields are same as Eq. (32). The other settings remain the same with the previous study. The results are shown in Table 6-8 and Figure 6-4. In previous study, we have already proved that the
reference temperature load for a 3-D temperature field is the mean temperature load along the thickness direction.

<table>
<thead>
<tr>
<th></th>
<th>Eigenvalue</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>uniform</td>
<td>10% 20% 30% 40%</td>
</tr>
<tr>
<td>Mode1</td>
<td>1.9314</td>
<td>2.0419 2.1658 2.3058 2.4650</td>
</tr>
<tr>
<td>Mode2</td>
<td>2.2399</td>
<td>2.3681 2.5118 2.6741 2.8587</td>
</tr>
<tr>
<td>Mode3</td>
<td>4.4724</td>
<td>4.7283 5.0152 5.3392 5.7080</td>
</tr>
</tbody>
</table>

Table 6-8: Eigenvalues and buckling temperatures of clutch disc with friction layer under linear temperature distribution along thickness direction
Figure 6-4: Buckling temperature variation of clutch disc with friction layer under linear temperature distribution along thickness direction

According to Figure 6-4, we can find that the critical buckling temperature increases linearly for each mode in this case. This result is different from the conclusion obtained by using the single material model. In Table 6-8, the maximum buckling temperature difference for Mode 1 is about 11°C which is small comparing with the critical buckling temperature. The effect of the temperature distribution along the thickness direction in this case is still not significant.

6.2 Effect of Non-Free Boundary Condition on Thermal Buckling

In the literature of thermal buckling studies in discs, the boundary conditions were always assumed free-free. In real applications, the clutch discs are fixed by a number of teeth around the outer edge. One type of real clutch discs is shown in Figure 6-5.

Figure 6-5: Clutch discs with teeth
An ABAQUS model is developed to study the effect of free-fixed boundary condition on thermal buckling. The tooth width is 40mm. The maximum diameter of the structure with teeth is 280mm. The rest geometry dimensions are shown in Table 6-2. The ABAQUS model is shown in Figure 6-6. The red part in the figure represents for the friction layer.

![Figure 6-6: ABAQUS model of clutch disc with teeth](image)

A set of studies have been performed via finite element method. Material A has been chosen in this study. The thickness of friction layer is 0.5mm. The reference temperature gradient is assumed as Eq. (22). The temperature distribution along the thickness direction is assumed uniform. The temperature field only covers the structure without teeth, which keeps the reference temperature load to be constant 250°C. The teeth are set free and fixed respectively in two groups of simulation for comparison. The results are shown in Table 6-9.
<table>
<thead>
<tr>
<th>Mode</th>
<th>Free teeth</th>
<th>Fixed teeth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Eigenvalue</td>
<td>T (°C)</td>
</tr>
<tr>
<td>Mode 1</td>
<td>2.1833</td>
<td>545.825</td>
</tr>
<tr>
<td>Mode 2</td>
<td>2.6164</td>
<td>654.1</td>
</tr>
<tr>
<td>Mode 3</td>
<td>5.3567</td>
<td>1339.175</td>
</tr>
</tbody>
</table>

Table 6-9: Eigenvalues and buckling temperatures of clutch disc with teeth under non-free boundary condition

Comparing Table 6-9 and Table 6-3, we can find that the model with teeth is harder to buckle comparing with the model without teeth under free-free boundary condition. The temperature difference is about 60°C for Mode 1. The explanation for the temperature difference is that the model with teeth has a larger volume and weight, which improves the critical buckling temperature. According to Table 6-9, the model under free-fixed boundary condition shows a great resistance to thermal buckling. The critical buckling temperature for Mode 1 is 1335°C which is rarely in real applications. The buckling modes of the model under free-free boundary condition are same with those in Figure 3-6 due to the same thermal load pattern and boundary condition. The buckling modes for the model under free-fixed boundary condition are shown in Figure 6-7.
According to Figure 6-7, the buckling Mode 1 of the model under free-fixed boundary condition still can be considered as a coning mode though it is not a perfect coning mode due to the fixed boundary. Mode 2 is a potato chip mode which is almost the same as Mode 2 in Figure 3-6. Therefore, we can conclude that the free-fixed
boundary condition greatly improves the critical buckling temperature. However, its effect on thermal buckling mode shapes is limited.

6.3 Summary

The friction performance of clutches is improved with the introduction of friction materials. However, it will significantly reduce the critical buckling temperature. Young’s modulus ratio between non-friction material and friction material is the key factor in this reduction. Also, with the increase of the thickness of friction layer, the critical buckling temperature will further decrease. The temperature reduction along the thickness direction can improve the critical buckling temperature. However, the effect is not significant. The free-fixed boundary condition improves the critical buckling temperature significantly comparing with the free-free boundary condition. Consequently, the friction material layer and the non-free boundary condition must be taken into consideration to make an accurate prediction on the thermal buckling load of clutch/brake discs.
CHAPTER SEVEN: CONCLUSION AND FUTURE WORK

7.1 Conclusions

This study provides a systematic method to study thermal buckling of real brake and clutch discs. We have investigated thermal buckling phenomenon of brake and clutch discs via a finite element analysis. It has been found that buckling can occur when the temperature distribution is uniform in the circumference but non-uniform in the radius. The critical buckling load and the dominant buckling mode shapes are significantly affected by the patterns of radial temperature distribution. For a linear profile of temperature, the axisymmetric coning mode has the lowest buckling temperature and the corresponding highest temperature always occurs at the inner radius. The non-axisymmetric potato chip modes also exist, but occur at higher buckling temperatures. For nonlinearly distributed temperature where the maximum temperature is located neither at the inner or outer radius, the non-axisymmetric buckling mode will be dominant and the corresponding buckling temperature is typically much higher than that of linearly distributed temperature profile.

We also found that slightly changed temperature distribution with the same temperature difference (e.g. linear, sinusoidal and exponential distributions with the same
magnitude and location of the maximum temperature) can greatly impact the thermal buckling mode shapes but they have very close eigenvalues, implying that their effects on the buckling temperatures are limited. On the other hand, a periodic temperature variation with multiple hot spots in the circumference can never cause buckling, since the thermal effects in the hot and cold regions eventually cancel out. Meanwhile, a non-periodic circumferential distribution of temperature may lead to buckling, although this scenario is very rare in practical applications. It should be pointed out that thermoelastic instability can be induced at the beginning of frictional sliding, and can result in both banding modes and focal hot spots. In comparison, thermal buckling takes place only when the temperature gradient exceeds a critical value, and therefore it can hardly occur at the beginning of a clutch engagement.

In the thermal buckling study on single material discs, non-uniform temperature distribution along the thickness direction was taken into consideration. Two possible temperature patterns were assumed in the study. It has been found that the effect of temperature distribution along the thickness direction can be neglect, which validated the thermal stress theory based on the thin plate geometry. This study simplified the temperature field setting for further works by ignoring the temperature distribution along the thickness direction.

We also studied the thermal buckling phenomenon on three real complicated models. The study on cone angle model showed that the manufactured cone angle can improve
the critical buckling temperature significantly. Through the study on the model with friction layer, it has been found that the introduction of friction material reduces the critical buckling temperature significantly. The effect was influenced by Young’s modulus ratio and the thickness of friction layer. By studying the model under non-free boundary condition, it has been found that the free-fixed boundary condition can improve the critical buckling temperature significantly but has limit effect on thermal buckling mode shapes.

7.2 Future Work

For computational efficiency, the brake and clutch discs are usually simplified as annular ring geometry. However, the geometries of real discs are complicated. Some of the clutch discs even have convection grooves. In some cases, porous materials are used for better performance. Sometimes, friction layers are applied to both sides of the discs. Therefore, to obtain a more accurate thermal buckling prediction, the geometrical effects must be taken into consideration.

In some situations, the friction discs have already deformed due to physical and thermal problems other than thermal buckling. These structure deformations can influence the thermal buckling. Thus, thermal buckling study on deformed discs is also a future research direction.

Also, due to the excessive heat generated in the frictional sliding process, the temperature could be substantially high, which may lead to the change in the material
properties. In the future research, temperature dependent material properties can also be taken into consideration.
REFERENCE


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